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On the resource allocation problems with interpersonal comparisons of welfare: Extended preference approach revisited*

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Abstract

This study introduces the extended preference approach studied in the literature of social welfare functionals into the pure exchange economy model. We clarify the relationship between equal income Walrasian allocations and leximin equitable ones, showing that two axioms defined by the leximin criterion can characterize the EIW rule. We also discuss its implication for the interpersonal comparisons of welfare.

Keywords: the equal income Walras rule; extended preference; the leximin criterion; interpersonal comparisons of utility levels

1 Introduction

This study applies the extended preference approach, a device used in social welfare functionals (SWFLs), to the resource allocation problem of exchange economies with a finite number of agents and goods.[1]

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[1] d'Aspremont (1985), d'Aspremont and Gevers (2002), Bossert and Weymark (2004), Sen (1970,1977,1979, 1982, 1986), and Suzumura (1983) surveyed this area. Fleurbaey and Hammond (2004) and Mongin and d'Aspremont (2004) investigated this field, considering a broader perspective on ethics and utility theories. Blackorby et al. (1984) provided a diagrammatic introduction. The recent contributions are Yamamura (2017) and Baccelli (2022).

An *extended preference* $\succcurlyeq_E$ can compare the welfare of different persons. Here, $(x, i) \succcurlyeq_E (y, j)$ implies that being agent i with consumption x is at least as well off as being agent j with consumption y. Using this concept, we consider leximin fairness, a lexicographic sophistication of the difference principle of Rawls (1971). A feasible allocation is *leximin equitable* if no other feasible ones that are superior to it from the leximin point of view exist. We impose three properties (E.1-3) on extended preferences, which have crucial implications for the existence and characteristics of leximin equitable allocations (Theorem 1 and 2).

Next, we study the relationship between the leximin equitable allocations and *equal income Walrasian* (EIW) ones. Theorem 3 shows that any two of the three concepts below imply the rest. The three concepts are the leximin equitable allocation, the EIW allocation, and *the price indicator type* of extended preferences. The price indicator type, a specific type of extended preference, measures the welfare levels of different agents with a given price.

The price indicator type is logically equivalent to *the locally created* extended preference, made with only local information of subjective preferences, applying the idea of local independence due to Nagahisa (1991) and Nagahisa and Suh (1995). The idea of locally created characterizes the price indicator type from the viewpoint of departure from Arrow's IIA. Theorem 4 shows that Theorem 3 still holds in most cases if we replace the price indicator type with a locally created one.

The final goal of this study is the axiomatization of the EIW rule using the leximin criterion. We assume that a rule respects the spirit of the leximin fairness. Every allocation selected by it must meet the leximin criterion in

Most of the literature studies SWFLs under an abstract framework of social choice. A few exceptions are Sen (1974a,b) and Deschamps and Gevers (1978), including an application to the income distribution problem in a single-commodity economy. We can regard the idea of egalitarian equivalence by Pazner and Schmeidler (1978) as defining a specific extended preference in commodity spaces.

some way. We define *Leximin Justification Possibility* (LMJP) and *Leximin All Unanimity* (LMAU). Take a feasible allocation arbitrarily. The former requires that if a rule selects it, at least one locally created extended preference must make the allocation leximin equitable. The latter says that if that holds for all the extended preferences, the rule must select it. We show that the EIW rule is the only rule meeting the two axioms (Theorem 5 and 6).

In addition to the importance of the leximin axiomatization, it is also worth noting the implication of Theorem 3 and 4, which says that if we hope that the set of EIW allocations is equal to the set of leximin equitable ones, we must use price indicator type of extended preferences or equivalently locally created ones. Thus, accepting the EIW rule while respecting the spirit of leximin fairness is equivalent to regarding interpersonal comparisons of welfare according to the idea of price indicator or equivalently locally created ones as correct, endorsing the validity of the suggestion proposed by Hammond (1991): The fact that people accept a rule is equivalent to their acknowledging the interpersonal comparisons of welfare based on that rule as correct. Thus, Theorems 3 and 4 imply that a solution based on Hammond's strategy exists when focusing on exchange economies.

The study uses the utility representation of extended preferences (Proposition 1) as a tool. It is only used for interpersonal comparisons of utility levels (ICUL), not for utility differences (ICUD). In this sense, the meaning of the utility representation does not go beyond convenience, as is the case with the typical usage of utility functions in microeconomic analysis.

The organization is as follows. Section 2 provides notation and definitions. Section 3 examines a few properties of the leximin equitable allocations. Sections 4 and 5 introduce the price indicator type and locally created one. After discussing a few properties of those preferences, we investigate the relationship between the leximin equitable allocations and the EIW ones using the two specific extended preferences. After formulating social choice rules defined on a new informational basis and introducing axioms, Section 6 states the axiomatization of the EIW rule. Section 7 discusses the implication of the results, reviewing the previous research on the SWFLs and the idea suggested by Hammond (1991). Section 8 concludes. The appendix summarizes subordinate matters.

2 Notation and definitions

2.1 Exchange Economies

The notation for vector inequalities is $\geq$, $>$, and $\gg$. Let $\Delta^l := \{p \in R^l_+ : \sum_{i=1}^{l} p_i = 1\}$ and $int.\Delta^l := \{p \in R^l_{++} : \sum_{i=1}^{l} p_i = 1\}$.

We consider exchange economies with a finite number of agents and a finite number of private goods. Let $N = \{1, 2, ..., n\}$ and $L = \{1, 2, ..., l\}$ be the set of agents and the set of private goods respectively. All agents have the same consumption set R^l_+. Let $z_i = (z_{i1}, ..., z_{il}) \in R^l_+$ and $z = (z_1, ..., z_n) \in R^{nl}_+$ be agent i's consumption and an allocation respectively. Let $\Omega \in R^l_{++}$ be the total endowment of the economy, owned collectively and fixed throughout the study. An allocation z is feasible if $\sum_{i \in N} z_i \leq \Omega$. Let Z be the set of feasible allocations.

Let $\succcurlyeq_i$ be agent i's preference on R^l_+, where $\succ_i$ and $\sim_i$ read as usual. We assume that $\succcurlyeq_i$ is continuous, convex, and monotonic on R^l_+. We say that $\succcurlyeq_i$ is convex if for any $x, y \in R^l_+$, $x \succ_i y$ implies $tx + (1-t)y \succ_i y$ for all $t \in (0, 1)$ and that $\succcurlyeq_i$ is monotonic if $x > y$ implies $x \succ_i y$. Let Q be the set of preferences satisfying all the conditions. A list of all agents' preferences, denoted $\succcurlyeq = (\succcurlyeq_i)_{i \in N}$, is called a profile. Let $Q^n = Q \times \cdots \times Q$, where Q appears n times, be the set of profiles.

Take a profile $\succcurlyeq$ arbitrarily. A feasible allocation z is Pareto optimal if and only if there are no other feasible ones z' with $z'_i \succ_i z_i$ for all $i \in N$.[2] Let

[2] The strong Pareto optimality reduces to the weak one because of the continuity and monotonicity of preferences on R^l_+.

$PO(\succcurlyeq)$ be the set of Pareto optimal allocations. Let $PO(\succcurlyeq, p)$ be the set of Pareto optimal allocations sharing with p as the supporting price vector. A feasible allocation z is an equal income Walrasian (EIW) allocation if there exists some $p \in int.\Delta^l$ such that $z_i \succcurlyeq_i x$ for all $x \in R^l_+$ with $px \leq p\left(\frac{\Omega}{n}\right)$. Let $EIW(\succcurlyeq)$ be the set of the EIW allocations. Let $EIW(\succcurlyeq, p)$ be a subset of $EIW(\succcurlyeq)$, where $p \in int.\Delta^l$ is an equilibrium price vector.

2.2 Extended Preferences

The notion of extended preferences is based on the principle of *extended sympathy* mentioned by Arrow (1963) and initiated by Suppes (1966) and Sen (1970). The basic idea is that a hypothetically existing ethical observer compares the welfare of different persons from a social point of view while respecting (or sympathizing with) their subjective preferences.

An extended preference $\succcurlyeq_E$ created from $\succcurlyeq \in Q^n$ is a complete and transitive binary relation on $R^l_+ \times N$. We read $(x, i) \succcurlyeq_E (y, j)$ as "being agent i with consumption x is at least as well off as being agent j with consumption y." We read $\succ_E$ and $\sim_E$ as usual.

We say that a set of continuous utility functions $(u_i)_{i\in N}$ represents $\succcurlyeq_E$ if $(x, i) \succcurlyeq_E (y, j) \Longleftrightarrow u_i(x) \geq u_j(y)$ for all $i, j \in N$ and all $x, y \in R^l_+$. We call $(u_i)_{i\in N}$ a *representation* of $\succcurlyeq_E$. We assume that an extended preference $\succcurlyeq_E$ satisfies the following properties.

