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The Weak Relationship Between Uncertainty and Incentives in Linear Contracts

Toshihiko Nanba

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関西大学ソシオネットワーク戦略研究機構

Research Institute for Socionetwork Strategies,
Kansai University
Joint Usage / Research Center, MEXT, Japan
Suita, Osaka, 564-8680, Japan
URL: https://www.kansai-u.ac.jp/riss/index.html
e-mail: riss@ml.kandai.jp
tel. 06-6368-1228
fax. 06-6330-3304

The Weak Relationship Between Uncertainty and Incentives in Linear Contracts*

Toshihiko Nanba †

Kansai University

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Abstract

Standard agency theory predicts that there is a negative trade-off between the amount of uncertainty and the incentive strength in linear contracts. However, this relationship is not empirically supported. In this paper, in order to re-examine this relationship, we investigate a contractual issue in which a risk neutral principal hires a risk averse agent who has the specific knowledge about the consequences of the agent's actions. The main result of this paper is that uncertainty does not affect the incentive strength in optimal linear contracts when the principal can use not only an output measure but also an input measure to evaluate the agent's performance.

Keywords: Uncertainty and incentives; Risk-averse agents; Specific knowledge

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†Correspondence to:The Research Institute for Socionetwork Strategies, Kansai University,3-3-35 Yamate-cho, Suita-shi, Osaka 564-8680, Japan. E-mail: toshihiko-nanba@gmail.com

1 Introduction

Choosing performance measures is an important and difficult problem for firms. The standard agency theory predicts that there is a negative trade-off between the amount of uncertainty and the strength of incentives in the optimal linear incentive contracts.[1] The intuition of this prediction is as follows. If a firm uses a linear contract, uncertainty in performance measures generates employees' income risk. Furthermore, the income risk decreases employees' utility if they are risk averse. It follows that when uncertainty increases, the firm may reduce performance pay in order to lower the risk premium generated from the income risk. That is, an increase in uncertainty decreases the equilibrium level of incentives because incentives become more costly to provide. As a result, uncertainty negativly related to the equilibrium level of incentives.

However, many empirical works do not support this relationship. For example, Prendergast (2002) points out that "*the data suggest a positive relationship between measures of uncertainty and incentives rather than the posited negative tradeoff*".

It is well known that the difference between the theoretical prediction and the empirical result is due to the difference in the definition of uncertainty. Whereas in theory the standard model focuses only on uncertainty that the agent can not control (e.g., measurement error in output), in practice there exists different types of uncertainty. In order for re-examination of the relationship between uncertainty and incentives, recent studies introduce different types of uncertainty to the standard moral hazard framework. Especially, many studies focus on uncertainty that generated from the agent's private information.[2]

For example, Baker and Jorgensen (2003) consider a moral hazard model in which the agent privately receives information about the principal's preferred action after the singing of the contract but before choosing action. This type of information can be interpreted as "*the specific knowledge*" in the sense of

[1] see Holmstrom (1979), Holmstrom and Milgrom (1987)

[2] see Zabojnik (1996), Baker and Jorgensen (2003), Raith (2008), Shi (2007) and Rantakari (2008)

Jensen and Meckling (1992). In their model, the agent is assumed to unable to communicate this knowledge to the principal. Therefore, the principal does not know which action are most profitable, and thus this asymmetry of information generates uncertainty for the principal. They refer this type of uncertainty to "*volatility*" while uncertainty in the standard model to "*noise*". Their main result is that the equilibrium incentive strength is negatively related to noise, whereas positively related to volatility.

Raith (2008) study a model similar to Baker and Jorgensen (2003) in the sense that the agent has specific knowledge. But his model is assumed that the principal can use multiple performance measures. Specifically, he focuses on linear incentive contracts which are based both on an output measure and an input measure. The output measure (e.g., total sales or firm's profit) is closely related to the principal's payoff and subjects to random influences outside the agent's control. Therefore, if the principal uses output-based pay, while the principal can motivate the agent to use specific knowledge, must compensate the agent for the income risk. On the other hand, the input measure (e.g., working hours) is closely related to the agent's action and less likely to be affected by random influences. That is, while input-based pay cannot motivate the agent to use specific knowledge, does not expose the agent to the income risk. Consequently, the principal faces a trade-off between using specific knowledge and reducing the agent's income risk.

