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文部科学大臣認定　共同利用・共同研究拠点

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Abstract

This paper studies the effect of the timing gaming by the agent in incentive contracts. It is well known that the agent has incentives to manipulate the timing of sales reports when the principal uses a nonlinear incentive contract such as a quota-based contract. However, it is not known how the agent's gaming activity affects the principal's profit.

We study a dynamic moral hazard model in which the agent can delay the timing of reporting. First, we compare linear contracts and quota-based contracts to examine how the timing gaming relates the optimal form of contracts. We show that when the agent's effort cost is relatively low, the principal prefers quota-based contracts to linear contracts. Second, we consider the non-gaming situation in which the agent cannot manipulate the timing of reporting to examine how the agent's gaming activity affects the principal's profit. We show that when the agent's effort cost is relatively high, gaming activities increase the principal's profit.

Keywords: Quota-based contract; Gaming; Moral-hazard

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1 Introduction

A quota-based contract is one of the most important forms of incentive contracts, especially for salespeople.[1] One reason for its popularity is that it can provide strong incentives to salespeople. Previous theoretical research on static moral hazard models also shows that a quota-based contract is the most efficient contract form from a firm's perspective, especially when a salesperson's liability is limited.[2]

Furthermore, it is well known that if an employment relationship between a firm and a salesperson exists over a long term (multi-period), quota-based contracts provide incentives for the salesperson to manipulate the timing of transactions to increase their wage ("timing gaming").[3] However, it is not clear how timing gaming affects a firm's objectives. In this study, I examine the effect of timing gaming on a theoretical framework.

Several empirical and theoretical studies have examined the effects of employees' gaming activities. For example, Larkin (2014) suggests that gaming is costly to firms. The author focuses on situations in which employees manipulate prices rather than the timing of sales reports. He uses a proprietary database of deals for a leading enterprise software vender and demonstrates that timing gaming by the vender's salesperson costs it 6-8% of revenue.

An example of a theoretical study is Au and Kawai (2019), who investigate the optimality of quota-based contracts. They examine a two-period moral hazard model in which the agent can carry over the first period's sales to the second round. Their main result is that if the principal does not have contractual commitment power (i.e., there is a possibility of renegotiation), a quota-based contract can be optimal for the principal.

To complement Au and Kawai (2019), we investigate a long-term employment relationship by considering an infinite-period moral hazard model wherein a risk-neutral principal (female) employs a risk-neutral agent (male) to sell her

[1]see Joseph and Kalwani (1998).

[2]see Kim (1997) and Oyer (2000).

[3]see Oyer (1998).

products or services. Every period consists of two rounds, and the agent exerts an effort to sell the product to one customer in each round.

The principal cannot observe not only the agent's effort level, but also whether the product has been sold or not. Instead, the principal can observe and verify the sales report, which is submitted by the agent at the end of each round. We assume that the agent can strategically manipulate the timing of sales reports. Specifically, the agent can report the current round's sales at the end of next round, that is, the agent can carry over the sales to the next round.

This study aims to answer two questions: (i) If the agent can manipulate the timing of sales reports, what form of contract is optimal for the principal? (ii) How does the gaming activity affect the principal's profit?

To answer the first question, we compare linear and quota-based contracts, which are the most common forms of sales force compensation. Linear contracts reward each sales equally, whereas quota-based contracts do not. Therefore, if the principal offers a linear contract, the agent has no incentive to manipulate the timing of reporting because it does not change his total wage. Meanwhile, a quota-based contract may incentivize the agent to game the timing because of nonlinearity of the compensation form. Specifically, the agent may have incentives to delay the sales reports when, at the beginning of the second round, (a) he has already made a quota, or (b) he cannot make a quota even if he could sell the product in that round.

We show that quota-based contracts are more profitable for the principal than linear contracts if and only if the cost per unit effort is relatively low. The intuition behind this result is as follows. Quota-based contracts can provide strong incentives to the agent, but the strength of these incentives depends on the achievement of quotas. Specifically, the agent exerts a high effort when he is likely to achieve a quota, but a low effort when he is not.

If marginal effort costs are not constant (i.e., the agent's cost function is nonlinear), these variations in the incentive strength generates inefficiency in effort costs. Furthermore, this inefficiency increases with the cost per unit effort. As a result, the negative effect of quota-based contract is more significant than the positive effect of that when the cost per unit effort is high.

Because linear contracts do not generate such inefficiency, they are more profitable for the principal than quota-based contracts when the cost per unit effort is high.

To answer the second question, we consider a situation in which the agent cannot game the timing of the sales reports (a non-gaming situation) and compare this and the gaming situation from the perspective of the principal's profits.

We show that timing gaming by the agent is profitable for the principal if and only if the cost per unit effort is relatively high. The logic behind this result is similar to that of the first question. In the non-gaming situation, quota-based contracts can provide stronger incentives than in the gaming situation, but such strong incentives also increase the variability in effort levels. Therefore, an increase in the cost per unit effort decreases the principal's profit in the non-gaming situation more sharply than in the gaming situation.

The rest of this paper is organized as follows. The next section describes our model; section 3 compares linear and quota-based contracts; section 4 compares gaming situation and non-gaming situation to analyze the effect of the timing gaming; section 5, we discuss our main results and potential extensions. The proofs of the lemmas and the results are provided in the appendices.

2 Model

We consider an infinite-period moral hazard model in which a risk-neutral principal (female) hires a risk-neutral agent (male) who is protected by limited liability (i.e., wages must be positive) to sell her products or services (hereafter "output"). They live in period $1, 2, \ldots$ until infinity and have a common discount factor $\delta \in [0, 1)$. Each period consists of two rounds (first round and second round). In each round, the agent exerts costly effort to sell the output to one customer. His effort level $e(\in [0, 1] \equiv E)$ is not observable and verifiable for the principal. The cost of exerting effort is denoted by $c(e) = ce^2$ where c $(\in \{0.01, 0.02, \cdots, 2.00\})$ is the cost per unit effort, which is commonly known.

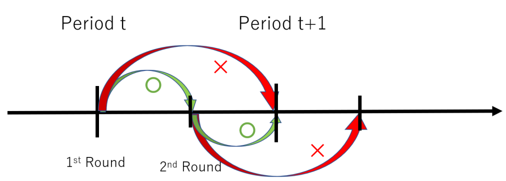
The agent's effort in each round stochastically determines the sales $y(\in \{0,1\} \equiv Y)$ for that round. Specifically, the probability of a successful transaction equals the effort level in that round, that is, $Prob(y = 1|e) = e$. We use $t(= 1, 2, 3 \cdots)$ and $R(\in \{1(st), 2(nd)\} \equiv \mathcal{R})$ to denote periods and rounds, respectively, that is, the sales in the round R of the period t is denoted by $y_{t,R}$.

We assume that the principal cannot observe either the agent's effort level or the sales. However, she can observe and verify the agent's self-reports (denoted by $r_{t,R} \in \{0,1\} \equiv \mathscr{R}$). This allows her to design enforceable contracts wherein payments are dependent on sales reports.

To examine the effect of the agent's timing gaming, we consider a situation wherein the agent can carry over sales from the current round to the next round by underreporting. The agent is assumed to be able to carry over the sales for one round only. Specifically, the constraint is given by $\theta_{t,R} \leq r_{t,R} \leq y_{t,R} + \theta_{t,R}$. $\theta_{t,R}(\in \{0,1\} \equiv \Theta)$ denotes carryover at the end of round R in period t, where it is defined as

$$\theta_{t,1} = \begin{cases} y_{t-1,2} + \theta_{t-1,2} - r_{t-1,2} & \text{if } \ \theta_{t-1,2} \leq r_{t-1,2} \\ y_{t-1,2} & \text{otherwise,} \end{cases}$$

$$\theta_{t,2} = \begin{cases} y_{t,1} + \theta_{t,1} - r_{t,1} & \text{if } \ \theta_{t,1} \leq r_{t,1} \\ y_{t,1} & \text{otherwise.} \end{cases}$$



In our model, a contract specifies the wage for each period (denoted by w_t). We impose several restrictions on contracts. First, the principal offers a contract at the beginning of the relationship and cannot change it afterwards (i.e., there is no possibility of renegotiation). Second, the wage for each period depends only on the sum of the sales reports for that period (hereafter referred to as the total sales report), but not on the history of sales reports, that is, wage in period t depends on $(r_{t,1} + r_{t,2})$. With these two assumptions, the wage scheme does not depend on the period; thus, the wage for period t can be expressed as $w(r_{t,1}, r_{t,2})$. Finally, we assume that the agent's liability is limited (i.e., $w(r_{t,1}, r_{t,2}) \geq 0$ for all $r_{t,1}, r_{t,2} \in \{0, 1\}$).

The timing of this game is as follows.

1. The principal offers a contract $w(r_1, r_2)$ and the agent chooses to accept or reject it. If the agent rejects the contract, the game ends and both the principal and the agent earn the outside option ($= 0$).

2. If the agent accepts the contract, he chooses effort level $e_{1,1} \in [0, 1]$ for the first round of period 1.

3. The output $y_{1,1}$ is realized, and it is observable for the agent but not for the principal.

4. The agent submits the sales report $r_{1,1}(\geq 0)$ and carries over $\theta_{1,1}(= y_{1,1} - r_{1,1})$ to the second round of period 1.

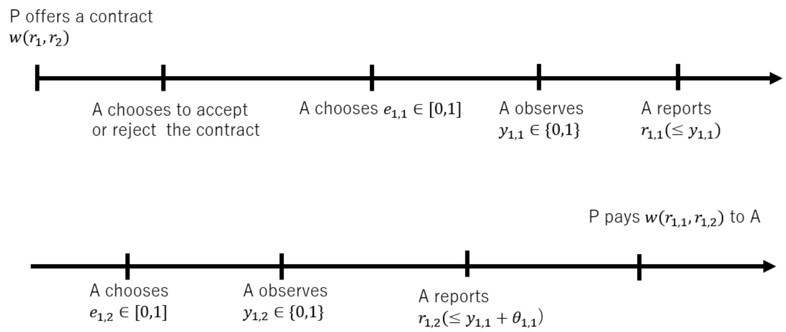
5. The agent chooses $e_{1,2} \in [0, 1]$.

6. The output of second round $y_{1,2}$ is realized, and it is observable for the agent but not for the principal.

7. The agent submits a sales report $r_{1,2}(\geq \theta_{1,1})$ and carries over $\theta_{1,2}(= y_{1,2} + \theta_{1,1} - r_{1,2})$ to the first round of period 2.

8. The principal pays the agent based on his total report in period 1.

9. Step 2-7 are repeated for period 2, $3, \cdots$.



We begin by considering the problem of the agent. The agent is assumed to maximize the expected total payoff, where period t's ex-post payoff is given by the wage minus the total effort cost in that period (i.e., $w_t(r_{t,1}, r_{t,2}) - (ce_{t,1}^2 + ce_{t,2}^2))$.

The agent's control variables in period t are $e_{t,1}, e_{t,2}, r_{t,1}$ and $r_{t,2}$. Thus, his problem is to choose a policy function for both effort level and sales reports (effort policy and report policy, respectively).