E.1. For any $i \in N$, $x \succcurlyeq_i y \Longleftrightarrow (x, i) \succcurlyeq_E (y, i)$ for all $x, y \in R^l_+$.

E.2. $\succcurlyeq_E$ has a representation.

E.3 There exists $z^* \in Z$ such that $z^*_k \neq 0$ for all k, and $(z^*_i, i) \sim_E (z^*_j, j)$ for all i, j.

E.1 is called *the axiom of identity*, assumed in almost all literature related to extended preferences with the abstract framework of social choice.[3] E.2 says

[3]Refer to Sen (1970) and d'Aspremont (1985) for more details.

that extended preferences should have continuous utility representations, which, as in many previous studies on SWFLs, allows us to assume that each agent has a utility function and that rules use information from utility comparisons to make social decisions. However, as clear later, only utility levels count, utility differences do not matter. Hence, this study is on interpersonal comparisons of utility levels (ICUL), not interpersonal comparisons of utility differences (ICUD).

Whether a given extended preference has a representation is as nonobvious as the problem of the existence of a continuous utility function representing a given preference, which is a subject in consumer theory. The appendix shows a necessary and sufficient condition for an extended preference that satisfies E.1 to have a representation.

In contrast with E.1, E.3 is an assumption specific to economic environments, assuring the existence of a reference point of egalitarian allocation for each extended preference. It covers many other similar assumptions. The assumption of $\left(\frac{\Omega}{n}, i\right) \sim_E \left(\frac{\Omega}{n}, j\right)$ for all i, j is the typical one. For issues with no ownership yet established, equal division of collective assets is likely fair.[4] A generalization of this assumption is $(s, i) \sim_E (s, j)$ for all i, j, where $s \in \left(0, \frac{\Omega}{n}\right]$. If every agent enjoys the same amount of goods (s) guaranteeing a minimum standard of living, interpreted as a counterpart of "the zero line of welfare" in List (2001), that allocation is considered fair. E.3 does not cover the case of $s = 0$ at first glance. However, it holds in this case as well: Using E.2, this case is equal to $u_1(0) = \cdots = u_n(0)$. Assuming $u_1\left(\frac{\Omega}{n}\right) \leq \cdots \leq u_n\left(\frac{\Omega}{n}\right)$ without loss of generality, and noting that utility functions are continuous, we can find z^* of E.3 on the set of $\left[0, \frac{\Omega}{n}\right] \times \cdots \times \left[0, \frac{\Omega}{n}\right]$, where $\left[0, \frac{\Omega}{n}\right]$ appears n time. Note also that under E.1, the $s = 0$ case is equivalent to $(x, i) \succcurlyeq_E (0, j)$ for all $i, j \in N$ and all $x \in R^l_+$, matching our intuition that as long as other conditions are equal, people without wealth are the most miserable.

[4] Carbon emission trading and the polar and space developments represent examples.

See the appendix regarding the independence of the three properties. Let $E(\succsim)$ be the set of extended preferences meeting E.1-3 created from $\succsim$. Representing preferences by utility functions and comparing the utilities among agents results in an extended preference. The two examples below are extended preferences meeting E.1-3 created according to this method.

Example 1 Take $p \in int\Delta^l$ arbitrarily. We invoke money-metric utility functions (Arrow and Hahn, 1971, p.106), formulated as $u_i^p(x) = \min\{pq : q \sim_i x\}$[5]. Let an extended preference $\succsim_E^p$ be such that for any $x, y \in R_+^l$ and any $i, j \in N$,

$$(x, i) \succsim_E^p (y, j) \Longleftrightarrow u_i^p(x) \geq u_j^p(y).$$

$\succsim_E^p$ meets E.3: Let $\frac{\Omega}{t}$ be each agent's endowment, where $t \geq n$. Let z_i^t be i's optimal consumption subject to the budget constraint $px \leq p\left(\frac{\Omega}{t}\right)$, and let $z^t = (z_i^t)_{i \in N}$. If we take a sufficiently large t, we can set $z^t \in Z$, thus meeting E.3.

Example 2 Let u_i^Ω be i's utility function such that $u_i^\Omega(x) = \lambda_x$, where λ_x meets $\lambda_x \Omega \sim_i x$. Then, the extended preference $\succsim_E^\Omega$ is such that for any $x, y \in R_+^l$ and any $i, j \in N$,

$$(x, i) \succsim_E^\Omega (y, j) \Longleftrightarrow u_i^\Omega(x) \geq u_j^\Omega(y).$$

The definition is due to Fleurbaey and Maniquet (2011), though they do not call it extended preference. A feasible allocation z is the egalitarian equivalent of Pazner and Schmeidler (1978) if and only if $(z_i, i) \sim_E^\Omega (z_j, j)$ for all i, j.

If we need to refer to multiple extended preferences, we use the notation $\succsim_{E'}$, $\succsim_{E''}$, and so on. Notice the difference between $\succsim_{E'}$ and $\succsim_E'$.

Last, we point out that E.1-3 means comparing the utilities of different agents by selecting special utility functions, not avoiding the usage of utility

[5] Weymark (1985) studied the properties of this function.

information. We will explain the utility functions later (Sections 4, 5, and 6). Our approach differs from the previous studies on SWFLs in many aspects. Section 7 discusses the significance of that research.

2.3 The leximin criterion

Let $\succcurlyeq_E$ be taken arbitrarily and fixed throughout the section. The *leximin criterion*, a lexicographic extension of the difference principle of Rawls (1971), is defined as follows. Given an allocation z, we arrange all (z_i, i) in ascending order such that $(z_{i_1}, i_1) \preccurlyeq_E (z_{i_2}, i_2) \preccurlyeq_E \cdots \preccurlyeq_E (z_{i_{n-1}}, i_{n-1}) \preccurlyeq_E (z_{i_n}, i_n)$, where tie is broken arbitrarily. The agent i_k $(k = 1, .., n)$ is the kth worst off agent in z. We denote i_k by $i_k(z)$. A lexicographic order $\geq_{L(E)}$ on the set of allocations is defined as follows:

$$
\begin{array}{lcc}
z >_{L(E)} z' & \iff & \begin{array}{c} \exists k \in \{1, ..., n\} \text{ s. t.} \\ (z_{i_\tau(z)}, i_\tau(z)) \sim_E (z'_{i_\tau(z')}, i_\tau(z')) \ \forall \tau \in \{1, ..., k-1\} \\ \& \\ (z_{i_k(z)}, i_k(z)) \succ_E (z'_{i_k(z')}, i_k(z')). \end{array} \\
z =_{L(E)} z' & \iff & (z_{i_k(z)}, i_k(z)) \sim_E (z'_{i_k(z')}, i_k(z')) \ \forall k = 1, ..., n. \\
z \geq_{L(E)} z' & \iff & z >_{L(E)} z' \vee z =_{L(E)} z'.
\end{array}
$$

Given z and z', z is *at least as leximin just as* z' if $z \geq_{L(E)} z'$. If this holds with $z >_{L(E)} z'$, z is *more leximin just than* z'. In contrast, if that does with $z =_{L(E)} z'$, z is equally as leximin just as z'. Note that all are well defined and transitive, and $\geq_{L(E)}$ is complete. A feasible allocation is leximin equitable if there is no other feasible one that is more leximin just than it. Let $LME(\succcurlyeq_E)$ be the set of those allocations. Note that $LME(\succcurlyeq_E) \subset PO(\succcurlyeq)$ holds because of E.1. An allocation z is *leximin justifiable* for $\succcurlyeq \in Q^n$ if there exists some $\succcurlyeq_E \in E(\succcurlyeq)$ such that $z \in LME(\succcurlyeq_E)$. The sequent section discusses the non-emptiness of $LME(\succcurlyeq_E)$ and a necessary and sufficient condition for an allocation to be leximin justifiable.

3 The properties of leximin equitable allocation

Two types of leximin equitable allocation exist. With the first type, every agent enjoys some goods, and their welfare is equal. This type is called *egalitarian*. With the other type, called *singular*, some agents enjoy nothing, and (oddly) their welfare is higher than or equal to that of agents that are not so. The formal definitions are: Take $z \in LME(\succcurlyeq_E)$ arbitrarily. Assume that $(z_1, 1) \preccurlyeq_E \cdots \preccurlyeq_E (z_n, n)$ without loss of generality. We say that z is egalitarian if

$$(z_1, 1) \sim_E \cdots \sim_E (z_n, n), \text{ and } z_i \neq 0 \text{ for all } i.$$

We say that z is singular if there exists some $k \geq 2$ such that

$$\overbrace{(z_1, 1) \sim_E \cdots \sim_E (z_{k-1}, k-1)}^{z_1, \ldots, z_{k-1} \neq 0} \preccurlyeq_E \overbrace{(z_k, k) \preccurlyeq_E \cdots \preccurlyeq_E (z_n, n)}^{z_k, \ldots, z_n = 0}.$$

The theorem below shows that under lacking E.3, two types coexist.