Raith (2008) shows that the optimal weight on output-based pay increases with the value of the agent's specific knowledge and decreases with random influences outside the agent's control. That is, his result implies that the equilibrium incentive strength of output measure is negatively related to noise and positively related to volatility.[3]

Whereas Raith (2008) examines how changes in these two types of uncertainty affect the design of the optimal linear contract, he does not explain the relationship between the amount of uncertainty and the strength of incentives

[3]Raith (2008) refers noise to environmental uncertainty and volatility to technological uncertainty.

in the optimal linear contract. The main purpose of our study is to complement his result by considering the relationship between uncertainty and incentives. Specifically, we will examine how these two types of uncertainty affect the equilibrium level of effort in the case where the agent has specific knowledge and the principal can use multiple performance measures.

In this paper, similar to Raith (2008), we consider a model in which the principal can use two types of performance measure (i.e., an output measure and an input measure) and the agent receives the information about productivity of activities after contracting but before choosing actions. It follows that the principal faces the same trade-off as Raith (2008).

Our main result is that both changes in the amount of noise and that of volatility do not affect the equilibrium level of incentives. Specifically, these two types of uncertainty do not change the sum of the equilibrium effort levels of each activity. The intuition of this result is as follows. As the amount of noise increases, the agent's income risk generated by output-based pay increases. Therefore, the principal reduces the weight on output-based pay to decrease the risk premium generated by the income risk, and then the agent's effort incentives generated by output-based pay decreases. On the other hand, as the amount of noise increases, the weight on input-based pay increases because input-based pay does not generates the income risk. Since these two effects on incentives cancel each other out, as a result, the amount of noise does not affect incentives.

Contrary, as the amount of volatility increases, the value of the agent's specific knowledge increases, and then the principal prefers to use this information by increasing the weight on output-based pay. Since an increase in the weight on output-based pay increases the agent's effort level, the principal can reduce input-based pay while keeping effort levels. As a result, the amount of volatility also does not affect incentives. From the above, both of noise and volatility do not affect the incentive strength in equilibrium.

Our results are consistent with sevral empirical works. First, our main result that uncertainty does not affect incentives is consistent with Prendargast's prediction, which is the "*tenuous*" trade-off between uncertainty and incentives.

Second, our other result that the parincipal keeps incentives by controlling the weight on performance measures is consistent with the empirical result by Gibbs et al (2009). They show that firms use several performance measures complementary to control uncertainty, using data from auto dealership manager incentive systems. That is, for example, if the first performance measure has a large noise, then firms add the other type of performance measures which has little noise to reduce uncertainty in performance evaluations.

This paper is organized as follows. We begin with formulating our model in section 2. Section 3 characterizes the optimal incentive linear contract in the model and examine the relationship between uncertainty and incentives. Finally, we conclude in section 4.

2 The Model

We consider a moral-hazard model in which a risk neutral principal (female) hires a risk averse agent (male) to exert effort on two activities.[4]

Production:

Output, which is denoted by y, is given by

$$y = \sum_{i=1}^{2} \theta_i e_i + \epsilon,$$

where $e_i(\geq 0)$ is a effort level of activity i, $\theta_i(\geq 0)$ is the marginal productivity of activity i's effort and ϵ is the random variable that is distributed by a normal distribution that has mean 0 and variance σ_ϵ^2, i.e., $\epsilon \sim N(0, \sigma_\epsilon^2)$.

Information about $\boldsymbol{\theta}$:

The principal knows only the distribution of $\boldsymbol{\theta} = (\theta_1, \theta_2)$ (i.e., she does not know the realization of $\boldsymbol{\theta}$), while the agent obtains the information about the realization of $\boldsymbol{\theta}$ after contracting but before choosing his effort. $\boldsymbol{\theta}$ and its distribution are characterized as follows.

[4] To keep the model simple, we consider the two-acitivity model. But we can easily extend this model to n-activity model.