The effort policy consists of the policy for the first and second rounds' efforts, which are functions from Θ to E and $\Theta \times \mathscr{R}$ to E, respectively. Meanwhile, the report policy consists of the policy for the sales reports of the first and second rounds, which are functions from $\Theta \times Y$ to $\mathscr{R}$ and $\Theta \times \mathscr{R} \times Y$ to $\mathscr{R}$, respectively. Using the dynamic programming framework, the optimal effort and report policies satisfy the following Bellman equations.

$$
\begin{aligned}
&V_1(\theta_1) \\
&= \max_{\substack{e_1(\theta_1) \\ r_1(\theta_1,1), r_1(\theta_1,0)}} \quad -c(e_1(\theta_1)) + e_1(\theta_1) V_2(\sigma_1(\theta_1, r_1(\theta_1, 1), 1)) \\
&\qquad\qquad\qquad + \{1 - e_1(\theta_1)\} V_2(\sigma_1(\theta_1, r_1(\theta_1, 0), 0))
\end{aligned}
$$

$$
\begin{aligned}
&V_2(\theta_2, r_1) = \\
&\max_{\substack{e_2(\theta_2, r_1)\\ r_2(\theta_2, r_1, 1)\\ r_2(\theta_2, r_1, 0)}} -c(e_2(\theta_2, r_1)) \\
&\qquad\qquad + e_2(\theta_2, r_1)[w(r_1, r_2(\theta_2, r_1, 1)) + \delta V_1(\sigma_2(\theta_2, r_2(\theta_2, r_1, 1), 1))] \\
&\qquad\qquad + \{1 - e_2(\theta_2, r_1)\}[w(r_1, r_2(\theta_2, r_1, 0)) + \delta V_1(\sigma_2(\theta_2, r_2(\theta_2, r_1, 0), 0))],
\end{aligned}
$$

where σ_1 and σ_2 are transition functions from $\Theta \times \mathscr{R} \times Y$ to $\Theta \times \mathscr{R}$ and from $\Theta \times \mathscr{R} \times Y$ to Θ, respectively.

Meanwhile, the principal's problem is to design the wage scheme to maximize her total expected profit, subject to incentive compatibility constraints for efforts and reports and limited liability constraint.[4] The principal's ex-post profit in period t is defined as the total sales reports minus the wage in that period, that is, $r_{t,1} + r_{t,2} - w(r_{t,1}, r_{t,2})$. Using a dynamic programing, the optimal wage scheme satisfies the following Bellman equations:

$$
\begin{aligned}
\Pi_1(\theta_1) =& e_1^*(\theta_1) r_1^*(\theta_1, 1) + \{1 - e_1^*(\theta_1)\} r_1^*(\theta_1, 1) \\
&+ e_1^*(\theta_1)\Pi_2(\sigma_1(\theta_1, r_1^*(\theta_1, 1), 1)) + \{1 - e_1^*(\theta_1)\}\Pi_2(\sigma_1(\theta_1, r_1^*(\theta_1, 0), 0))
\end{aligned}
$$

$$
\begin{aligned}
&\Pi_2(\theta_2, r_1) = \\
&\max_{w(r_1, r_2)} e_2^*(\theta_2, r_1) r_2^*(\theta_1, 1) + \{1 - e_1^*(\theta_1)\} r_1^*(\theta_1, 1) \\
&\qquad\qquad + e_2^*(\theta_2, r_1)[w(r_1, r_2^*(\theta_2, r_1, 1)) + \delta\Pi_1(\sigma_2(\theta_2, r_2^*(\theta_2, r_1, 1), 1))] \\
&\qquad\qquad + \{1 - e_2^*(\theta_2, r_1)\}[w(r_1, r_2^*(\theta_2, r_1, 0)) + \delta\Pi_1(\sigma_2(\theta_2, r_2^*(\theta_2, r_1, 0), 0))],
\end{aligned}
$$

where e^* and r^* are the optimal effort and report policies under the contract offered by the principal, respectively.

We focus on three types of contracts: linear, high-quota, and low-quota.

Definition 1. *A contract is linear if wage scheme is*

$$
w(r_1, r_2, b) = b(r_1 + r_2)
$$

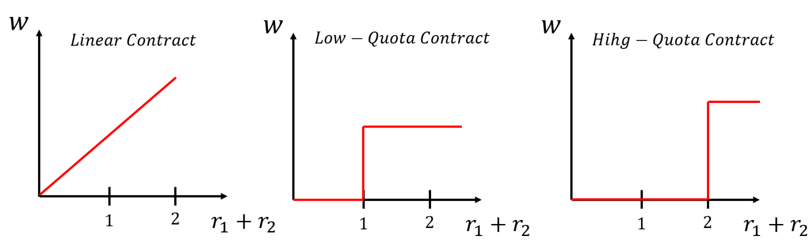
[4] Because we assume that the agent's outside option is 0 and the minimum wage is 0, he always accepts the principal's offer. That is, we can drop the participation constraint without loss of generality.

Definition 2. *A contract is low-quota if wage scheme is*

$$w(r_1, r_2, b) = \begin{cases} b & \textit{if} \quad r_1 + r_2 \geq 1 \\ 0 & \textit{otherwise} \end{cases}$$

Definition 3. *A contract is high-quota if wage scheme is*

$$w(r_1, r_2, b) = \begin{cases} b & \textit{if} \quad r_1 + r_2 = 2 \\ 0 & \textit{otherwise} \end{cases}$$



Under a linear contract, the agent receives a constant amount for each unit of sales report. Thus, this type of contract provides linear incentives to the agent. However, under a low-quota contract, the agent receives b if and only if the total sales report in that period is higher than 1; thus, low-quota contracts provide nonlinear incentives to the agent because marginal wages from reporting additional sales are not constant. Therefore, the agent may have an incentive to manipulate the timing of sales reporting. Specifically, he has incentives to delay the reports when he cannot make quotas even though he reports true sales. Finally, under a high-quota contract, the agent receives b if and only if the total sales report in that period is 2. This type also provides nonlinear incentives to the agent, and thus, he has incentives to manipulate the timing in this case too.

3 Linear vs Quota

In this section, we compare the three types of contracts with respect to the principal's profit. We examine which types efficiently incentivize the agent when he can manipulate the timing of reports ("gaming situation"). We derive the necessary and sufficient conditions for each contract to be optimal for the principal.

3.1 Linear contracts vs Low-quota contracts

First, we characterize the optimal linear contract and derive the principal's profit. To do this, we characterize the agent's optimal effort and report policies. The following lemma characterizes the optimal effort and report policies in a linear contract.

Lemma 1. *Assume the principal chooses a linear contract.*

(i) The optimal report policy (r_1^P, r_2^P) *satisfies*

$$r_1^P(\theta_1, y_1) = y_1, \ \ r_2^P(\theta_2, r_1, y_2) = y_2$$

for all $\theta_1, \theta_2 \in \Theta$*,* $y_1, y_2 \in Y$ *and* $r_1 \in \mathscr{R}$*.*

(ii) The optimal effort policy (e_1^P, e_2^P) *satisfies*

$$e_1^P(\theta_1) = e_2^P(\theta_2, r_1) = \min\{\frac{b}{2c}, 1\}$$

for all $\theta_1, \theta_2 \in \Theta$ *and* $r_1 \in \mathscr{R}$*.*

Proof. See Appendix. □

(i) in Lemma 1 implies that under a linear contract, the agent always reports true sales in every round (i.e., $r_{t,R} = y_{t,R}$ for all t and R). This is because the agent always receives b per unit of sales, so he has no incentive to manipulate the timing of the sales reporting. (ii) in lemma 1 implies that the agent exerts the same efforts in every round (i.e., $e_{t,R} = \frac{b}{2c}$ *or* 1).

Given lemma 1, the principal's problem becomes static. Hence, her problem is expressed as

$$\begin{aligned} & \max_b E[r_1^P + r_2^P - w_P(r_1^P, r_2^P, b)] \\ = & \max_b 2(1 - e^P). \end{aligned}$$

Solving this problem, we can characterize the optimal linear contract and derive the principal's profit.

Lemma 2. *Assume the principal chooses a linear contract.*

(i) The optimal linear contract $w^P(r_1, r_2, b^P)$ *satisfies*

$$b^P = \min\{\frac{1}{2}, 2c\}.$$

Then, the agent's optimal effort level is given by

$$e^P = \begin{cases} 1 & \textit{if} \quad c \geq \frac{1}{4} \\ \frac{1}{4c} & \textit{otherwise} \end{cases}$$

(ii) The principal's expected profit under the optimal linear contract Π^P *is given by*

$$\Pi^P = \min\{\frac{1}{8c}, 1 - 2c\}$$

Proof. See Appendix. □

Next, we characterize the optimal low-quota contract and derive the principal's profit. For simplicity in the following analysis, we assume that both the principal and the agent are sufficiently patient, that is, $\delta \to 1$. The following lemma describes the agent's optimal effort and report policies in low-quota contracts. [5]

Lemma 3. *Assume the principal chooses a low-quota contract and* $\delta \to 1$*.*

(i) The optimal report policy $\{r_1^L, r_2^L\}$ *satisfies*

$$r_1^L(\theta_1, y_1) = \begin{cases} 1 & \textit{if} \quad \theta_1 + y_1 \geq 1 \\ 0 & \textit{otherwise} \end{cases}$$

$$r_2^L(\theta_2, r_1, y_2) = \begin{cases} 1 & \textit{if} \quad \theta_2 = 1 \quad \textit{or} \quad (\theta_2, r_1, y_2) = (0, 0, 1) \\ 0 & \textit{otherwise} \end{cases}$$

(ii) The optimal effort policy $\{e_1^L(0), e_1^L(1), e_2^L(0,0), e_2^L(0,1), e_2^L(1,1)\}$ *is expressed as follows.*

[5] There is another type of optimal policy apart from one characterized in this lemma. These two types of policies are the same in the terms of expected level of effort and payment. That is, the optimal low-quota contract is the same under these optimal policies. Hence, we focus on only one these two. In Appendix A, we provide the characteristics of both optimal policies.

Case 1: $2c \leq b$

$$e_1^L(0) = 0, \quad e_1^L(1) = 0,$$

$$e_2^L(0,0) = 1, \quad e_2^L(0,1) = 1, \quad e_2^L(1,1) \in [0,1]$$

or

$$e_1^L(0) = 2 - \sqrt{2}, \quad e_1^L(1) = 0,$$

$$e_2^L(0,0) = 1, \quad e_2^L(0,1) = \sqrt{2} - 1, \quad e_2^L(1,1) \in [0,1].$$

Case 2: $2c > b$

$$e_1^L(0) : c^2(e_1^L(0))^4 - 4c^2(e_1^L(0))^3 + 4c^2(e_1^L(0))^2 - 8c^2 e_1^L(0) + 4bc - b^2 = 0$$

$$e_1^L(1) = 0, \quad e_2^L(0,0) = \frac{b}{2c}, \quad e_2^L(1,1) \in [0,1]$$

$$\begin{aligned} e_2^L(0,1) &= e_1^L(0) - \frac{e_1^L(0)}{2} \\ &\quad - \frac{1}{2}\sqrt{-\frac{b(b-4c)}{c^2} + e_1^L(0)(e_1^L(0)((e_1^L(0))^2 - 2) - 8)} \end{aligned}$$

Proof. See Appendix. □

(i) in lemma 3 means that the agent underreports at the end of the second round when he reports 1 in the first round and successfully sells the output in the second round. Given the optimal report policy r^L, the state transitions are shown in Figure 1.