Theorem 1 *Assume E.1 and E.2. For any $z \in LME(\succcurlyeq_E)$, z is either egalitarian or singular.*

Proof. Take $z \in LME(\succcurlyeq_E)$ arbitrarily. We assume $(z_1, 1) \preccurlyeq_E \cdots \preccurlyeq_E (z_n, n)$ without loss of generality, which is rewritten

$$u_1(z_1) \leq \cdots \leq u_n(z_n),$$

using $(u_i)_{i \in N}$, a representation of $\succcurlyeq_E$. There are two cases to discuss.

Case 1: $z_i \neq 0$ for all i.

If $u_j(z_j) < u_{j+1}(z_{j+1})$ for some j, a sufficiently small amount of the transfer (ε) from $j+1$ to j leads to

$$u_1(z_1) \leq \cdots \leq u_j(z_j + \varepsilon) < u_{j+1}(z_{j+1} - \varepsilon) \leq \cdots \leq u_n(z_n).$$

This transfer improves z from the leximin point of view, which is a contradiction. Thus, we have $u_1(z_1) = \cdots = u_n(z_n)$, which means that z is egalitarian.

Case 2: $z_i = 0$ for some i.

Note that no agent j with $z_j \neq 0$ ranks (strictly) higher in the order than i. If so, the same proof as in Case 1 leads to a contradiction. Thus, we can assume

$$\overbrace{u_1(z_1) \leq \cdots \leq u_{k-1}(z_{k-1})}^{z_1,\ldots,z_{k-1}\neq 0} \leq \overbrace{u_k(z_k) \leq \cdots \leq u_n(z_n)}^{z_k,\ldots,z_n=0}.$$

Here, we have $u_1(z_1) = \cdots = u_{k-1}(z_{k-1})$ as **in** Case 1, which implies that z is singular. ■

The singular type is implausible from any standpoint of fairness, let alone leximin. We need and suffice E.3 besides E.1 and E.2 to eliminate singular types.

Theorem 2 *Assume E.1 and E.2. For leximin equitable allocations to exist and for all of them to be egalitarian, holding E.3 is necessary and sufficient.*

Proof. As the necessity part is apparent, we show sufficiency.

Existence: Take $(u_i)_{i\in N}$, a representation of $\succcurlyeq_E$ arbitrarily. Let E be a set of utility vectors such that

$$(u_1, \ldots, u_n) \in E \Longleftrightarrow u_1 = \cdots = u_n \text{ and } \exists z \in Z \text{ s.t. } u_i = u_i(z_i)\ \forall i$$

Thanks to E.3, E is well-defined. Imagine a 45-degree line on R^n passing through the origin. All points of E are on this line. The far upper-right point $\widehat{u}$ of E exists on this line because E is closed and upper-bounded. We show that $\widehat{u}$ corresponds to an egalitarian allocation, which completes the proof of existence.

Let $\widehat{z}$ be the feasible allocation attaining $\widehat{u}$. Letting z_i^* meet E.3, we have $u_i(\widehat{z}_i) \geq u_i(z_i^*)$. As $z_i^* \neq 0$, the monotonicity of preference implies $u_i(\widehat{z}_i) > u_i(0)$. Now, we show $\widehat{z} \in PO(\succcurlyeq)$. Suppose not. By noting the monotonicity of preferences, there exists some $z' \in Z$ such that $u_i(z_i') > u_i(\widehat{z}_i)$ for all i. Without loss of generality, we assume that agent n has the lowest utility in z'. Thus, for any $i \neq n$, $u_i(z'_i) \geq u_n(z'_n) > u_i(\widehat{z}_i)$. Taking away some amount (ε_i) of goods from z'_i, we can have $u_i(z'_i - \varepsilon_i) = u_n(z'_n)$, which is possible by applying the intermediate value theorem on $[z'_i, 0]$ with noting $u_i(\widehat{z}_i) > u_i(0)$ proved above. Let $z^{-\varepsilon_i}$ be the feasible allocation created this way. By definition, the utility vector attained by $z^{-\varepsilon_i}$ lies on E, and to the upper right of $\widehat{u}$, which is a contradiction.

Next, we show that $\widehat{z}$ is leximin equitable. Suppose not. Then, there exists some $z \in Z$ that is more leximin just than $\widehat{z}$. The diagram below illustrates the comparison of utilities in $\widehat{z}$ and z.

$$\begin{array}{ccccccccccc} u_1(\widehat{z}_1) & = & \cdots & = & u_{k-1}(\widehat{z}_{k-1}) & = & u_k(\widehat{z}_k) & = & \cdots & = & u_n(\widehat{z}_n) \\ \shortparallel & & & & \shortparallel & & \wedge & & & & \\ u_{i_1}(z_{i_1}) & \leq & \cdots & \leq & u_{i_{k-1}}(z_{i_{k-1}}) & \leq & u_{i_k}(z_{i_k}) & \leq & \cdots & \leq & u_{i_n}(z_{i_n}) \end{array}$$

The diagram reveals $u_i(z_i) \geq u_i(\widehat{z}_i)$ for all i, with inequality for $i_k, ..., i_n$, which contradicts $\widehat{z} \in PO(\succcurlyeq)$.

To ensure that all of them are egalitarian, take $z \in LME(\succcurlyeq_E)$ arbitrarily. Suppose that $(z_1, 1) \preccurlyeq_E \cdots \preccurlyeq_E (z_n, n)$ without loss of generality. On the other hand, E.3 means $(z^*_1, 1) \sim_E \cdots \sim_E (z^*_n, n)$. As z is at least as leximin just as z^*, we have $(z^*_1, 1) \preccurlyeq_E (z_1, 1)$, which implies $(z^*_i, i) \preccurlyeq_E (z_i, i)$ for all i. Thus, E.1 implies $z^*_i \preccurlyeq_i z_i$ for all i. As $z^*_i \neq 0$, z_i is so as well, which holds for all i. Thus, Theorem 1 shows that z is egalitarian. ∎

From now on, we assume E.1-3.

The leximin criterion satisfies Hammond equity or its variants.[6] Hammond equity states that a smaller utility disparity between individuals is more desirable. Generally, we cannot eliminate those gaps when the alternatives are finite. However, we can do it because of the complete divisibility of goods and continuity of preferences. Consequently, all leximin equitable allocations are

[6] Hammond equity is due to Hammond (1976). A bit stronger version (XE) appeared in Deschamps and Gevers (1978) and d'Aspremont and Gevers (1977). The weakest version is minimal equity (ME), introduced by d'Aspremont and Gevers (1977).

egalitarian.

The existence of leximin equitable allocations is a welcome result. Moreover, its complete characterization is impeccable. However, two problems below result in, as a direct consequence of Theorem 2.

Corollary 1 *If allocations z and z' are leximin equitable for $\succsim_E$, then they are identical on a preference basis, i.e., $z_i \sim_i z'_i$ for all i.*

Proof. A direct consequence of Theorem 2. ■

Corollary 2 *If $z \in LME(\succsim_E)$, then there exists $\succsim_{E'}$ such that $z \notin LME(\succsim_{E'}$).*

Proof. As $z_i = 0$ for some i contradicts Theorem 2, we suppose $z_i \neq 0$ for all i. Let u_i be a utility function representing $\succsim_i$ such that $u_i\left(\frac{z_i}{2}\right) = 0$, and let not all $u_i(z_i)$ be equal. Let $\succsim_{E'}$be an extended preference represented by $(u_i)_{i \in N}$. Note that $\succsim_{E'}$ meets E.3 because of $u_i\left(\frac{z_i}{2}\right) = 0$. Theorem 2 implies $z \notin LME(\succsim_{E'})$. ■

These results complicate the task of the axiomatization of the EIW rule using the leximin criterion. In most cases, a single extended preference cannot make all EIW allocations leximin justifiable (Corollary 1). Thus, we have no choice but to use multiple ones. However, these will likely create contradictory leximin judgments (Corollary 2). We will discuss this issue later.