- For all $i(=1,2)$, $\theta_i \in \{\theta^L, \theta^H\}$, where $0 < \theta^L \leq \theta^H$.
- Each activity's productivity does not overlap, that is, if $\theta_1 = \theta^L$, then $\theta_2 = \theta^H$.
- For all $i(=1,2)$ and $k(=L,H)$, $Prob(\theta_i = \theta^k) = \frac{1}{2}$.
- Mean and variance of θ are denoted by $M(>0)$ and $\sigma_\theta^2(>0)$, respectively. That is, $\frac{1}{2}(\theta^L + \theta^H) = M$ and $4(\theta^L - \theta^H)^2 = \sigma_\theta^2$.

We assume that the agent can not communicate this information to the principal. It follows that this information can be interpreted as the agent's specific knowledge in the sense of Jensen and Meckling (1992). On the other hand, σ_θ^2 can be interpreted as volatility in the sense of Baker and Jorgensen (2003). Furthermore, since the specific knowledge is more valuable for the principal when σ_θ^2 increases, we can use σ_θ^2 as the measure regarding the value of the agent's specific knowledge.

Uncertainty:
Both of the principal and the agent know the distribution of ϵ before contracting, whereas they do not know the realization of ϵ before the agent chooses his effort level. It follows that the agent chooses an effort level before he knows the realization of ϵ, and thus the realization of ϵ does not affect the agent's effort level. Therefore, σ_ϵ^2 can be interpreted as noise in the sense of Baker and Jorgensen (2003).

Agent's utility:
The agent has a constant absolute risk-aversion (CARA) utility function given by

$$u(w, e_1, e_2) = -exp[-R(w - \sum_{i=1}^{2} \frac{d}{2} e_i^2)],$$

where $R(>0)$ represents the degree of absolute risk-aversion, w is the principal's payment to the agent and $de_i^2/2$ is the agent's disutility of exerting activity i's effort.[5]

Performance measurement:

We assume that the principal can observe and verify not only the output (y) but also the effort level of each activity ($e_1 + e_2$). That is, the principal can use output-based pay that depends on y and input-based pay that depends on (e_1, e_2). If the principal uses output-based pay, then the principal can give the agent strong incentives to use his specific knowledge because he chooses the effort level of each activity (e_1, e_2) to maximize y, whereas this type of payment imposes the income risk to the agent because output contains random variable ϵ. On the other hand, input-based pay does not impose the income risk to the agent because it does not have uncertainty for the agent, while does not give the agent incentives to use the specific knowledge.

Compensation:

We focus on linear contracts that combine output-based pay and input-based pay. For simplicity, we assume that input-based pay depends on the total effort of each activity (i.e., $e_1 + e_2$).

Hence, the principal's payment to the agent is characterized as follows.

$$w = \alpha + \beta y + \gamma(e_1 + e_2).$$

where α is a fixed payment, β is a payment rate for output y and γ is a payment rate for the total input $e_1 + e_2$. In this notations, β can be interpreted as the weight on output-based pay, while γ is the weight on input-based pay.

Timing:

The timing of this game is as follows.

1. The principal offers a compensation plan (α, β, γ) to the agent.

2. The agent chooses to accept or reject the offer. If he chooses rejection, then he receives reservation wage $\bar{W}$ (we normalize it to 0) from his outside

[5]The agent's disutility function is based on Baker (2002) in the sense that the disutility from exerting effort of each activity is additively separable.

option.

3. The agent recieves the information about the realization of $\boldsymbol{\theta}$.

4. The agent chooses the effort level of each activity (e_1, e_2).

5. The output y realizes and the agent is compensated accordingly.

The principal's problem:

The principal's problem is to design a linear contract that depends on y and $e_1 + e_2$ to maximize her expected value of $y - w(y)$ by giving the agent incentives to accept the contract and to choose his effort levels optimally. Hence, her problem can be described as follows.

$$\max_{\alpha,\beta,\gamma,e_1,e_2} E[y - w|\epsilon, \theta_1, \theta_2]$$

subject to

$$e_i \in \arg\max_{e_i} E[u(w, e_1, e_2)|\epsilon] \quad for\ i = 1, 2, \tag{1}$$

$$E[u(w, e_1, e_2))|\epsilon, \theta_1, \theta_2] \geq 0. \tag{2}$$

Here, constraints (**??**) and (**??**) imply the incentive compatibility constraint and the participation constraint, respectively.

3 The optimal contract

In this section, we analyze our model to examine the relationship between uncertainty (σ_ϵ^2 or σ_θ^2) and the equilibrium level of incentives. We begin with considering the agent's problem in order to solve the principal's problem described above.