(ii) in lemma 3 implies that $e_2^L(0,0) \geq e_1^L(0) \geq e_2^L(0,1) \geq e_1^L(1)$. The agent has strong incentives to exert effort when he does not have a carryover, especially for the second round.

Given lemma 3, the principal's problem is to choose b so that it satisfies following equations.

$$\Pi_1(0) = e_1^L(0) + e_1^L(0)\Pi_2(0,1) + (1 - e_1^L(0))\Pi_2(0,0)$$

$$\Pi_1(1) = 1 + e_1^L(1)\Pi_2(1,1) + (1 - e_1^L(1))\Pi_2(0,1)$$

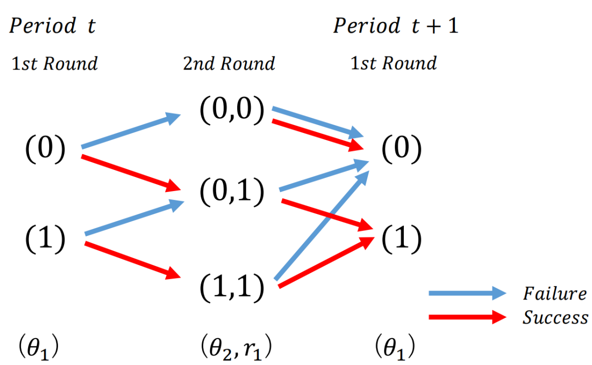


Figure 1:

$$\Pi_2(0,0) = (1-b)e_2^L(0,0) + \delta\Pi_1(0)$$

$$\Pi_2(0,1) = -b + \delta\{e_2^L(0,1)\Pi_1(1) + (1-e_2^L(0,1))\Pi_1(0)\}$$

$$\Pi_2(1,1) = 1 - b + \delta\{e_2^L(1,1)\Pi_1(1) + (1-e_2^L(1,1))\Pi_1(0)\}$$

When $\delta \to 1$, the problem can be expressed as the following maximization problem:

$$\begin{aligned}\max_b&[(1-b)\{e_1^L(0)(1-e_2^L(0,0)) + e_2^L(0,0)\} + \{(2-b)e_1^L(0)e_1^L(1)\\ &- (1-b)(1-e_1^L(0))1 - e_1^L(1))e_2^L(0,0)\} - (1-b)e_1^L(1)\{e_1^L(0)\\ &(1-e_2^L(0,0)) + e_2^L(0,0)\}e_2^L(1,1)]/[1-(1-e_1^L(0)-e_1^L(1))e_2^L(0,0)\\ &- e_1^L(1)e_2^L(1,1)].\end{aligned}$$

Denote the optimal low-quota contract and the bonus level in it as $w_L(r_1^L, r_2^L, b^L)$ and b^L, respectively. Furthermore, the principal's expected profit under $w_L(r_1^L, r_2^L, b^L)$ is denoted by Π^L.

We derive b^L and Π^L by numerical calculation using "Mathematica". The following result shows the relationship between Π^P (characterized in lemma 2) and Π^L.

Result 1. *Assume $\delta \to 1$. For all $b(\geq 0)$ and all c, $\Pi^P > \Pi^L$.*

Proof. See Appendix. $\square$

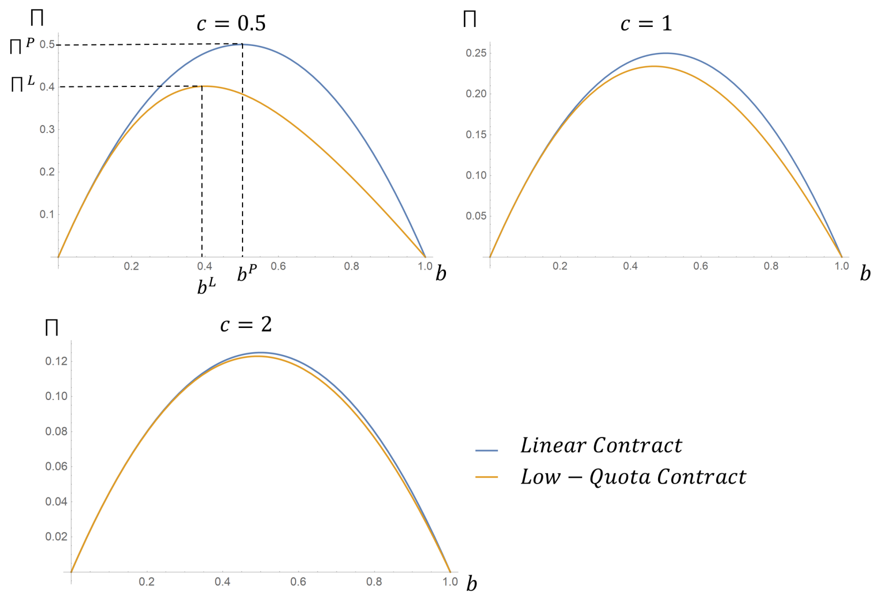


Figure 2:

Result 1 implies that linear contracts are more profitable for the principal than low-quotas contracts for all b and c. The intuition behind this result is as follows: Low-quota contracts do not provide strong incentives to the agent because it is easy for him to make quotas. Meanwhile, this type of contract induces the inefficiency in effort costs due to nonlinearity of compensation.

Linear contracts do not generate such inefficiency because the agent chooses the same effort level in every round, although they do not provide strong incentives. As a result, the principal always prefers linear contracts to low-quota contracts.

Furthermore, the difference between the principal's profit in the optimal linear contract and the optimal low-quota contract decreases as c increase (see figure 2).

3.2 Linear vs High-quota Contract

We have confirmed that the optimal linear contract is more profitable for the principal than low-quota contracts. Now, we compare linear contracts and high-quota contracts with respect to the principal's profit. We begin by considering the problem of the agent. The following lemma characterizes the optimal effort and report policies under high-quota contracts.

Lemma 4. *Assume $\delta \to 1$ and the principal chooses high-quota contracts.*

(i) The optimal report policy (r_1^H, r_2^H) satisfies

$$r_1^H(\theta_1, y_1) = \begin{cases} 1 & \textit{if} \quad \theta_1 + y_1 \geq 1 \\ 0 & \textit{otherwise} \end{cases}$$

$$r_2^H(\theta_2, r_1, y_2) = \begin{cases} 1 & \textit{if} \quad \theta_2 = 1 \quad \textit{or} \quad (\theta_2, r_1, y_2) = (0, 1, 1) \\ 0 & \textit{otherwise} \end{cases}$$

(ii) The optimal effort policy $\{e_1^H(0), e_1^H(1), e_2^H(0,0), e_2^H(0,1), e_2^H(1,1)\}$ are expressed as follows.

Case 1: $b \leq 2c$

$$e_1^H(0) = \frac{b^3 - 2b^2c + 4c(\sqrt{b^4 - 6b^3c + 8b^2c^2 + 16c^4} - 4c^2}{4(b-2c)^2c},$$

$$e_1^H(1) = b^3 - 6b^2c + 8bc^2 + 16c^3 - 4c\sqrt{b^4 - 6b^3c + 8b^2c^2 + 16d^4},$$

$$e_2^H(0,0) = e_2^H(1,1) = \frac{\sqrt{b^4 - 6b^3c + 8b^2c^2 + 16c^4} - 4c^2}{c(4c-b)},$$

$$e_2^H(0,1) = \frac{b}{2c}$$

Case 2: $2c < b \leq 4c$

$$e_1^H(0) = \frac{b^3 - b^2c - 16c^3 - 4\sqrt{2}(bc)^{\frac{3}{2}} + 16\sqrt{2}\sqrt{bc^5}}{4(b-2c)^2c},$$

$$e_1^H(1) = \frac{1}{4(b-2c)^2c}(b^3 - 6b^2c + 8bc^2 + 16c^3 + 4\sqrt{2}(bc)^{\frac{3}{2}} - 16\sqrt{2}\sqrt{bc^5}),$$

$$e_2^H(0,0) = e_2^H(1,1) = \frac{\frac{\sqrt{2}b^{\frac{3}{2}}}{\sqrt{c}} + 4c - 4\sqrt{2}\sqrt{bc}}{2b-4c},$$

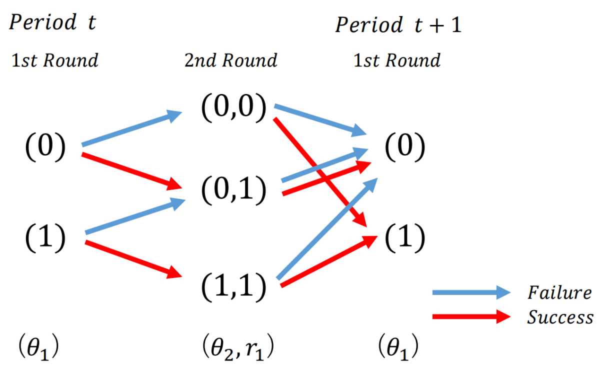


Figure 3:

$$e_2^H(0,1) = 1$$

Case 3: $b > 4c$

$$e_1^H(0) = 1, e_1^H(1) \in [0,1],$$

$$e_2^H(0,0) \in [0,1], e_2^H(0,1) = 1, e_2^H(1,1) \in [0,1].$$

Proof. See Appendix. □

(i) in lemma 4 implies that there are two cases in which the agent carries over sales (see figure 3). The first case is that the agent can make quotas without reporting the second round's sales. In the second case, the agent cannot make quotas even if he reports the sales for that round. Given the optimal report policy (r^H), the state transitions are shown in Figure 3. (ii) in lemma 4 implies that $e_2^H(0,1) \geq e_1^H(1) \geq e_2^H(0,0) \geq e_1^H(0)$. Figure plots effort levels of each state.

Given lemma 4, the principal's problem is to choose b so that it satisfies following equations:

$$\Pi_1(0) = e_1^H(0) + e_1^H(0)\Pi_2(0,1) + (1 - e_1^H(0))\Pi_2(0,0)$$

$$\Pi_1(1) = 1 + e_1^H(1)\Pi_2(1,1) + (1 - e_1^H(1))\Pi_2(0,1)$$

$$\Pi_2(0,0) = \delta\{(1 - e_2^H(0,1))\Pi_1(0) + e_2^H(0,1)\Pi_1(1)$$

$$\Pi_2(0,1) = (1-b)e_2^H(0,1) + \delta\{\Pi_1(0)\}$$

$$\Pi_2(1,1) = 1 - b + \delta\{(1 - e_2^H(1,1))\Pi_1(0) + e_2^H(1,1)\Pi_1(1)\}$$

When $\delta \to 1$, her problem can be expressed as the following maximization problem.

$$\begin{aligned}
&\max_b[(1-b)\{e_1^H(0)(1+e_2^H(0,1) - be_2^H(0,1))+ \\
&\quad (1-e_1^H(0))e_2^H(0,0)(1+(1-b)e_1^H(1)(1-e_2^H(0,1)) + (1-b)e_2^H(0,1)+ \\
&\quad e_1^H(0)e_1^H(1)((b-1)e_2^H(0,1))e_2^H(1,1) - 1) \\
&\quad /(1+e_2^H(0,0) - e_1^H(0)e_2^H(0,0) - e_1^H(1)e_2^H(1,1)).
\end{aligned}$$

Denote the optimal high-quota contract and the bonus level in it as $w_H(r_1^H, r_2^H, b^H)$ and b^H, respectively. Furthermore, the principal's expected profit under $w^H(r_1^H, r_2^H, b^H)$ is denoted by Π^H.