4 Price indicator type of extended preferences

Suppose $z \in PO(\succsim, p)$. An extended preference $\succsim_E$ is a *price indicator type* at z and p if for any $i, j \in N$ and any $x \in R^l_+$,

$$(x, j) \succsim_E (z_i, i) \text{ implies } px \geq pz_i, \text{ and } (x, j) \succ_E (z_i, i) \text{ implies } px > pz_i.$$

Note that $(z_i, i) \succsim_E (z_j, j)$ holds if and only if $pz_i \geq pz_j$. The price indicator type compares the welfare of different agents at z using p, judging that the richer, the better, where we interpret pz_i as the wealth of agent i.

Remark 1 *For any $z \in PO(\succsim, p)$, the extended preference illustrated by Example 1 is a price indicator type at z and p. In contrast, the extended preference of Example 2 is not so, as demonstrated later.*

Theorem 3 below shows that the price indicator type relates leximin equitable allocations with EIW allocations.

Theorem 3 *Take $z \in PO(\succsim, p)$ and $\succsim_E \in E(\succsim)$ arbitrarily. Any two of the three below imply the rest.*

(i) $z \in EIW(\succsim, p)$.

(ii) $z \in LME(\succsim_E)$.

(iii) $\succsim_E$ is a price indicator type at p and z.

Proof. (i) and (ii) imply (iii): Suppose $(x, j) \succsim_E (z_i, i)$. As Theorem 2 and (ii) show $(z_i, i) \sim_E (z_j, j)$ for all i, j, we have $(x, j) \succsim_E (z_j, j)$, which implies $x \succsim_j z_j$ because of E.1. Thus, (i) implies $px \geq pz_j$. We can show that $(x, j) \succ_E (z_i, i)$ implies $px > pz_j$ as well, which completes the proof.

(ii) and (iii) imply (i): Theorem 2 and (ii) show $(z_i, i) \sim_E (z_j, j)$ for all i, j. As $\succsim_E$ is the price indicator type at p and z, this implies $pz_i = pz_j$ for all i, j. Noting $z \in PO(\succsim, p)$, we have $z \in EIW(\succsim)$.

(iii) and (i) imply (ii): Suppose $(z_j, j) \succ_E (z_i, i)$ for some i, j. By (iii), this implies $pz_j > pz_i$, which contradicts $pz_i = pz_j$, a direct consequence of (i). Thus, we complete $(z_i, i) \sim_E (z_j, j)$ for all i, j. Noting $z \in PO(\succsim)$, Theorem 2 implies $z \in LME(\succsim_E)$. ■

Do not misunderstand that Theorem 3 implies the equivalence of (i)-(iii). What it implies is (i)+(ii)=(ii)+(iii)=(iii)+(i). It should be intuitively clear that (i) alone does not imply (ii) or (iii) as $E(\succcurlyeq)$ has a rich structure. The other cases are so as well. Refer to the Appendix for details.

Theorem 3 says the following.

(i) and (ii) imply (iii): If an extended preference makes an EIW allocation leximin equitable, it must be a price indicator type.

(ii) and (iii) imply (i): If a feasible allocation is leximin justifiable for an extended preference, which is a price indicator type, it is an EIW allocation.

(iii) and (i) imply (ii): For any EIW allocation, if we make an extended preference, which is a price indicator type using the associated equilibrium prices, that allocation is leximin justifiable for that preference.

Preceding us, Fleurbaey and Maniquet (2008, 2011) studied links between the leximin equitable and EIW allocations. Their leximin, called Ω-Equivalent Leximin ($R^{\Omega lex}$, p.7), is defined using $\succcurlyeq_E^{\Omega}$ of Example 2. Note that $\succcurlyeq_E^{\Omega}$ is not necessarily a price indicator type. Take $z \in EIW(\succcurlyeq, p)$ arbitrarily. If $\succcurlyeq_E^{\Omega}$ is a price indicator type at p and z, Theorem 3 implies z is a leximin equitable allocation, which Theorem 2 with the definition of $\succcurlyeq_E^{\Omega}$ implies that for all i, the indifference curve passing through z_i must pass through the same $\lambda\Omega$, which is an unlikely result. The same result follows if we start with the supposition that z is a leximin equitable allocation. Thus, the leximin equitable allocations defined with $\succcurlyeq_E^{\Omega}$ do not necessarily coincide with the EIW allocations. This conclusion is consistent with Tab.5.1 in Fleurbaey and Maniquet (2011, p.95) which clarifies whether the EIW rule and the Leximin rule satisfy several axioms (+) or not (-). As seen in the table, the signs of the two rules are not the same.

On the other hand, Fleurbaey and Maniquet (2008) show that the Ω-Implicit income maximin relation makes the EIW rule rationalizable. This relation is defined using a money-metric utility function, just like the extended preference in Example 1. However, this is a social ordering on the set of allocations, not an extended preference or the social judgment that uses it.

5 Locally created extended preferences

Next, we consider extended preferences created with only local information. The meaning of locally creating is similar to local independence (Nagahisa 1991, and Nagahisa and Suh 1995). The formal definition is as follows.

Let $p_N = (p_i)_{i \in N}$, where $p_i \in int.\Delta^l$, and $z \in Z$ be given. We say that $\succcurlyeq$ and $\succcurlyeq'$ are p_N identical at z if for any i, $x \succcurlyeq_i z_i$ implies $p_i x_i \geq p_i z_i$ and $x \succcurlyeq'_i z_i$ implies $p_i x_i \geq p_i z_i$. Here, p_i is a common supporting price vector for $\succcurlyeq_i$ and $\succcurlyeq'_i$. Let $p_N(\succcurlyeq, z)$ be the set of profiles that are p_N identical to $\succcurlyeq$ at z. If $\succcurlyeq$ and $\succcurlyeq'$ are p_N identical at z, then $p_N(\succcurlyeq, z) = p_N(\succcurlyeq', z)$. Thus, we write $p_N(\cdot, z)$ if there is no need to specify $\succcurlyeq$.

The welfare ranking of $\succcurlyeq_E$ at z, denoted by $\geq_{(\succcurlyeq_E, z)}$, is the order on N such that $i \geq_{(\succcurlyeq_E, z)} j \iff (z_i, i) \succcurlyeq_E (z_j, j)$ for all i, j, which is a list that indicates who is the wealthiest at z, who is the second, ..., and who is the worse off at z.

Now, we define

$$\bigcap_{\succcurlyeq \in p_N(\cdot, z)} \left\{ \geq_{(\succcurlyeq_E, z)} : \succcurlyeq_E \in E(\succcurlyeq) \right\} \tag{1}$$

or equivalently

$$\bigcap_{\succcurlyeq' \in p_N(\succcurlyeq, z)} \left\{ \geq_{(\succcurlyeq'_E, z)} : \succcurlyeq'_E \in E(\succcurlyeq') \right\}.$$

We say that an extended preference $\succcurlyeq_E$ is locally created at p_N and z if and only if $\geq_{(\succcurlyeq_E, z)}$ belongs to (1). Then, $\geq_{(\succcurlyeq_E, z)}$, the welfare ranking at z, is determined only by preferences around z, not depending on global information far away from z. To simplify the discussion, we assume that individual preferences are smooth and z is an interior point. Then, p_N matches marginal rates of substitution (MRS) at z, equaling across all $\succcurlyeq' \in p_N(\succcurlyeq, z)$, which are therefore approximately identical to each other when we focus on only around z. If MRSs at z are the only information available to create extended preferences, they have the same information for deciding the welfare ranking at z. Thus, we conclude that the set of welfare rankings at z is equal across all $\succcurlyeq' \in p_N(\succcurlyeq, z)$. As a result, the welfare rankings made from a locally created extended preference belong to (1).

As we assume neither smoothness nor interiority, different supporting prices $p'_N = (p'_i)_{i \in N}$ may define different sets (1). We say an extended preference is locally created at z if, for some p_N, it becomes a locally created one at p_N and z. If z also does not matter, we omit "at z" either. If preferences are assumed to be smooth, this diversity disappears. We will also discuss this case later.

The locally created extended preferences remind us of local independence (LI) (Nagahisa 1991, Nagahisa and Suh 1995) for characterizing the Walras rule. LI is a choice consistency requirement of social choice: It requires the decision to choose z to be the same between $\succcurlyeq$ and $\succcurlyeq'$ if MRS equal condition holds for z. On the other hand, the idea of locally created extended preferences does not require that much. It only demands the information about welfare ranking at z to remain unchanged, not necessarily implying that the decision to choose z must be the same.

From now on, all discussions need the following additional assumption regarding $E(\succcurlyeq)$. If every agent has the same preference, we write $\succcurlyeq = (\succcurlyeq, ..., \succcurlyeq)$, permitting abuse of notation. Let $\succcurlyeq_{E^*}$ be the extended preference such that for any $i, j \in N$ and any $x, y \in R^l_+$, $(x, i) \succcurlyeq_{E^*} (y, j) \Leftrightarrow x \succcurlyeq y$. We modify $E(\succcurlyeq)$ as follows.