The agent's problem is to decide whether to accept or reject the principal's offer and to choose his effort level to maximize his expected utility. First, we consider the agent's problem for choosing the optimal effort level of each activity. Given the compensation scheme w, the agent chooses his effort level to maximize his certainty equivalent after $\boldsymbol{\theta}$ is realized. Therefore, to derive the optimal effort levels (denoted by $e_1^*(\theta_1)$ and $e_2^*(\theta_2)$), we need to characterize the agent's certainty equivalent after he observes the realization of $\boldsymbol{\theta}$, which is denoted by CE_{ac}. Following lemma characterizes CE_{ac}.

Lemma 1 *The agent's certainty equivalent after he observed the realization of $\boldsymbol{\theta}$ is given by*

$$CE_{ac} = \alpha + \sum_{i=1}^{2}\{\beta(\theta_i e_i) + \gamma e_i - \frac{d}{2}e_i^2\}) - \frac{1}{2}R\beta^2\sigma_\epsilon^2.$$

Proof. see the Appendix.

Given lemma 1, the agent's problem for choosing the optimal effort level can be expressed as

$$\begin{aligned} &\max_{e_1,e_2} CE_{ac} \\ = \quad &\max_{e_1,e_2} \ \alpha + \sum_{i=1}^{2}\{\beta(\theta_i e_i) + \gamma e_i - \frac{d}{2}e_i^2\}) - \frac{1}{2}R\beta^2\sigma_\epsilon^2. \end{aligned}$$

By differentiating the agent's objective function with respect to e_i, the first-order condition for activity i's optimal effort level is

$$\beta\theta_i + \gamma - de_i^* = 0.$$

Hence, activity i's optimal effort level is given by

$$e_i^*(\theta_i) = \frac{\beta\theta_i + \gamma}{d}. \tag{3}$$

(**??**) implies that the optimal effort level of activity i depends on the realization of θ_i. Therefore, we denoted it as $e^*(\theta_i)$.

Second, we consider the agent's decision regarding participation. The agent will choose acceptance if his expected wage at the time of receiving the offer is higher than reservation wage. Specifically, the certainty equivalent before he observes the realization of $\boldsymbol{\theta}$, which is denoted by CE_{bc}, is higher than 0. Note that the agent chooses the optimal effort levels by using the information of the realization of $\boldsymbol{\theta}$. Hence, we need to derive CE_{bc} by assuming that $e_1 = e^*(\theta_1)$ and $e_2 = e^*(\theta_2)$. However, CE_{bc} is mathematically more complex than CE_{ac}

because the risk-averse agent calculates CE_{bc} taking into account not only noise σ_ϵ^2 but also volatility σ_θ^2. Following lemma characterizes CE_{bc}.

Lemma 2 *The agent's certainty equivalent before he observes the realization of $\boldsymbol{\theta}$ is given by*

$$CE_{bc} = \alpha - \frac{1}{2}R\beta^2\sigma_\epsilon^2 + \sum_{k\in\{H,L\}} \{\beta\theta^k e^*(\theta^k) + \gamma e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k))^2\}$$

Proof. see the Appendix.

To satisfy the participation constraint, CE_{bc} must be equal or higher than 0. Therefore, it is optimal for the principal to set α so that CE_{bc} equals 0.

Specifically, α^* is determined to satisfy the following equation.

$$\alpha^* = \frac{1}{2}R\beta^2\sigma_\epsilon^2 - \sum_{k\in\{H,L\}} \{\beta\theta^k e^*(\theta^k) + \gamma e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k))^2\}.$$

On the other hand, the principal's certainty equivalent (denoted by CE_P) is given by

$$\begin{aligned} CE_P &= E[y - w|\epsilon, \boldsymbol{\theta}] \\ &= \frac{1}{2}\sum_{i=1}^{2}\sum_{k\in\{H,L\}} \theta^k e_i^*(\theta^k) \\ &\quad -(\alpha^* + \frac{1}{2}\sum_{i=1}^{2}\sum_{k\in\{H,L\}} \{\beta\theta^k e_i^*(\theta^k) + \gamma e_i^*(\theta^k)\}) \\ &= \sum_{k\in\{H,L\}} \{\theta^k e^*(\theta^k) - (\alpha^* + \beta\theta^k e^*(\theta^k) + \gamma e^*(\theta^k))\} \end{aligned}$$