Similar to result 2, we derive b^H and Π^H using numerical calculation. The following result shows the relationship between Π^P and Π^H.

Result 2. *Assume $\delta \to 1$. If $c < (>)1.47$, then $\Pi_{HighQ} > (<)\Pi_{Linear}$.*

Result 2 implies that when c is low (high), the high-quota contract (the linear contract) is more profitable for the principal than the linear contract (the high-quota contract). Figure 4 shows the comparison between the optimal linear contract and the optimal high-quota contract with respect to the principal's profit. The logic behind this result is as follows. The high-quota contract provides stronger incentives than the linear contract, but induce the inefficiency in effort cost. Furthermore, an increase in c increases this inefficiency. To see this, we compare these two contracts with respect to the equilibrium level of expected total effort and expected payment. Figure 5 and Figure 6 show the comparison with respect to expected total effort per period and expected payment per period, respectively. These two figures imply that when c is high, the optimal linear contract can incentivize the agent more efficiently than the optimal high-quota contract. This is because the inefficiency in high-quota contracts increases as c increases. That is, the strong incentives in high-quota

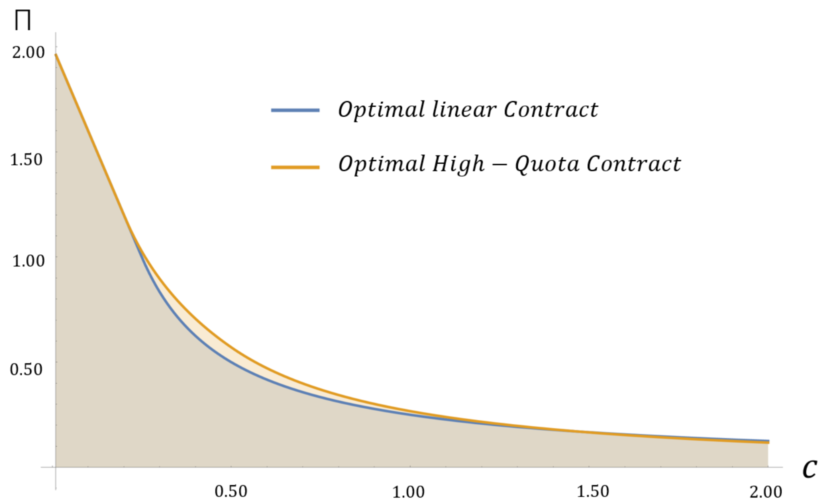


Figure 4:

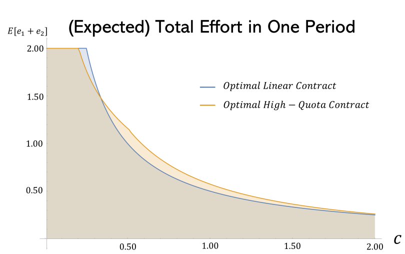


Figure 5:

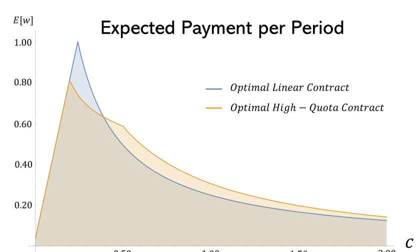


Figure 6:

contracts becomes costlier to provide. Consequence, the principal prefers the optimal high-quota contract to the optimal linear contract if c is high.

4 The Effect of Gaming

In this section, we examine the effect of gaming. To do this, we modify the basic model to a new model wherein the agent cannot carryover the sales to the next round. In the new model, the agent has to report the true sales in each round, that is, $r_{t,1} = y_{t,1}$ and $r_{t,2} = y_{t,2}$ for all t. We begin by deriving the agent's optimal effort policy under low-quota contracts in this new situation (non-gaming situation).

Lemma 5. *Assume the agent cannot carry over the sales, and the principal chooses low-quota contracts.*
The optimal effort policy $e^{L(NG)}$ *satisfies*

$$\{e_1^{L(NG)}(0), e_2^{L(NG)}(1), e_2^{L(NG)}(0)\} = \begin{cases} \{\frac{b(b+4c)}{8c^2}, 0, \frac{b}{2c}\} & if \quad \frac{b}{2c} \leq 1 \\ \{0, 0, 1\} & otherwise. \end{cases}$$

Proof. See Appendix. □

The principal's problem becomes static because the agent cannot manipulate the timing of sales reports. In other words, the problem is to choose b to maximize the expected profit for a period. Given lemma 5, it is expressed as

$$\begin{aligned} &\max_b E[y_1 - y_2 - w^{L(NG)}(y_1, y_2, b^{L(NG)})|e^{L(NG)}] \\ &= \max_b \ (1-b)\{e_1^{L(NG)} + (1 - e_1^{L(NG)})e_2^{L(NG)}\} \end{aligned}$$

We denote $\Pi^{L(NG)}$ as the principal's profit per period under the optimal low-quota contract in a non-gaming situation. The following result shows the comparison between Π^L and $\Pi^{L(NG)}$ We denote the optimal high-quota contract and the bonus level in it as $w_H(r_1^H, r_2^H, b^H)$ and b^H, respectively. Furthermore, the principal's expected profit under $w_H(r_1^H, r_2^H, b^H)$ is denoted by Π^H.

Similar to result 2, we derive b^H and Π^H using numerical calculation. The following result shows the relationship between Π^P and Π^H.

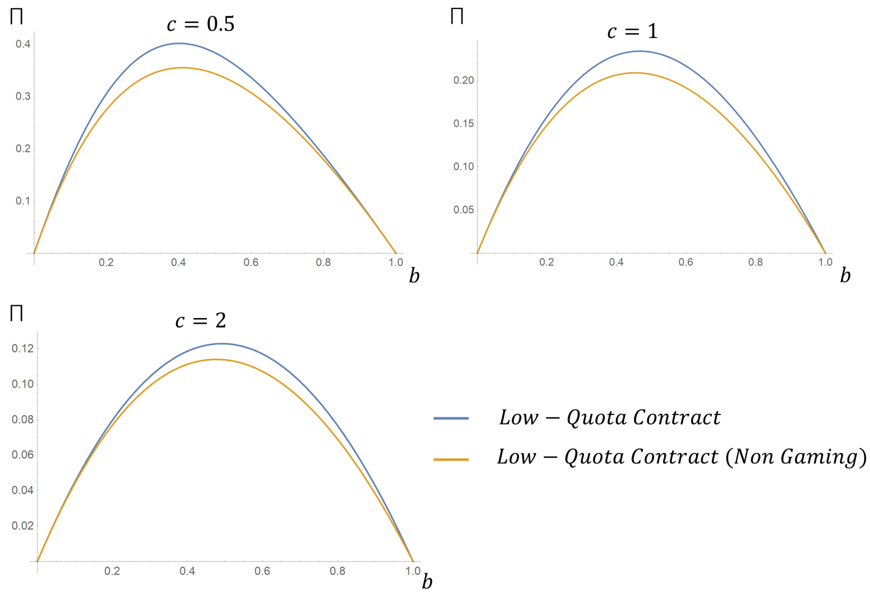


Figure 7:

Result 3. *Assume* $\delta \to 1$. *For all* $c(>0)$ *and* $b(>0)$, $\Pi^L > \Pi^{L(NG)}$.

Proof. See Appendix. □

Result 3 implies that the agent's gaming activity is profitable for the principal in a low-quota contract. The logic behind this result is as follows: In the second round, when the quota has been achieved at the end of the first round, the agent has no incentive to exert efforts in the non gaming situation, but he does have incentives in the gaming situation. It follows that the inefficiency in effort costs is large in the non-gaming situation. Consequently, low-quota contracts incentivize the agent to be more efficient in the gaming situation.

Next, we examine the effect of gaming on high-quota contracts. The following lemma characterizes the agent's optimal effort policy in a non-gaming situation.

Lemma 6. *Assume that the agent cannot carry over the sales, and the principal*

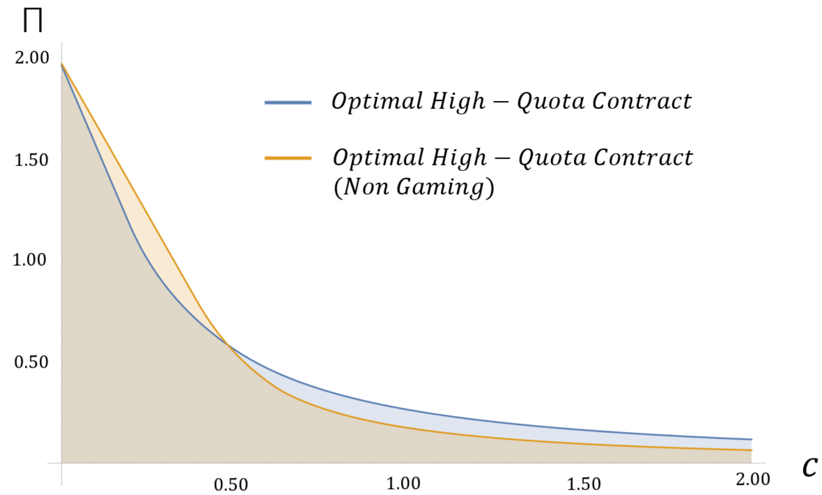


Figure 8:

chooses high-quota contracts. Then, the agent's effort policy $e^{H(NG)}$ *is*

$$\{e_1^{H(NG)}(0), e_2^{H(NG)}(1), e_2^{H(NG)}(0)\} = \begin{cases} \{\frac{b^2}{8c^2}, 0, \frac{b}{2c}\} & if \quad \frac{b}{2c} \leq 1 \\ \{\frac{b-c}{2c}, 0, 1\} & if \quad \frac{b-c}{2c} < 1 \leq \frac{b}{2c}. \\ \{1, 0, 1\} & if \quad 1 < \frac{b-c}{2c}. \end{cases}$$

Proof. See Appendix. □

Given lemma 6, the principal's problem is expressed as

$$\begin{aligned} &\max_b E[y_1 - y_2 - w^{H(NG)}(y_1, y_2, b^{H(NG)})|e^{H(NG)}] \\ &= \max_b \; e_1^{H(NG)} + (2-b)e_1^{H(NG)} e_2^{H(NG)}(1) \end{aligned}$$

We denote $\Pi^{H(NG)}$ as the principal's profit per period in the optimal high-quota contract under a non-gaming situation. The following result shows that the relationship between Π^H and $\Pi^{H(NG)}$

Result 4. *Assume* $\delta \to 1$. *If* $c > (<) 0.49$, *then* $\Pi^H > (<) \Pi^{H(NG)}$.