D.1. If every agent has the same preference, then $E(\succcurlyeq) = \{\succcurlyeq_{E^*}\}$.[7]

There is no compelling reason to dismiss D.1. If everyone has the same preference, that preference should be the only extended preference, and no other extended one is considered possible. We leave $E(\succcurlyeq)$ unchanged except for this

[7] Note that $\succcurlyeq_{E^*}$ meets E.1-3. D.1 and E.1-3 are in different categories: E.1-3 are properties of extended preferences, while D.1 are properties of the *set* of extended preferences.

peculiar case.

Now, we prove two lemmas for locally created extended preferences.

Lemma 1 *Suppose $z \in PO(\succsim, p)$ and $pz_1 \leq pz_2 \leq \cdots \leq pz_n$ without loss of generality. Then, locally created extended preferences exist at p and z, all of which make the same welfare ranking at z as $pz_1 \leq pz_2 \leq \cdots \leq pz_n$, that is*

$$pz_1 \leq pz_2 \leq \cdots \leq pz_n \Leftrightarrow (z_1, 1) \precsim_E (z_2, 2) \precsim_E \cdots \precsim_E (z_n, n). \tag{2}$$

Proof. To simplify the notation, we use $\geq^p$ as the welfare ranking of (2). Let us show

$$\{\geq^p\} = \overbrace{\bigcap_{\succsim' \in p(\succsim, z)} \left\{\geq_{(\succsim'_E, z)} : \succsim'_E \in E(\succsim')\right\}}^{\text{the set of (1)}}. \tag{3}$$

(3) completes the proof. First, we show the following.

$$\geq^p \in \overbrace{\bigcap_{\succsim' \in p(\succsim, z)} \left\{\geq_{(\succsim'_E, z)} : \succsim'_E \in E(\succsim')\right\}}^{\text{the set of (1)}}. \tag{4}$$

Take $\succsim' \in p(\succsim, z)$ arbitrarily. Let $\succsim'_{E^*} \in E(\succsim')$ be an extended preference defined by money-metric utility functions made from using p (See Example 1). Then, $\succsim'_{E^*}$ makes the same welfare ranking as $\geq^p$ at z. Thus, we have

$$\geq^p \in \left\{\geq_{(\succsim'_E, z)} : \succsim'_E \in E(\succsim')\right\}$$

As this holds for all $\succsim' \in p(\succsim, z)$, we have (4).

Next, let $\succsim^p$ be a profile such that every agent has the same preference represented by the utility function $u(x) = px$. Because of D.1, $\succsim^p$ itself is the only possible extended preference for $\succsim^p$, and the welfare ranking at z is the same as $\geq^p$. Thus, we have

$$\left\{\geq_{(\succsim^p_E, z)} : \succsim^p_E \in E(\succsim^p)\right\} = \{\geq^p\} \tag{5}$$

By noting $\overbrace{\bigcap_{\succcurlyeq' \in p(\succcurlyeq, z)} \left\{ \geq_{(\succcurlyeq'_E, z)} : \succcurlyeq'_E \in E(\succcurlyeq') \right\}}^{\text{the set of (1)}} \subset \left\{ \geq_{(\succcurlyeq^p_E, z)} : \succcurlyeq^p_E \in E(\succcurlyeq^p) \right\}$, (4) and (5) complete (3). ■

Lemma 2 *Suppose $z \in EIW(\succcurlyeq, p)$. The three statements below are equivalent to each other.*

(i) $\succcurlyeq_E \in E(\succcurlyeq)$ is locally created at p and z.

(ii) $\geq_{(\succcurlyeq_E, z)}$ is to weigh every agent well off equally: $(z_1, 1) \sim_E (z_2, 2) \sim_E \cdots \sim_E (z_n, n)$.

(iii) $\succcurlyeq_E \in E(\succcurlyeq)$ is a price indicator type at p and z.

Proof. As (ii)$\Longleftrightarrow$(iii) is straightforward, the only remaining is (i)$\Longleftrightarrow$(ii). Let $\geq^p$ be the welfare ranking at z, which is the same as (ii). Following the same procedure as Lemma 1, we have

$$\{\geq^p\} = \overbrace{\bigcap_{\succcurlyeq' \in p(\succcurlyeq, z)} \left\{ \geq_{(\succcurlyeq'_E, z)} : \succcurlyeq'_E \in E(\succcurlyeq') \right\}}^{\text{the set of (1)}}. \tag{6}$$

(6) completes (i)$\Longleftrightarrow$(ii). ■

Replacing "a price indicator type" in Theorem 3 with "locally created," we obtain Theorem 4 below.

Theorem 4 *Take $z \in PO(\succcurlyeq, p)$ and $\succcurlyeq_E \in E(\succcurlyeq)$ arbitrarily. Any two of the three below imply the rest.*

(i) $z \in EIW(\succcurlyeq, p)$.

(ii) $z \in LME(\succcurlyeq_E)$.

(iii) $\succcurlyeq_E$ is locally created at p and z.

Proof. As Theorem 3 and Lemma 2 imply that (i)+(ii)$\Longrightarrow$(iii) and (iii)+(i)$\Longrightarrow$(ii), the only remaining is (ii)+(iii)$\Longrightarrow$(i). Theorem 2 and (ii) imply $(z_1, 1) \sim_E (z_2, 2) \sim_E \cdots \sim_E (z_n, n)$, which together with Lemma 1 shows $pz_1 = pz_2 = \cdots = pz_n$. Noting $z \in PO(\succcurlyeq, p)$, we have $z \in EIW(\succcurlyeq, p)$. ■

6 Informational Basis for Rules

A rule F is a mapping associating with each profile $\succcurlyeq \in Q^n$ a non-empty subset $F(\succcurlyeq)$ of Z. This is a standard definition. However, it decides $F(\succcurlyeq)$ by comparing the welfare of different agents. We assume that the agent's preferences are the only entity existing and that extended preferences are an artificial concept created from those, though indispensable tools to decide $F(\succcurlyeq)$. The figure below illustrates this relation.

$$\succcurlyeq \qquad \overset{\succcurlyeq_E, \succcurlyeq_{E'}, \ldots \in E(\succcurlyeq)}{\overset{\downarrow}{\longrightarrow}} \qquad F(\succcurlyeq) \tag{7}$$

To make the leximin axiomatization of the EIW rule, we need to solve two problems pointed out earlier: A single extended preference is insufficient, necessitating multiple ones (Corollary 1); however**,** these would generate multiple mutually contradictory leximin judgments simultaneously (Corollary 2). The two devices below settle the problems.

The first is the concept of locally created extended preference, which, as previously discussed, is a requirement of informational economization when comparing the welfare of different persons. Let $LC(\succcurlyeq, z)$, a subset of $E(\succcurlyeq)$, consist of, extended preferences locally created at z, though not needed to contain them all**.** We interpret $LC(\succcurlyeq, z)$ as the *informational basis* used for whether $z \in F(\succcurlyeq)$.[8] Note that $LC(\succcurlyeq, z)$ is well defined: Take $p_N = (p_i)_{i \in N}$, where p_i is a supporting price vector for $\succcurlyeq_i$ at z_i. For any $\succcurlyeq' \in p_N(\succcurlyeq, z)$, we define $\succcurlyeq'_{E^*}$ by

$$(x, i) \succcurlyeq'_{E^*} (y, j) \Longleftrightarrow u_i^{p_i}(x) \geq u_j^{p_j}(y), \tag{8}$$

where $u_i^{p_i}$ and $u_j^{p_j}$ are money-metric utility functions defined with p_i and p_j.[9]

[8] An alternative idea is to a price indicator type as an informational basis. However, the price indicator type cannot cover the case of z not being Pareto optimal. It is more appropriate to define informational bases using locally created extended preferences

[9] Refer back to Example 1 for its definition. The similar procedure as in the example shows that $\succcurlyeq'_{E^*}$ meets E.3..

For all $\succcurlyeq' \in p_N(\succcurlyeq, z)$, $\geq_{(\succcurlyeq'_{E^*}, z)}$ coincides with $\geq_{(\succcurlyeq_{E^*}, z)}$, thus (1) is nonempty. We can define $LC(\succcurlyeq, z)$ with $\succcurlyeq_{E^*} \in LC(\succcurlyeq, z)$.

Second, we consider how to synthesize multiple leximin judgments created from those preferences. There may be various ways of judging, but we must observe the following two principles.

1. If we want to select z, at least one extended preference must make it leximin equitable. Otherwise, we would always do so regardless of the leximin criterion.