Proof. See Appendix. □

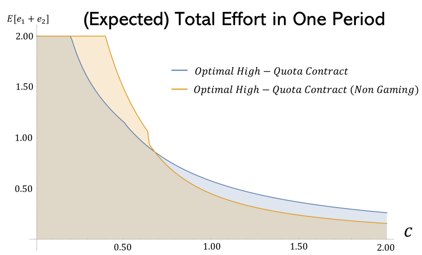


Figure 9:

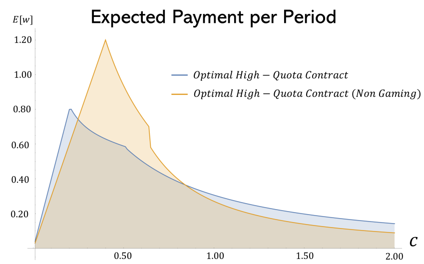


Figure 10:

Result 4 implies that the agent's gaming activity is profitable for the principal when c is high. Figure 8 plots the principal's profit under the optimal high-quota contract in the gaming and non-gaming situations. This shows that, as c increases, the optimal profit decrease more sharply in the non-gaming situation than the gaming situation. To analyze this further, we compare these two situations with respect to expected efforts and expected payment.

The relationship between these two situation is similar to that between linear and high-quota contracts in the gaming situation. High-quota contracts can provide stronger incentives to the agent in the non-gaming situation because if the quota is not achieved, all sales reports for that period will be useless, as a result, the agent will exerts efforts to make his quota (Figure 9). However, these strong incentives induce inefficient effort cost. Becase an increase in c increases this inefficiency, incentives becomes costlier to provide (Figure 10). Consequently, gaming activity is profitable for the principal when c is high.

5 Conclusion

This paper considered an infinite-period moral hazard model in which the agent can underreport the sales in order to increase his wage. We first showed that when the effort cost is relatively low, the principal prefers quota-based contracts to linear contracts. This result implies that quota-based contract can be more profitable than linear contracts even if the agent can manipulate the report timing.

In addition, we investigate the effect of timing gaming by considering the situation where the agent does not allow to manipulate the timing of sales reports. Then, we showed that the agent's gaming activities increases the principal's profit in quota-based contracts when the effort cost is relatively high.

However, our model needs improvement because we impose several strong assumptions on the model. For example, we assume that wage contracts do not change in the infinite repeated relationship and depend on the history of sales, i.e., wage depends only on the current period's sales. Therefore, these assumptions need to be relaxed in order to obtain more general results.

Appendix A

In appendix A, we give proofs of lemma 1-6.

Proof of lemma 1

The optimal report policy:

Since linear contracts provide linear incentives to the agent, there is no incentive to carry over sales from the first round to the second round. Therefore, he always reports honestly in linear contract. It means that $r_1^P(\theta_1, y_1) = y_1$, $r_2^P(\theta_2, r_1, y_2) = y_2$ for all $\theta_1, \theta_2 \in \Theta$, $y_1, y_2 \in Y$ and $r_1 \in \mathscr{R}$.

The optimal effort policy:

Given the optimal report rule (r_1^P, r_2^P), the agent's problem of choosing the level of effort in each round becomes stationary. Specifically, his problem is expressed as

$$\max_{e \in E} eb - ce^2.$$

Solving above the agent's problem, his optimal effort in both rounds (e_1^P, e_2^P) are given by

$$e_1^P = e_2^P = \min\{1, \frac{b}{2c}\}$$

Proof of lemma 2

First, we derive the optimal bonus in linear contracts (denoted by b^P). Given the agent's optimal policy for effort level and report ((e_1^P, e_2^P) and (r_1^P, r_2^P)), the principal's profit maximization problem regarding b in every round is expressed by

$$\max_b 2(1-b) \times \min\{1, \frac{b}{2c}\}$$

Solving this problem, we obtain

$$b^P = \min\{\frac{1}{2}, 2c\}.$$

Second, we calculate the principal's revenue for each round in the optimal linear contract (denoted by Π^P). Given the optimal effort policy (e_1^P, e_2^P) and bonus b^P, Π^P can be expressed as

$$2(1-b^P) \times \min\{1, \frac{b^P}{2c}\}.$$

We can derive Π^P as follows.

Case 1 ($c > \frac{1}{4}$)

Note that $b^P = \frac{1}{2}$ since $\frac{1}{2} < 2c$. Then,

$$\Pi^P = 2(1-\frac{1}{2}) \times \frac{1}{4c} = \frac{1}{4c}.$$

Case 2 ($c \leq \frac{1}{4}$)

Note that $b^P = 2c$ since $\frac{1}{2} \geq 2c$. Then,

$$\Pi^P = 2(1-2c) \times 1 = 2(1-2c).$$

As a result, we obtain

$$\Pi^P = \max\{\frac{1}{4c}, 2(1-2c)\}.$$

Proof of lemma 3

The optimal report policy:

First, note that the agent cannot overreport the sales (i.e., $r_1 \leq \theta_1 + y_1$, $r_2 \leq \theta_2 + y_2$). He must report 0 when $r_1 = y_1 = 0$ or $r_2 = y_2 = 0$. Therefore, $r_1^L(0,0) = r_2^L(0,0,0) = r_2^L(0,1,0) = 0$.

Second, note that he can carry over the sales for only one period. Then, he must report 1 if he has a carryover from the previous round (i.e., $\theta_{t,R} =$

1). Therefore, $r_1^L(1,0) = r_1^L(1,1) = r_2^L(1,0,0) = r_2^L(1,1,0) = r_2^L(1,0,1) = r_2^L(1,1,1) = 1$.

Next, we consider the optimal report at the state $(0,0,1)$. At this state, underreporting decreases his total payoff since $V(1) - V(0) < b$. Hence, it is optimal for him to report honestly, that is, $r_2^L(0,0,1) = 1$.

Then, we consider the optimal report at the sate $(0,1,1)$. At this state, the agent has already achieved his quota in the first round. Then, truth reporting decreases his total profit since $V(0) < V(1)$. Therefore, it is optimal for him to carry over the sales at this state, that is, $r_2^L(0,1,1) = 0$.

Finally, we consider the optimal report at the sate $(0,1)$. To derive $r_1^L(0,1)$, we compare the agent's total payoff in both $r(0,1) = 0$ and $r(0,1) = 1$ cases, where he reports sales optimally at other states and exerts optimal efforts in every states. Note that the agent has to report the sales either in the first or second round. Then, whether he delays his sales report or not does not change his profit in this case. Therefore, $r_1^L(0,1) \in \{0,1\}$.

As a result, the optimal report policies are the following two types.

Type 1

$$r_1^L(\theta_1, y_1) = \begin{cases} 1 & \text{if} \quad \theta_1 + y_1 \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$r_2^L(\theta_1, r_1, y_2) = \begin{cases} 1 & \text{if} \quad \theta_1 = 1 \ \ \textit{or} \ \ (\theta_2, r_1, y_2) = (0,0,1) \\ 0 & \text{otherwise} \end{cases}$$

Type 2

$$r_1^L(\theta_1, y_1) = \begin{cases} 1 & \text{if} \quad (\theta_1, y_1) = (0,1) \text{ or } (1,1) \\ 0 & \text{otherwise} \end{cases}$$

$$r_2^L(\theta_1, r_1, y_2) = \begin{cases} 1 & \text{if} \quad \theta_1 = 1 \ \ \textit{or} \ \ (\theta_2, r_1, y_2) = (0,0,1) \\ 0 & \text{otherwise} \end{cases}$$

The optimal effort policy:

To derive the agent's optimal effort policy in low quota contracts, we solve the Bellman equation characterized in our model.

Type 1

First, we derive an optimal effort policy when the agent chooses *type 1 report policy*. Under this report policy, the state $(1,0)$ is unreachable. Therefore, it is enough to derive the optimal effort level in reachable states (i.e., {(0), (1), (0,0), (0,1),(1,1)}). We derive $\{e_1^L(0), e_1^L(1), e_2^L(0,0), e_2^L(0,1), e_2^L(1,1)\}$ by solving following equations.

$$V_1(0) = -c(e_1^L(0))^2 + e_1^L(0)V_2(0,1) + (1 - e_1^L(0))V_2(0,0)$$

$$V_1(1) = -c(e_1^L(1))^2 + e_1^L(1)V_2(1,1) + (1 - e_1^L(1))V_2(0,1)$$

$$V_2(0,0) = be_2^L(0,0) - c(e_2^L(0,0))^2 + \delta V_1(0)$$

$$V_2(0,1) = b - c(e_2^L(0,1))^2 + \delta\{e_2^L(0,1)V_1(1) + (1 - e_2^L(0,1))V_1(0)\}$$

$$V_2(1,1) = b - c(e_2^L(1,1))^2 + \delta\{e_2^L(1,1)V_1(1) + (1 - e_2^L(1,1))V_1(0)\}$$

To solve these equations, we first consider $e^L(1)$. In this state, the agent has no incentive to exert efforts because selling the output in that round is not profitable. That is, $e_1^L(1) = 0$. Furthermore, $e_1^L(1) = 0$ means that the state $\{(1,1)\}$ is not reachable, i.e., $e^L(1,1) \in [0,1]$.

Next, we consider $e^L(0,0)$. Note that the agent's profit after the next period does not depend on whether he can sell the output or not, in that round. Therefore, his problem at the state $\{(0,0)\}$ is

$$\max_{e_2(0,0)\in E} be_2(0,0) - c(e_2(0,0))^2$$

By solving above the problem, we obtain $e_2^L(0,0) = \min\{1, \frac{b}{2c}\}$.

In the following analysis, we divide the cases into case 1 ($2c \leq b$) and case 2 ($2c > b$) in order to derive $\{e_1^L(0), e_2^L(0,1)\}$.

Case 1 ($2c \leq b$)

Note that $e_1^L(1) = 0$, $e_2^L(0,0) = 1$. Then, $\{e_1^L(0), e_2^L(0,1)\}$ must satisfy following equations.

$$V_1(0) = -c(e_1^L(0))^2 + e_1^L(0)V_2(0,1) + (1 - e_1^L(0))V_2(0,0)$$

$$V_1(1) = V_2(0,1)$$

$$V_2(0,0) = b - c + \delta V_1(0)$$

$$V_2(0,1) = b - c(e_2^L(0,1))^2 + \delta\{e_2^L(0,1)V_1(1) + (1 - e_2^L(0,1))V_1(0)\}$$

Solving the above equations for $e_1^L(0)$ and $e_2^L(0,1)$, we obtain

$$\{e_1^L(0), e_2^L(0,1)\} = \{0,1\} \text{ } or \text{ } \{2-\sqrt{2}, \sqrt{2}-1\}.$$

Case 2 ($2c > b$)

Note that $e_1^L(1) = 0$, $e_2^L(0,0) = \frac{b}{2c}$. $\{e_1^L(0), e_2^L(0,1)\}$ must satisfy following equations.

$$V_1(0) = -c(e_1^L(0))^2 + e_1^L(0)V_2(0,1) + (1 - e_1^L(0))V_2(0,0)$$

$$V_1(1) = V_2(0,1)$$

$$V_2(0,0) = \frac{b^2}{2c} - \frac{b^2}{4c} + \delta V_1(0)$$

$$V_2(0,1) = b - c(e_2^L(0,1))^2 + \delta\{e_2^L(0,1)V_1(1) + (1 - e_2^L(0,1))V_1(0)\}$$

By solving above equations under the constraint of ($2c > b$), we can obtain the following results.

$$e_1^L(0) : c^2(e_1^L(0))^4 - 4c^2(e_1^L(0))^3 + 4c^2(e_1^L(0))^2 - 8c^2e_1^L(0) + 4bc - b^2 = 0$$

$$e_1^L(1) = 0, \quad e_2^L(0,0) = \frac{b}{2c}, \quad e_2^L(1,1) \in [0,1]$$

$$\begin{aligned} e_2^L(0,1) &= e_1^L(0) - \frac{e_1^L(0)}{2} \\ &\quad -\frac{1}{2}\sqrt{-\frac{b(b-4c)}{c^2} + e_1^L(0)(e_1^L(0)((e_1^L(0))^2 - 2) - 8)} \end{aligned}$$

Type 2

Second, we derive an optimal effort policy when the agent chooses *type 2 report policy*. Under this report policy, the state $(0,1)$ is unreachable. Therefore, it is enough to derive the optimal effort level in reachable states (i.e., {(0), (1), (0,0), (1,0),(1,1)}). We derive $\{e_1^L(0), e_1^L(1), e_2^L(0,0), e_2^L(1,0), e_2^L(1,1)\}$ by solving following equations.

$$V_1(0) = -c(e_1^L(0))^2 + e_1^L(0)V_2(1,0) + (1 - e_1^L(0))V_2(0,0)$$

$$V_1(1) = -c(e_1^L(1))^2 + e_1^L(1)V_2(1,1) + (1 - e_1^L(1))V_2(1,0)$$

$$V_2(0,0) = be_2^L(0,0) - c(e_2^L(0,0))^2 + \delta V_1(0)$$

$$V_2(1,0) = b - c(e_2^L(1,0))^2 + \delta\{e_2^L(1,0)V_1(1) + (1 - e_2^L(1,0))V_1(0)\}$$

$$V_2(1,1) = b - c(e_2^L(1,1))^2 + \delta\{e_2^L(1,1)V_1(1) + (1 - e_2^L(1,1))V_1(0)\}$$

Similar to Type 1, the following optimal effort policy is obtained.

Case 1 ($2c \leq b$)

$$e_1^L(0) = 0, e_1^L(1) = 0, e_2^L(0,0) = 1, e_2^L(1,0) = 1, e_2^L(1,1) \in [0,1]$$

or

$$e_1^L(0) = 2 - \sqrt{2}, e_1^L(1) = 0, e_2^L(0,0) = 1, e_2^L(1,0) = \sqrt{2} - 1, e_2^L(1,1) \in [0,1].$$

Case 2 ($2c > b$)

$$e_1^L(0) : c^2(e_1^L(0))^4 - 4c^2(e_1^L(0))^3 + 4c^2(e_1^L(0))^2 - 8c^2 e_1^L(0) + 4bc - b^2 = 0$$

$$e_1^L(1) = 0, \quad e_2^L(0,0) = \frac{b}{2c}, \quad e_2^L(1,1) \in [0,1]$$

$$\begin{aligned} e_2^L(1,0) &= e_1^L(0) - \frac{e_1^L(0)}{2} \\ &\quad -\frac{1}{2}\sqrt{-\frac{b(b-4c)}{c^2} + e_1^L(0)(e_1^L(0)((e_1^L(0))^2 - 2) - 8)} \end{aligned}$$

Proof of lemma 4

The optimal report rule:

Note that the agent cannot overreport the sales (i.e., $r_1 \leq \theta_1 + y_1$, $r_2 \leq \theta_2 + y_2$). He must report 0 when $r_1 = y_1 = 0$ or $r_2 = y_2 = 0$. Therefore, $r_1^H(0,0) = r_2^H(0,0,0) = r_2^H(0,1,0) = 0$.

Note that he can carry over the sales for only one period. Then, he must report 1 if he has a carryover (i.e., $\theta_{t,R} = 1$). Therefore, $r_1^H(1,0) = r_1^H(1,1) = r_2^L(1,0,0) = r_2^H(1,1,0) = r_2^H(1,0,1) = r_2^H(1,1,1) = 1$.

Then, we consider the optimal report at the state $(0,1)$. Note that if the agent underreports his sales for the first round, he will never receive the bonus.

Furthermore, he has to report the carryover in the second round of that period, even though he cannot make his quota. It follows that carrying over the first round's sales does not increase the agent's profit. Hence, the optimal report at $(0, 1)$ is to report his true sales, that is, $r_1^H(0, 1) = 1$.

Next, we consider $r_2^H(0, 0, 1)$. At this state, truth reporting decrease the agent's profit since $V_1(0) < V_1(1)$. Therefore, it is optimal for him to underreport the sales, i.e., $r_2^H(0, 0, 1) = 0$

Finally, we consider $r_2^H(0, 1, 1)$. Note that $V_1(0) - V_1(1) < b$. Then, truth reporting is optimal for the agent. That is, $r_2^H(0, 1, 1) = 1$.

As a result, we obtain

$$r_1^H(\theta_1, y_1) = \begin{cases} 1 & \text{if} \quad \theta_1 + y_1 \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$r_2^H(\theta_2, r_1, y_2) = \begin{cases} 1 & \text{if} \quad \theta_2 = 1 \ \ \mathit{or} \ \ (\theta_2, r_1, y_2) = (0, 1, 1) \\ 0 & \text{otherwise} \end{cases}$$

The optimal effort policy:

To derive the agent's optimal effort policy under a high-quota contract, we solve the Bellman equation. When the agent follows the optimal report policy $\{r_1^H, r_2^H\}$, then the state $(1, 0)$ is unreachable. Therefore, it is enough to derive the optimal effort level at reachable states (i.e., {(0), (1), (0,0), (0,1),(1,1)}). Specifically, we can derive $\{e_1^H(0), e_1^H(1), e_2^H(0, 0), e_2^H(0, 1), e_2^H(1, 1)\}$ by solving following equations.

$$V_1(0) = -c(e_1^H(0))^2 + e_1^H(0)V_2(0, 1) + (1 - e_1^H(0))V_2(0, 0)$$

$$V_1(1) = -c(e_1^H(1))^2 + e_1^H(1)V_2(1, 1) + (1 - e_1^H(1))V_2(0, 1)$$

$$\begin{aligned} V_2(0, 0) =& be_2^H(0, 0) - c(e_2^H(0, 0))^2 \\ &+ \delta\{e_2^H(0, 0)V_1(1) + (1 - e_2^H(0, 0))V_1(0)\} \end{aligned}$$

$$V_2(0, 1) = b - c(e_2^H(0, 1))^2 + \delta e_2^H(0, 1)V_1(0)$$

$$V_2(1, 1) = b - c(e_2^H(1, 1))^2 + \delta\{e_2^H(1, 1)V_1(1) + (1 - e_2^H(1, 1))V_1(0)\}$$

To solve these equations, we first consider $e_2^H(0,1)$. Given the optimal report policy, $(0,1)$ transitions to (0) regardless of sales in that round. It follows that the effort level at $(0,1)$ (i.e., $e_2(0,1)$) only affects his current period's profit. Then, his effort choise problem at $(0,1)$ is

$$\max_{e_2(0,1)\in E} be_2(0,1) - c(e_2(0,1))^2.$$

Solving this problem, we obtain $e_2^H(0,1) = \min\{1, \frac{b}{2c}\}$. Next, we consider the case where $2c < b$.

Case 1 ($b \leq 2c$)

Note that $e_2^H(0,1) = \frac{b}{2c}$. Then, $\{e_1^H(0), e_1^H(1), e^H(0,0), e^H(1,1)\}$ must satisfy following equations.

$$V_1(0) = -c(e_1^H(0))^2 + e_1^H(0)V_2(0,1) + (1 - e_1^H(0))V_2(0,0)$$

$$V_1(1) = -c(e_1^H(1))^2 + e_1^H(1)V_2(1,1) + (1 - e_1^H(1))V_2(0,1)$$

$$\begin{aligned} V_2(0,0) =& be_2^H(0,0) - c(e_2^H(0,0))^2 \\ &+ \delta\{e_2^H(0,0)V_1(1) + (1 - e_2^H(0,0))V_1(0)\} \end{aligned}$$

$$V_2(0,1) = -\frac{b^2}{4c} + b + \frac{b\delta}{2c}V_1(0)$$

$$V_2(1,1) = b - c(e_2^H(1,1))^2 + \delta\{e_2^H(1,1)V_1(1) + (1 - e_2^H(1,1))V_1(0)\}$$

Solving these equations, we obtain

$$e_1^H(0) = \frac{b^3 - 2b^2c + 4c(\sqrt{b^4 - 6b^3c + 8b^2c^2 + 16c^4} - 4c^2}{4(b-2c)^2c},$$

$$e_1^H(1) = b^3 - 6b^2c + 8bc^2 + 16c^3 - 4c\sqrt{b^4 - 6b^3c + 8b^2c^2 + 16d^4},$$

$$e_2^H(0,0) = e_2^H(1,1) = \frac{\sqrt{b^4 - 6b^3c + 8b^2c^2 + 16c^4} - 4c^2}{c(4c-b)},$$

$$e_2^H(0,1) = \frac{b}{2c}$$

Next, we consider the case where $2c > b$, (i.e., $e_2^H(0,1) = 1$). In this case, the agent's optimal effort at (0) is given by

$$\min\{\frac{b^3 - b^2c - 16c^3 - 4\sqrt{2}(bc)^{\frac{3}{2}} + 16\sqrt{2}\sqrt{bc^5}}{4(b-2c)^2c}, 1\}.$$

In the following analysis, we divide the cases into case 2 ($2c \le b < 4c$) and case 3 ($4c < b$).

Case 2 ($2c \le b < 4c$)

Note that $e_1^H(0) = \frac{b^3 - b^2c - 16c^3 - 4\sqrt{2}(bc)^{\frac{3}{2}} + 16\sqrt{2}\sqrt{bc^5}}{4(b-2c)^2c}$ and $e_2^H(0,1) = 1$. Substituting these into the above equations, we obtain

$$e_1^H(0) = \frac{b^3 - b^2c - 16c^3 - 4\sqrt{2}(bc)^{\frac{3}{2}} + 16\sqrt{2}\sqrt{bc^5}}{4(b-2c)^2c},$$

$$e_1^H(1) = \frac{1}{4(b-2c)^2c}(b^3 - 6b^2c + 8bc^2 + 16c^3 + 4\sqrt{2}(bc)^{\frac{3}{2}} - 16\sqrt{2}\sqrt{bc^5}),$$

$$e_2^H(0,0) = e_2^H(1,1) = \frac{\frac{\sqrt{2}b^{\frac{3}{2}}}{\sqrt{c}} + 4c - 4\sqrt{2}\sqrt{bc}}{2b-4c},$$

$$e_2^H(0,1) = 1$$

Case 3 ($4c < b$)

Note that $e_1^H(0) = e_2^H(0,1) = 1$. Then, $e_1^H(1), e_2^H(0,0), e_2^H(1,1)$ must satisfy following equations.

$$V_1(0) = -c + V_2(0,1)$$

$$V_1(1) = -c(e_1^H(1))^2 + e_1^H(1)V_2(1,1) + (1 - e_1^H(1))V_2(0,1)$$

$$\begin{aligned} V_2(0,0) =& be_2^H(0,0) - c(e_2^H(0,0))^2 \\ &+ \delta\{e_2^H(0,0)V_1(1) + (1 - e_2^H(0,0))V_1(0)\} \end{aligned}$$

$$V_2(0,1) = b - c + \delta V_1(0)$$

$$V_2(1,1) = b - c(e_2^H(1,1))^2 + \delta\{e_2^H(1,1)V_1(1) + (1 - e_2^H(1,1))V_1(0)\}$$

Solving these equations, we obtain

$$e_1^H(0) = 1, e_1^H(1) \in [0,1],$$

$$e_2^H(0,0) = 1, e_2^H(0,1) \in [0,1], e_2^H(1,1) \in [0,1].$$

Proof of lemma 5

We derive the optimal effort policy under a low-quota contract in non-gaming situation. Using backward induction, we first analyze the optimal effort policy in the second round.