2. If z becomes leximin equitable for all extended preferences, we must select it. Otherwise, z would never be selectable, and the leximin criterion would be useless.

Leximin Justification Possibility (LMJP) crystallizes the first idea, and Leximin All Unanimity (LMAU) embodies the second. A rule F satisfies LMJP if for any $z \in F(\succcurlyeq)$, there exists some $\succcurlyeq_E \in LC(\succcurlyeq, z)$ and $z \in LME(\succcurlyeq_E)$. A rule F meets LMAU if $z \in F(\succcurlyeq)$ holds whenever $z \in LME(\succcurlyeq_E)$ is true for all $\succcurlyeq_E \in LC(\succcurlyeq, z)$. Both are the most appropriate ways to integrate leximin judgments in cases where one judgment is insufficient, but multiple judgments may contradict each other. Let

$$LME^{\exists}(\succcurlyeq) = \left\{ z \in Z : z \in \bigcup_{\succcurlyeq_E \in LC(\succcurlyeq, z)} LME(\succcurlyeq_E) \right\}$$

$$LME^{\forall}(\succcurlyeq) = \left\{ z \in Z : z \in \bigcap_{\succcurlyeq_E \in LC(\succcurlyeq, z)} LME(\succcurlyeq_E) \right\}.$$

Using those notations, we can make more concise definitions of LMJP and LMAU.

$$\begin{array}{lcl} \text{LMJP} & : & F(\succcurlyeq) \subset LME^{\exists}(\succcurlyeq) \text{ for all } \succcurlyeq \in Q^n. \\ \text{LMAU} & : & LME^{\forall}(\succcurlyeq) \subset F(\succcurlyeq) \text{ for all } \succcurlyeq \in Q^n. \end{array}$$

Theorem 2 and (8) assure that $LME^{\exists}(\succsim)$ is nonempty. Theorem 5 below ensures that $LME^{\forall}(\succsim)$ is nonempty, strengthening so much the equivalence relation between EIW allocations and leximin equitable ones clarified by Theorem 3 and 4.

Theorem 5 *$LME^{\forall}(\succsim) = EIW(\succsim) = LME^{\exists}(\succsim)$ for all $\succsim \in Q^n$.*

Proof. The (i)-(iii) below completes the proof.

(i) $LME^{\forall}(\succsim) \subset EIW(\succsim)$: Suppose not. Then, there exists $z \in Z$ such that $z \in \bigcap_{\succsim_E \in LC(\succsim, z)} LME(\succsim_E)$ and $z \notin EIW(\succsim)$. Let $z \in PO(\succsim, p)$, and the ordering be $pz_1 \geq pz_2 \geq \cdots \geq pz_n$ without loss of generality. Lemma 1 implies that there exists $\succsim_E \in LC(\succsim, z)$ such that $\geq_{(\succsim_E, z)}$is the same as $pz_1 \geq pz_2 \geq \cdots \geq pz_n$. Here, noting $z \notin EIW(\succsim)$, $\geq$ holds with strict relation somewhere in the ordering, which contradicts $z \in LME(\succsim_E)$, considering Theorem 2.

(ii) $EIW(\succsim) \subset LME^{\exists}(\succsim)$: Take $z \in EIW(\succsim, p)$. Take $\succsim_E \in E(\succsim)$, a price indicator type at p and z, arbitrarily. Refer to Remark 1 for the existence. Then, Theorem 3 and 4 show that $z \in LME(\succsim_E)$ and $\succsim_E \in LC(\succsim, z)$, which implies $z \in LME^{\exists}(\succsim)$.

(iii) $LME^{\exists}(\succsim) \subset LME^{\forall}(\succsim)$: Take $z \in LME^{\exists}(\succsim)$. There exists $\succsim_E \in LC(\succsim, z)$ with $z \in LME(\succsim_E)$. Let $p \in int.\Delta^l$ be a supporting price vector associated with z. As $\succsim_E \in LC(\succsim, z)$, $\geq_{(\succsim_E, z)}$ is contained in $\overbrace{\bigcap_{\succsim' \in p(\succsim, z)} \left\{ \geq_{(\succsim'_E, z)} : \succsim'_E \in E(\succsim) \right\}}^{\text{the set of (1)}}$.
Let $\succsim^p$ be a profile where every agent has the same preference represented by the utility function $u(x) = px$. Then, we have

$$\geq_{(\succsim_E, z)} \in \overbrace{\bigcap_{\succsim' \in p(\succsim, z)} \left\{ \geq_{(\succsim'_E, z)} : \succsim'_E \in E(\succsim) \right\}}^{\text{the set of (1)}} \subset \left\{ \geq_{(\succsim^p_E, z)} : \succsim^p_E \in E(\succsim^p) \right\}.$$

Because of D.1, the right-hand set is singleton. Thus, we have

$$\left\{ \geq_{(\succsim_E, z)} \right\} = \overbrace{\bigcap_{\succsim' \in p(\succsim, z)} \left\{ \geq_{(\succsim'_E, z)} : \succsim'_E \in E(\succsim) \right\}}^{\text{the set of (1)}}.$$

The above relation implies that every $\succsim_{E'} \in LC(\succsim, z)$ creates the same welfare ranking at z, and the same as $\geq_{(\succsim_E, z)}$. Thus, Theorem 2 implies $z \in \bigcap_{\succsim_E \in LC(\succsim, z)} LME(\succsim_E)$, which completes the proof. ■

Theorem 4 shows that restricting the range of LMJP and LMAU to only over locally created extended preferences escapes the general tension brought to the fore by Corollary 2.

Theorem 5 demonstrates that by restricting extended preferences to those created locally, we escape the difficulties displayed in Corollary 1 and 2. Different extended preferences are in charge of different EIW allocations, and this setup avoids the problem in Corollary 1. All locally created preferences produce **a** unanimous leximin judgment, thereby resolving the contradiction displayed in Corollary 2.

Now, we state the leximin axiomatization of the EIW rule. We omit the proof, as it is a direct consequence of Theorem 5.

Theorem 6 *The EIW rule is the only rule satisfying LMJP and LMAU.*

The independence of the axioms is easy to show. Any strict subcorrespondence of the EIW rule satisfies LMJP but not LMAU. The all-select rule, selecting all feasible allocations for any case, meets LMAU but not LMJP.

Let us consider the case of smoothness of preferences. Let $\widehat{Q}$ be a subset of Q consisting of smooth preferences subject to a boundary condition such that $\left\{0 \leq x \leq \Omega : x \succsim \frac{\Omega}{n}\right\} \subset R^l_{++}$.The boundary conditions assure the interiority of EIW allocations. A rule F is a mapping that associates with each profile $\succsim \in \widehat{Q}^n$ a nonempty subset of Z. We strengthen E.3 as follows.

E.3' There exists $z^* \in Z$ such that $(z^*_1, 1) \sim_E (z^*_2, 2) \sim_E \cdots \sim_E (z^*_n, n)$ and $\{0 \leq x \leq \Omega : x \succsim_i z^*_i\} \subset R^l_{++}$ for all i.

E.3' assures every leximin equitable allocation is in the interior, $LME(\succsim_E) \subset R^{nl}_{++}$ for all $\succsim_E \in E(\succsim)$: Take $z \in LME(\succsim_E)$ arbitrarily. Theorem 2 assures

$(z_1, 1) \sim_E (z_2, 2) \sim_E \cdots \sim_E (z_n, n)$. Comparing z and z^* using the leximin criterion, we have $(z_i, i) \succcurlyeq_E (z_i^*, i)$, and hence $z_i \succcurlyeq_i z_i^*$ for all i, which completes the proof.

Let us reinterpret $E(\succcurlyeq)$ as the set of extended preferences meeting E.1, E.2, and E.3'. Note that D.1 still holds under this reinterpretation. Remark 1 assures the existence of such extended preferences.

The results survive under this slight modification. The only necessity in proofs is replacing $\succcurlyeq^p$ with a profile represented by the same CES utility function, choosing parameters appropriately to fit boundary conditions.

7 Discussion

We discuss two issues here. The first is the relevance of this study to previous research on SWFLs. Let $\widehat{u} : R_+^l \times N \to R$ be an extended utility function, associating with each (x, i) the utility $\widehat{u}(x, i)$. A rule F in that research is modified as follows:

$$\widehat{u} \longrightarrow F(\widehat{u}). \tag{9}$$

Here, F associates with each $\widehat{u}$ a nonempty subset $F(\widehat{u})$ of Z. The relation between utility functions and preferences in standard microeconomic analysis holds between $\widehat{u}$ and $\succcurlyeq_E$ as well: Each extended utility function induces an extended preference, but different functions are likely to cause the same one. Thus, the rule F may associate different sets of allocations with the same extended preference $\succcurlyeq_E$, depending on the invariance axiom the rule satisfies.