The problem for the agent in the second round is to choose the effort level for each sales (0 or 1) in the first round. When the sales in the first round is 1, the agent has no incentive to effort in the second round because the effort does not increase the wage, but does increase effort costs. That is, $e_2^{L(NG)}(1) = 0$.

We consider the case where the sales in the first round is 0. The agent's problem is expressed as

$$\max_{e_2(0)} be_2(0) - c(e_2(0))^2$$

Solving this problem, we obtain $e_2^{L(NG)}(0) = \min\{1, \frac{b}{2c}\}$

Next, we consider the first round. Given the agent's effort choice in the second round, the effort choise problem in the first round is

$$\max_{e_1} e_1[b - ce_1^2 - c(e_2(1))^2] + (1 - e_1)[-ce_1^2 + be_2(0) - c(e_2(0))^2].$$

To solve this problem, we divide it into two cases.

Case 1 : $2c > b$

Since $e_2^{L(NG)}(0) = \frac{b}{2c}$ and $e_2^{L(NG)}(1) = 0$, then the problem is given by

$$\max_{e_1} be_1 + (1 - e_1)\frac{b^2}{2c} - c(e_1)^2.$$

Solving the above, we obtain $e_1^{L(NG)} = \frac{b(b+4c)}{8c^2}$.

Case 2 : $2c \leq b$

Since $e_2^{L(NG)}(0) = 1$, the agent makes his quota with probability 1 regardless of sales in the first round. He has no incentive to exert effort in the first round, that is, $e_1^{L(NG)} = 0$. As a result, we obtain

$$\{e_1^{L(NG)}(0), e_2^{L(NG)}(1), e_2^{L(NG)}(0)\} = \begin{cases} \{\frac{b(b+4c)}{8c^2}, 0, \frac{b}{2c}\} & if \ \ \frac{b}{2c} \leq 1 \\ \{0, 0, 1\} & otherwise. \end{cases}$$

Proof of lemma 6

Similar to lemma 5, we begin with analyzing the second round's effort policy. If the agent sells the output in the first round, then his problem in the second round is

$$\max_{e_2(1)} be_2(1) - c(e_2(1))^2$$

Solving the above, we obtain $e_2^{H(NG)}(1) = \min\{1, \frac{b}{2c}\}$.

On the other hand, if the agent cannot sell the output in the first round, he has no incentive to exert effort because he cannot make quota in that period even if he sells the output in the second round. That is, $e_2^{H(NG)}(0) = 0$.

Next, we analyze the optimal effort policy for the first round. This problem is given by

$$\max_{e_1} e_1[-ce_1^2 + be_2(1) - c(e_2(1))^2] + (1 - e_1)[-ce_1^2 - c(e_2(0))^2]$$

We divide it into following two cases.

Case 1 : $b \leq 2c$

Since $e_2^{H(NG)}(1) = \frac{b}{2c}$, the agent's problem for choosing the first round's effort is

$$\max_{e_1} e_1[-ce_1^2 + be_2(1) - c(e_2(1))^2] + (1 - e_1)[-ce_1^2 - c(e_2(0))^2]$$

Solving the above, we obtain $e_1^{H(NG)} = \frac{b^2}{8c^2}$.

Case 2 : $b > 2c$

Since $e_2^{H(NG)}(1) = 1$, the agent's problem for choosing the first round's effort is

$$\max_{e_1} (b - c)e_1 - c(e_1)^2$$

Solving the above, we obtain $e_1^{H(NG)} = \min\{1, \frac{b-c}{2c}\}$.

As a result, we obtain

$$\{e_1^{H(NG)}(0), e_2^{H(NG)}(1), e_2^{H(NG)}(0)\} = \begin{cases} \{\frac{b^2}{8c^2}, 0, \frac{b}{2c}\} & if \;\; \frac{b}{2c} \leq 1 \\ \{\frac{b-c}{2c}, 0, 1\} & if \;\; \frac{b-c}{2c} < 1 \leq \frac{b}{2c}. \\ \{1, 0, 1\} & if \;\; 1 < \frac{b-c}{2c}. \end{cases}$$

Appendix B

In appendix B, we will give "Mathematica's codes" that derive result 1-6.

Linear Contract

The agent's optimal effort

```
LinearOptimalEffort[b_, c_] := Min[1, b / (2 * c)]
```
最小

The optimal bonus (the agent's optimal effort is given)

```
LinearOptimalBonus[c_] := ArgMax[{2 * (1 - b) * LinearOptimalEffort[b, c], 0 ≤ b}, b]
```
関数が最大となる変数の位置

The principal's optimal profit

```
LinearOptimalΠ[c_] :=
 2 * (1 - LinearOptimalBonus[c]) * LinearOptimalEffort[LinearOptimalBonus[c], c]
```

The total effort in one period

```
LinearTotalOptimalEffort[c_] := 2 * LinearOptimalEffort[LinearOptimalBonus[c], c]
```

The expected payment per period

```
LinearExpectedPayment[c_] :=
 2 * LinearOptimalEffort[LinearOptimalBonus[c], c] * LinearOptimalBonus[c]
```

High-Quota Contract

The principal profit

```
HighQuotaΠ[e10_, e11_, e200_, e201_, e211_, c_, b_] :=
  (e10 (1 + e201 - b e201) - (-1 + e10) e200 (1 + (-1 + b) e11 (-1 + e201) + e201 - b e201) +
     e10 e11 (-1 + (-1 + b) e201) e211) / (1 + e200 - e10 e200 - e11 e211)
```

The principal profit when the agent chooses optimal effort given by Lemma 4

HIGHQuotaΠ[b_, c_] := Piecewise[
区分

$$\Big\{\Big\{\mathrm{HighQuota\Pi}\Big[\left(b^3-2b^2c+4c\left(-4c^2+\sqrt{b^4-6b^3c+8b^2c^2+16c^4}\right)\right)\Big/\left(4(b-2c)^2c\right),$$
$$\left(b^3-6b^2c+8bc^2+16c^3-4c\sqrt{b^4-6b^3c+8b^2c^2+16c^4}\right)\Big/\left(4(b-2c)^2c\right),$$
$$\frac{-4c+\frac{\sqrt{b^4-6b^3c+8b^2c^2+16c^4}}{c}}{-2b+4c},\ \frac{b}{2c},\ \frac{-4c+\frac{\sqrt{b^4-6b^3c+8b^2c^2+16c^4}}{c}}{-2b+4c},\ c,\ b\Big],\ 0\le b<2c\Big\},$$
$$\Big\{\mathrm{HighQuota\Pi}\Big[\frac{b^3-2b^2c-16c^3-4\sqrt{2}\,(b\,c)^{3/2}+16\sqrt{2}\sqrt{b\,c^5}}{4(b-2c)^2c},$$
$$\left(b^3-6b^2c+8bc^2+16c^3+4\sqrt{2}\,(b\,c)^{3/2}-16\sqrt{2}\sqrt{b\,c^5}\right)\Big/\left(4(b-2c)^2c\right),$$
$$\frac{\frac{\sqrt{2}\,b^{3/2}}{\sqrt{c}}+4c-4\sqrt{2}\sqrt{b\,c}}{2b-4c},\ 1,\ \frac{\frac{\sqrt{2}\,b^{3/2}}{\sqrt{c}}+4c-4\sqrt{2}\sqrt{b\,c}}{2b-4c},\ c,\ b\Big],$$
$$2c\le b\le 4c\Big\},\ \{\mathrm{HighQuota\Pi}[1,0,0,1,0,c,b],\ 4*c<b\}\Big\}\Big]$$

The optimal bonus

```
HIGHOptimalBonus[c_] := NArgMax[{HIGHQuotaΠ[b, c], 0 ≤ b ≤ 2}, b]
```
関数が最大となる点

The principal’s optimal profit

```
HIGHOptimalΠ[c_] := HIGHQuotaΠ[HIGHOptimalBonus[c], c]
```

The expected effort at the first round when the agent chooses the optimal report policy given by Lemma 4

$$\text{High1stEffort}[\text{e10\_}, \text{e11\_}, \text{e200\_}, \text{e201\_}, \text{e211\_}] := \frac{-1 + \text{e11 e211}}{-1 + (-1 + \text{e10})\ \text{e200} + \text{e11 e211}} * \text{e10} + \frac{(-1 + \text{e10})\ \text{e200}}{-1 + (-1 + \text{e10})\ \text{e200} + \text{e11 e211}} * \text{e11}$$

The expected effort at the first round when the agent chooses the optimal report policy and the effort policy given by Lemma 4

$$\text{HIGH1stEffort}[\text{b\_}, \text{c\_}] := \text{FullSimplify}\Big[$$
完全に簡約

$$\text{Piecewise}\Big[\Big\{\Big\{\text{High1stEffort}\Big[\frac{b^3 - 2 b^2 c + 4 c \left(-4 c^2 + \sqrt{b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4}\right)}{4 (b - 2 c)^2 c},$$
区分

$$\frac{b^3 - 6 b^2 c + 8 b c^2 + 16 c^3 - 4 c \sqrt{b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4}}{4 (b - 2 c)^2 c},$$

$$\frac{-4 c + \frac{\sqrt{b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4}}{c}}{-2 b + 4 c}, \frac{b}{2 c}, \frac{-4 c + \frac{\sqrt{b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4}}{c}}{-2 b + 4 c}\Big], 0 \le b \le 2 c\Big\},$$

$$\Big\{\text{High1stEffort}\Big[\frac{b^3 - 2 b^2 c - 16 c^3 - 4 \sqrt{2}\ (b c)^{3/2} + 16 \sqrt{2} \sqrt{b c^5}}{4 (b - 2 c)^2 c},$$

$$\frac{b^3 - 6 b^2 c + 8 b c^2 + 16 c^3 + 4 \sqrt{2}\ (b c)^{3/2} - 16 \sqrt{2} \sqrt{b c^5}}{4 (b - 2 c)^2 c}, \frac{\frac{\sqrt{2}\ b^{3/2}}{\sqrt{c}} + 4 c - 4 \sqrt{2} \sqrt{b c}}{2 b - 4 c}, 1,$$

$$\frac{\frac{\sqrt{2}\ b^{3/2}}{\sqrt{c}} + 4 c - 4 \sqrt{2} \sqrt{b c}}{2 b - 4 c}\Big], 2 c \le b \le 4 c\Big\}, \{\text{High1stEffort}[1, 0, 0, 1, 0], 4 * c < b\}\Big\}\Big]\Big]$$

The expected effort at the second round when the agent chooses the optimal report policy given by Lemma 4

$$\text{High2ndEffort}[\text{e10\_}, \text{e11\_}, \text{e200\_}, \text{e201\_}, \text{e211\_}] := -\frac{(-1 + \text{e10})\ (-1 + \text{e11 e211})}{-1 + (-1 + \text{e10})\ \text{e200} + \text{e11 e211}} * \text{e200} +$$

$$(((-1 + \text{e11})\ \text{e200} + \text{e10}\ (-1 + \text{e200} - \text{e11 e200} + \text{e11 e211}))\ /$$

$$(-1 + (-1 + \text{e10})\ \text{e200} + \text{e11 e211})) * \text{e201} + \frac{(-1 + \text{e10})\ \text{e11 e200}}{-1 + (-1 + \text{e10})\ \text{e200} + \text{e11 e211}} * \text{e211}$$

The expected effort at the second round when the agent chooses the optimal report policy and the effort policy given by Lemma 4

```
HIGH2ndEffort[b_, c_] := FullSimplify[Piecewise[
  {{High2ndEffort[(b^3 - 2 b^2 c + 4 c (-4 c^2 + √(b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4))) / (4 (b - 2 c)^2 c),
     (b^3 - 6 b^2 c + 8 b c^2 + 16 c^3 - 4 c √(b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4)) / (4 (b - 2 c)^2 c),
     (-4 c + √(b^4-6 b^3 c+8 b^2 c^2+16 c^4)/c) / (-2 b + 4 c), b/(2 c), (-4 c + √(b^4-6 b^3 c+8 b^2 c^2+16 c^4)/c) / (-2 b + 4 c)], 0 ≤ b ≤ 2 c},
   {High2ndEffort[(b^3 - 2 b^2 c - 16 c^3 - 4 √2 (b c)^(3/2) + 16 √2 √(b c^5)) / (4 (b - 2 c)^2 c),
     (b^3 - 6 b^2 c + 8 b c^2 + 16 c^3 + 4 √2 (b c)^(3/2) - 16 √2 √(b c^5)) / (4 (b - 2 c)^2 c),
     (√2 b^(3/2)/√c + 4 c - 4 √2 √(b c)) / (2 b - 4 c), 1, (√2 b^(3/2)/√c + 4 c - 4 √2 √(b c)) / (2 b - 4 c)],
    2 c ≤ b ≤ 4 c}, {High2ndEffort[1, 0, 0, 1, 0], 4 * c < b}}]]
```

The total expected effort in one period

```
HIGHTotalEffort[b_, c_] := HIGH1stEffort[b, c] + HIGH2ndEffort[b, c]
```

The total expected effort in one period when the principal chooses the optimal bonus

```
HIGHTotalEffortUnderOptimalbonus[c_] := HIGHTotalEffort[HIGHOptimalBonus[c], c]
```

The expected payment

```
HighPayment[e10_, e11_, e200_, e201_, e211_, b_] :=
 (((-1 + e11) e200 + e10 (-1 + e200 - e11 e200 + e11 e211)) / (-1 + (-1 + e10) e200 + e11 e211) * e201 +
   ((-1 + e10) e11 e200) / (-1 + (-1 + e10) e200 + e11 e211)) * b
```

The expected payment when the agent chooses the optimal policy and the optimal effort policy given by Lemma 4

```
HIGHPayment[b_, c_] :=
 FullSimplify[Piecewise[{{HighPayment[
 完全に簡約        区分
```

$$\text{FullSimplify}\Big[\text{Piecewise}\Big[\Big\{\Big\{\text{HighPayment}\Big[\frac{b^3 - 2 b^2 c + 4 c \left(-4 c^2 + \sqrt{b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4}\right)}{4 (b - 2 c)^2 c},$$

$$\frac{b^3 - 6 b^2 c + 8 b c^2 + 16 c^3 - 4 c \sqrt{b^4 - 6 b^3 c + 8 b^2 c^2 + 16 c^4}}{4 (b - 2 c)^2 c},$$

$$\frac{-4 c + \frac{\sqrt{b^4-6 b^3 c+8 b^2 c^2+16 c^4}}{c}}{-2 b + 4 c}, \frac{b}{2 c}, \frac{-4 c + \frac{\sqrt{b^4-6 b^3 c+8 b^2 c^2+16 c^4}}{c}}{-2 b + 4 c}, b\Big], 0 \le b \le 2 c\Big\},$$

$$\Big\{\text{HighPayment}\Big[\frac{b^3 - 2 b^2 c - 16 c^3 - 4 \sqrt{2} (b c)^{3/2} + 16 \sqrt{2} \sqrt{b c^5}}{4 (b - 2 c)^2 c},$$

$$\frac{b^3 - 6 b^2 c + 8 b c^2 + 16 c^3 + 4 \sqrt{2} (b c)^{3/2} - 16 \sqrt{2} \sqrt{b c^5}}{4 (b - 2 c)^2 c},$$

$$\frac{\frac{\sqrt{2} b^{3/2}}{\sqrt{c}} + 4 c - 4 \sqrt{2} \sqrt{b c}}{2 b - 4 c}, 1, \frac{\frac{\sqrt{2} b^{3/2}}{\sqrt{c}} + 4 c - 4 \sqrt{2} \sqrt{b c}}{2 b - 4 c}, b\Big],$$

$$2 c \le b \le 4 c\Big\}, \{\text{HighPayment}[1, 0, 0, 1, 0, b], 4 * c < b\}\Big\}\Big]\Big]$$

The expected payment when the principal chooses the optimal bonus

```
HIGHOptimalPayment[c_] := HIGHPayment[HIGHOptimalBonus[c], c]
```

High-Quota contract (Non-Gaming)

The agent’s optimal effort at the second round

```
NonGamingHigh2ndEffort[b_, c_] := Min[1, b / (2 * c)]
                                  最小
```

The agent’s optimal effort at the first round

```
NonGamingHigh1stEffort[b_, c_] :=
 ArgMax[{e1 * (NonGamingHigh2ndEffort[b, c] * b - c * NonGamingHigh2ndEffort[b, c]^2) -
 関数が最大となる変数の位置
    c * e1^2, 0 ≤ e1 ≤ 1}, e1]
```

The principal’s profit when the agent chooses the optimal policy and the optimal effort policy given by Lemma 4

```
NonGamingHighΠ[b_, c_] :=
 NonGamingHigh1stEffort[b, c] * NonGamingHigh2ndEffort[b, c] * (2 - b) +
  NonGamingHigh1stEffort[b, c] * (1 - NonGamingHigh2ndEffort[b, c])
```

The optimal bonus

```
NonGamingHighOptimalBonus[c_] := NArgMax[{NonGamingHighΠ[b, c], 0 ≤ b}, b]
                                 関数が最大となる点
```

The principal’s optimal profit

```
NonGamingHighOptimalΠ[c_] := NonGamingHighΠ[NonGamingHighOptimalBonus[c], c]
```

The expected total effort in one period

```
NonGamingHighTotalEffort[c_] :=
 NonGamingHigh1stEffort[NonGamingHighOptimalBonus[c], c] +
  NonGamingHigh1stEffort[NonGamingHighOptimalBonus[c], c] *
   NonGamingHigh2ndEffort[NonGamingHighOptimalBonus[c], c]
```

The optimal expected payment when the agent chooses the optimal policy and the optimal effort policy given by Lemma 4

```
NonGamingHighOptimalPayment[c_] :=
 NonGamingHigh1stEffort[NonGamingHighOptimalBonus[c], c] * NonGamingHigh2ndEffort[
   NonGamingHighOptimalBonus[c], c] * NonGamingHighOptimalBonus[c]
```

Low-Quota Contract

The agent's expected utility

```
LowU[e10_, e11_, e200_, e201_, e211_, b_, c_] :=
 (b (e200 + (-1 + e11) e200 e201 - e11 e200 e211 +
       e10 (1 + e11 e201 + e200 (-1 + e201 - e11 e201) + e11 (-1 + e200) e211)) -
    c (e10^2 (1 + (-1 + e11) e201 - e11 e211) + e200^2 (1 + (-1 + e11) e201 - e11 e211) +
       e10 (e200^2 (-1 + e201 - e11 e201 + e11 e211) + e201
            (e201 + e11 (e11 - e201 e211 + e211^2))))) / (1 + (-1 + e10 + e11) e201 - e11 e211)
```

The agent's optimal effort at the first round when he does not have carryover

```
Low1stOptimalEffort0[b_, c_] :=
 ArgMax[{LowU[e10, 0, e200, e201, 0, b, c], 0 ≤ e10 ≤ 1, 0 ≤ e200 ≤ 1, 0 ≤ e201 ≤ 1},
 関数が最大となる変数の位置
   {e10, e200, e201}][[1]]
```

The agent's optimal effort at the second round when he does not have carryover and does not report the sales at the first round

```
Low2ndOptimalEffort00[b_, c_] :=
 ArgMax[{LowU[e10, 0, e200, e201, 0, b, c], 0 ≤ e10 ≤ 1, 0 ≤ e200 ≤ 1, 0 ≤ e201 ≤ 1},
 関数が最大となる変数の位置
   {e10, e200, e201}][[2]]
```

The agent's optimal effort at the second round when he have carryover but does not report the sales at the first round

```
Low2ndOptimalEffort01[b_, c_] :=
 ArgMax[{LowU[e10, 0, e200, e201, 0, b, c], 0 ≤ e10 ≤ 1, 0 ≤ e200 ≤ 1, 0 ≤ e201 ≤ 1},
 関数が最大となる変数の位置
   {e10, e200, e201}][[3]]
```

The principal's profit

```
LowΠ[e10_, e11_, e200_, e201_, e211_, b_, c_] :=
 ((1 - b) (e10 (-1 + e200) - e200) + ((-2 + b) e10 e11 - (-1 + b) (-1 + e10) (-1 + e11) e200)
     e201 + (-1 + b) e11 (e10 (-1 + e200) - e200) e211) / (-1 - (-1 + e10 + e11) e201 + e11 e211)
```

The principal's profit when the agent chooses the optimal report policy and the effort policy given by Lemma 4

```
LOWΠ[b_, c_] := LowΠ[Low1stOptimalEffort0[b, c], 0,
  Low2ndOptimalEffort00[b, c], Low2ndOptimalEffort01[b, c], 0, b, c]
```

The principal's optimal profit

```
LOptimalΠ[c_] := NMaximize[{LOWΠ[b, c], 0 ≤ b < 1}, b]
                 数値最大化
```

Low-Quota Contract (Non-Gaming)

The agent's optimal effort at the second round

```
NonGamingLow2ndEffort[b_, c_] := Min[1, b / (2 * c)]
                                 最小
```

The agent's optimal effort at the first round

```
NonGamingLow1stEffort[b_, c_] := ArgMax[
                                 関数が最大となる変数の位置
  {e1 * b + (1 - e1) * (NonGamingLow2ndEffort[b, c] * b - c * NonGamingLow2ndEffort[b, c] ^2) -
    c * e1^2, 0 ≤ e1 ≤ 1}, e1]
```

The principal's profit when the agent chooses optimal effort given by Lemma 5.

```
NonGagmingLowΠ[b_, c_] := (NonGamingLow1stEffort[b, c] +
     (1 - NonGamingLow1stEffort[b, c]) * NonGamingLow2ndEffort[b, c]) * (1 - b)
```

The principal's optimal profit

```
NonGagmingLowOptimalΠ[c_] := NArgMax[{NonGagmingLowΠ[b, c], 0 ≤ b}, b]
                             関数が最大となる点
```

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