As any function $\widehat{u} : R_+^l \times N \to R$ is permissible for an extended utility function, the extended preference induced from $\widehat{u}$ does not necessarily meet E.3, which differs from us. However, the remarkable difference is the usage and interpretation of extended preferences, which is made clear by comparing

(7) and (9). For the previous research on SWFLs, extended preferences $\succsim_E$ (induced from $\widehat{u}$) are *actual*, not interpreted as hypothetically created concepts (or tools) used for social decisions as we did. Thus, they regard $F(\succsim_E)$ as the social decision.

If each person's subjective preferences are the only ones we know, our setting will appeal more than that of the SWFL's research. To declare in such situations that "the state in which agent i enjoys consumption x is better than the state in which agent j enjoys consumption y, i.e., $(x,i) \succsim_E (y,j)$," we would have to bring in strong value judgments. We aim to make welfare comparisons without resorting to such value judgments as much as possible. To accomplish it, we have no choice but to admit that multiple equally valid extended preferences will inevitably emerge, and social decisions should be made by appropriately compromising and integrating them (LMJP and LMAU are concrete methods). However, if we can use (non-welfare or other) information beyond preferences, we could identify the real extended preference. In this situation, the formulation in the research on the SWFLs is more plausible than ours. Thus, which one is appropriate depends on the situation we described above.

In the previous research on SWFLs, the leximin and EIW rules are defined as $\widehat{u} \longrightarrow LME(\widehat{u})$ and $\widehat{u} \longrightarrow EIW(\widehat{u})$, respectively. Both rules are irrelevant in general; $LME(\widehat{u}) = EIW(\widehat{u})$ is not necessarily true, as discussed in Fleurbaey and Maniquet (2011), because the extended preference induced from $\widehat{u}$ is not necessarily of price indicator type.

The incoincidence of the two rules is due to invariance axioms, saying that if two extended utility functions $\widehat{u}$ and $\widehat{u}'$ are informationally equivalent, then $F(\widehat{u}) = F(\widehat{u}')$, the social decisions being unchanged. What is "informationally equivalent" depends on axioms. The most demanding version, ordinal non-comparability (ONC), is equivalent to the statement that $\widehat{u}$ and $\widehat{u}'$ are informationally equivalent if $\widehat{u}$ and $\widehat{u}'$ are the same in ordinal meaning. When $\widehat{u}$ changes to $\widehat{u}'$, the EIW allocations are unchanged, but the same is not necessarily true for leximin equitable allocations. Thus, the EIW rule satisfies ONC, but the leximin rule (criterion) does not.

What puzzles us here is why LMJP and LMAU, defined only with the leximin criterion, can axiomatize the EIW rule. Why is the axiomatization unaffected by the differences that appear when we view from the invariance axioms? The answer is that we defined informational equivalence with the price indicator type or equivalently locally created. The changes in extended preferences are only within the price indicator type; $\succsim_E \rightarrow \succsim'_E$ is allowed only if $\succsim'_E$ is a price indicator type. Although this is apparent from Theorem 5, the discussion below clarifies it: Let $\succsim_E$ and $\succsim'_E$ be price indicator types at z and p. Then, $z \in LME(\succsim_E)$ implies $z \in LME(\succsim'_E)$. The proof is easy: Thanks to Theorem 2, $z \in LME(\succsim_E)$ implies $(z_i, i) \sim_E (z_j, j)$ for all i, j, which further implies $pz_i = pz_j$ for all i, j because $\succsim_E$ is a price indicator type at z and p. Suppose $z \notin LME(\succsim'_E)$. Then, Theorem 2 implies $(z_j, j) \succ'_E (z_i, i)$ for some i, j. As $\succsim'_E$ is a price indicator type at z and p, we have $pz_j > pz_i$, a contradiction.

As already discussed, limiting the class of extended preferences to price indicator types, i.e., locally created, is semantically equivalent to relaxing Arrow's IIA in SWFLs, which opens another route for interpersonal welfare comparisons, ushering in a way that differs from invariance axioms. Baccelli (2022) suggested this possibility in the context of the Arrovian framework.[10]

Second, Theorem 4 is especially valuable, having three implications. Of these, "if an EIW allocation is leximin justifiable, then that extended preference is a locally created one" is the most remarkable. The message is that if we wish to value the EIW rule and the leximin criterion, we must only use the locally created extended preferences. In connection with this argument, Ham-

[10] Baccelli (2022) gives the Borda and relative utilitarian rules as examples and examines them in detail. See also Fleurbaey (2003), studying informational basis using the concept of data filter.

mond (1991) stated a thought-provoking idea about interpersonal comparisons of utilities (ICUs):

But if it is easier to think what is a good social welfare ordering, rather than how to make ICUs, why should we not start with the ordering and have it reveal the ICUs, instead of starting with ICUs and trying to derive a social ordering? Especially if it is not at all clear anyway how to incorporate ICUs into a social ordering even if we believe we have made securely founded and ethically relevant interpersonal comparisons of both utility levels and utility differences.

-Hammond (1991, p226-227)

This study reveals one relationship, which Hammond remarked "not at all clear," between ICUs and resource allocation rules. Though only dealing with exchange economies, thus a limited argument, our result answers Hammond's question. The fact that people accepted the rules corresponds to the fact that people agreed with making welfare comparisons based on the rules because each person receives their utility according to the rules. In this sense, we are making welfare comparisons between individuals, as discussed elsewhere (Section 6.3) in Hammond. Accepting the EIW rule and the leximin fairness as valuable is equivalent to justifying only locally created extended preferences or equivalently regarding welfare comparisons by those extended preferences as correct.

8 Conclusion

This paper combines two studies advanced independently in social choice. One is the study of interpersonal comparisons of welfare, and the other is the study of axiomatic analysis of resource allocation problems.[11] Several studies have proved the advantages of the EIW rule from normative points of view.[12] This

[11] Chambers and Hayashi (2017) proposed an alternative axiomatization of the Walras rule considering income distribution problems.

[12] Thomson (2007) includes a comprehensive survey on the justifications with no envy concept. Fleurbaey and Maniquet (2011) study the normative aspect of the equal income Walras rule with social ordering functions, a rational choice function version of Arrow's social welfare

paper also belongs to that stream of research.

We proposed a new information basis for social decisions, different from the previous studies of SWFLs, closely related to Hammond's idea on ICU, and further research is worth doing in this direction.

9 Appendix

The independence of E.1-E.3:

E.1: Take a profile $\succcurlyeq$ such that the preferences are not representable by linear utility functions. Let $\succcurlyeq_E$ be such that $(x,i) \succcurlyeq_E (y,j) \iff px \geq py$, where $p \in int.\Delta^l$. Then, $\succcurlyeq_E$ satisfies E.2 and E.3 but not E.1.

E.2: Let utility functions be such that $v_i(x) = \sum_{h=1}^{l} x_{ih} + 1$. Let $\succcurlyeq$ be the profile induced from those utility functions.

Now we change agent 1's utility function in the following:

$$v_1'(x) = \begin{cases} 0 & \text{if } x = 0 \\ \sum_{h=1}^{l} x_{1h} + 1 & \text{otherwise} \end{cases}$$

Let $\succcurlyeq_E$ be the extended preference made by utility comparison using $v_1'$and v_i $(i = 2, ..., n)$. Then, $\succcurlyeq_E$ satisfies E.1 and E.3. In contrast, $\succcurlyeq_E$ violates E.2: Suppose, on the contrary, that $\succcurlyeq_E$ has a representation $(u_i)_{i \in N}$. We have for all $x \neq 0$, $(x,1) \sim_E (x,2)$, and hence $u_1(x) = u_2(x)$. As u_1 and u_2 are continuous functions, approaching x to 0 implies $(0,1) \sim_E (0,2)$, which contradicts the definition of $\succcurlyeq_E$.

E.3: Let u_i be agent i's continuous utility function representing $\succcurlyeq_i$ and $u_1(0) > u_2(\Omega)$. Let $\succcurlyeq_E$ be the extended preference made by utility comparison using the u_is. Then, $\succcurlyeq_E$ satisfies E.1 and E.2 but not E.3.

Independence of (i)-(iii) in Theorem 3:

function.

(i)$\not\Rightarrow$(ii): Let $z \in EIW(\succcurlyeq, p)$ and $(u_i)_{i \in N}$ represent $\succcurlyeq_E$ such that not all $u_i(z_i)$ are equal. Theorem 2 implies $z \notin LME(\succcurlyeq_E)$.

(i)$\not\Rightarrow$(iii): Let us use the example of (i)$\not\Rightarrow$(ii). If $\succcurlyeq_E$ is a price indicator type at z and p, Theorem 3 implies $z_i \sim_E z_j$ for all i, j, a contradiction.

(ii)$\not\Rightarrow$(i): Let z be such that $z \in PO(\succcurlyeq)$ with $z_i \neq 0$ for all i, and $z \notin EIW(\succcurlyeq$). We define a representation of $\succcurlyeq_E$ such that $u_i(z_i) = 1$ for all i. Theorem 2 implies $z \in LME(\succcurlyeq_E)$.

(ii)$\not\Rightarrow$(iii): Let z and p be such that $z \in PO(\succcurlyeq, p)$, $z_i \neq 0$ for all i, and $pz_1 > pz_2$. Using money-metric utility functions u_i^p, we define an extended preference such that

$$(x, i) \succcurlyeq_E (y, j) \Longleftrightarrow \alpha_i u_i^p(x) \geq \alpha_j u_j^p(y),$$

where α_i and α_j are positive constants such that $\alpha_i pz_i = \alpha_j pz_j$. Theorem 2 shows $z \in LME(\succcurlyeq_E)$. Suppose that $\succcurlyeq_E$ is a price indicator type at z and p. As Theorem 2 says $z \in EIW(\succcurlyeq, p)$, i.e., $pz_1 = pz_2$, a contradiction.

(iii)$\not\Rightarrow$(i) : Let $z = (\Omega, 0, ..., 0)$ and $p \in int.\Delta^l$ be such that $z \in PO(\succcurlyeq, p)$. We invoke $\succcurlyeq_E^p$ of Example 1. Then, we have $z \notin EIW(\succcurlyeq, p)$.

(iii)$\not\Rightarrow$(ii): We use the same example as the case of (iii)$\not\Rightarrow$(i). We have $(z_1, 1) \succ_E (z_i, i)$ for all $i \neq 1$. Invoking Theorem 2, we have $z \notin LME(\succcurlyeq_E)$.

A necessary and sufficient condition for an extended preference to have a representation:

Take an extended preference $\succcurlyeq_E$ arbitrarily, fixed throughout this subsection. We propose a property below.

Continuity among agents: For any $i, j \in N$, and any $x^\nu, y^\nu \in R_+^l (\nu = 1, 2, ...,)$, if $(x^\nu, i) \succcurlyeq_E (y^\nu, j)$ for all ν and $x^\nu \longrightarrow x$, $y^\nu \longrightarrow y$, then $(x, i) \succcurlyeq_E (y, j)$.

This condition makes the continuity of a single agent's preference applicable between different agents.

Proposition 1 *Suppose that $\succcurlyeq_E$ meets E.1. If $\succcurlyeq_E$ satisfies continuity among agents, then it meets E.2, and the converse is also true.*

Proof. The converse is apparent. Let us show the "if" part. As the indifference relation $\sim_E$ is equivalent (reflective, symmetric, and transitive), it makes a partition $\{C_\lambda\}_{\lambda\in\Lambda}$ of $R_+^l \times N$, each C_λ corresponding to an indifference curve of $\succcurlyeq_E$. Just as preference order ranks the indifference curves in the usual sense, $\succcurlyeq_E$ ranks C_λ as well, like $\cdots \prec_E C_\lambda \prec_E C_{\lambda'} \prec_E \cdots$. Permitting the abuse of notation, we write $i \in C_\lambda$ if C_λ passes through some (x,i), i.e., there exists some x such that $(x,i) \in C_\lambda$. Now, we show the following.

$$C_{\lambda_1} \prec_E C_\lambda \prec_E C_{\lambda_2} \text{ and } i \in C_{\lambda_1} \cap C_{\lambda_2} \Longrightarrow i \in C_\lambda. \tag{A-1}$$

Suppose not. Take $(y,j) \in C_\lambda$, $j \neq i$, arbitrarily. Then, we have

$$R_+^l = \left\{x \in R_+^l : (x,i) \succ_E (y,j)\right\} \bigcup \left\{x \in R_+^l : (x,i) \prec_E (y,j)\right\}.$$

The two sets on the right side are disjoint, open due to continuity among agents, and non-empty because of $C_{\lambda_1} \prec_E C_\lambda \prec_E C_{\lambda_2}$ and $i \in C_{\lambda_1} \cap C_{\lambda_2}$, which contradicts the connectedness of R_+^l, completing the proof of (A-1).

Let $\mathbb{C}(i)$ be the set of indifference curves C_λ such that $i \in C_\lambda$, defined formally as $\mathbb{C}(i) = \bigcup\limits_{i\in C_\lambda} C_\lambda$. Let $C_{(0,i)}$ be the indifference curve passing through $(0,i)$. Then, considering (A-1), $\mathbb{C}(i)$ is either bounded above or not. For the former case, $\mathbb{C}(i)$ consists of all C_λ with $C_{(0,i)} \preccurlyeq_E C_\lambda$. For the latter, there exists some C_{λ_i} such that $\mathbb{C}(i)$ consists of all C_λ with $C_{(0,i)} \preccurlyeq_E C_\lambda \prec_E C_{\lambda_i}$ or all C_λ. Here, note that C_{λ_i} does not belong to $\mathbb{C}(i)$. Let us show this. Suppose, on the contrary, that $i \in C_{\lambda_i}$, i.e., $(x,i) \in C_{\lambda_i}$ for some x. Then, taking $x^* \gg x$, there exists C_{λ^*} such that $(x^*,i) \in C_{\lambda^*}$ and $C_{\lambda_i} \prec_E C_{\lambda^*}$, contradicting the definition of $\mathbb{C}(i)$.

We write the two cases as $\left[C_{(0,i)}, +\infty\right)$ and $\left[C_{(0,i)}, C_{\lambda_i}\right)$ respectively, all called segments. Notice also that for $\mathbb{C}(i) = \left[C_{(0,i)}, C_{\lambda_i}\right)$ case, $\mathbb{C}(i)$ is nondegenerate, never shrinking to one point.

Then, there exists a sequence of agents $i_1, i_2, ..., i_k$ such that their segments cover all indifference curves;

$$\left[C_{(0,i_1)}, C_{\lambda_{i_1}}\right) \bigcup \left[C_{(0,i_2)}, C_{\lambda_{i_2}}\right) \bigcup \cdots \bigcup \left[C_{(0,i_k)}, +\infty\right) = \bigcup_{\lambda \in \Lambda} C_\lambda,$$

where the segments are placed from left to right according to the ascending order of the extended preference and **f**or any two adjacent segments, either they have overlap or the rightmost point of the left segment is the same as the leftmost point of the right segment; for example, $\left[C_{(0,i_1)}, C_{\lambda_{i_1}}\right) \bigcap \left[C_{(0,i_2)}, C_{\lambda_{i_2}}\right) = \emptyset \Longrightarrow C_{\lambda_{i_1}} = C_{(0,i_2)}$. The diagram below illustrates this.

$$\overbrace{C_r \prec_E \cdots \prec_E C_{r'} \prec_E C_{r''}}^{\left[C_{(0,i_1)}, C_{\lambda_{i_1}}\right)}$$

$$\overbrace{C_{r'} \prec_E C_{r''} \cdots \prec_E C_{r'''}}^{\left[C_{(0,i_2)}, C_{\lambda_{i_2}}\right)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\overbrace{C_{r'\cdots'} \prec_E \cdots \prec_E \cdots}^{\left[C_{(0,i_k)}, +\infty\right)}$$

Now, we form a representation $(u_i)_{i \in N}$, using those segments. As the figure shows, we can create the representation by representing $\succcurlyeq_E$ with someone's utility function for each segment and connecting them. Note also that we use E.1 here; $\succcurlyeq_E$ on the segment is representable by that agent's utility function.

Let v_{i_1} be utility functions representing $\succcurlyeq_{i_1}$. For each $C_\lambda \in \left[C_{(0,i_1)}, C_{\lambda_{i_1}}\right)$, we define the utility as $v_{i_1}(C_\lambda)$. Precisely, we define for any $(x, i) \in C_\lambda$, $u_i(x) = v_{i_1}(y)$, where $(y, i_1) \in C_\lambda$. Next, let v_{i_2} be utility functions representing $\succcurlyeq_{i_2}$, thus meeting $v_{i_2}(C_{r''}) = v_{i_1}(C_{r''})$. For each $C_\lambda \in \left[C_{r''}, C_{\lambda_{i_2}}\right)$, we define the utility as $v_{i_2}(C_\lambda)$. The procedure ends and gives $v_{i_k}(C_\lambda)$ to each $C_\lambda \in [C_{r'\cdots'}, +\infty)$. This procedure defines a representation of $\succcurlyeq_E$. ■

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