Then, the total amount of the certainty equivalent (denoted by CE), which is defined $CE_P + CE_{bc}$, is given by

$$CE = \sum_{k\in\{H,L\}} \{\theta^k e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k)^2\} - \frac{1}{2}R\beta^2\sigma_\epsilon^2$$

By maximization principle, the principal's problem is to design the compensation scheme that maximizes CE. That is, her problem can be rewritten as

$$\max_{\beta,\gamma} \sum_{k\in\{H,L\}} \{\theta^k e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k)^2\} - \frac{1}{2}R\beta^2\sigma_\epsilon^2 \tag{4}$$

Following proposition characterizes the optimal weight on output-based pay and input-based pay by solving above problem.

Proposition 1 *In the equilibrium, the optimal weight on output-based pay (denoted by β^*) and input-based pay (denoted by γ^*) are given by*

$$\beta^* = \frac{2\sigma_\theta^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2}, \quad \gamma^* = \frac{dMR\sigma_\epsilon^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2}.$$

Furthermore,

$$\beta^* + \frac{1}{M}\gamma^* = 1. \tag{5}$$

Proof. see the Appendix.

(**??**) implies that R, σ_ϵ^2 and σ_θ^2 do not change the sum of the weight on input and output-based payment.

Furthermore, the following proposition shows that the sum of equilibrium effort level of two activities also is not affected by R, σ_ϵ^2 and σ_θ^2.

Proposition 2 *In the equilibrium, the agent's total effort (i.e., $e_1^* + e_2^*$) is given by $\frac{2M}{d}$.*

Proof. see the Appendix.

This proposition implies that the agent's risk attitude, the amount of volatility and the amount of noise do not affect the agent's total effort level in the equilibrium. Futhermore, the results of proposition 1 and 2 can be interpreted that uncertainty does not affect incentives.

Next, we check how changes in R, σ_ϵ^2 and σ_θ^2 affect β^* and γ^* in order to confirm the individual effects. Following proposition shows that the results of our model are consistent with previous studies such as Baker and Jorgensen (2003), Raith (2008) and Gibbs et al (2009).

Proposition 3 *In the equilibrium, following inequality must be hold.*

$$\frac{\partial \beta^*}{\partial \sigma_\epsilon^2} < 0, \ \frac{\partial \beta^*}{\partial \sigma_\theta^2} > 0, \ \frac{\partial \beta^*}{\partial R} < 0 \tag{6}$$

$$\frac{\partial \gamma^*}{\partial \sigma_\epsilon^2} > 0, \ \frac{\partial \gamma^*}{\partial \sigma_\theta^2} < 0, \ \frac{\partial \gamma^*}{\partial R} > 0. \tag{7}$$

(**??**) implies that β^* is increasing in σ_θ^2 but decreasing in σ_ϵ^2 and R. First and third inequality ($\frac{\partial \beta^*}{\partial \sigma_\epsilon^2} < 0$ and $\frac{\partial \beta^*}{\partial \sigma_\theta^2} > 0$, respectively) in (**??**) can be interpreted that the amount of noise and the degree of the agent's risk aversion are negatively related to the incentives of output-based pay. These relationships are consistent with standard agency theory such as Holmstrom (1979) in the sense that an increase risk or noise decreases incentives generated provided by output-based pay. Second inequality ($\frac{\partial \gamma^*}{\partial \sigma_\epsilon^2} > 0$) in (**??**) can be interpreted that the amount of volatility is positively related to the incentives in output-based pay. This result is consistent with Raith (2008) in the sense that an increase in the value of the agent's specific knowledge increases the incentives provided by output-based pay.

On the other hand, (**??**) implies that the optimal weight on input-based pay decreases with the amount of noise and the degree of risk-averse, while increases with the amount of volatility. That is, γ^* is affected by these three factors (i.e., $\sigma_\epsilon^2, \sigma_\theta^2, R$) in the opposite way to β^*.

From proposition 1, 2 and 3, we can say that the principal keeps the amount of incentives given to the agent constant by controlling the weights on output-based pay and input-based pay for each type of uncertainty.

4 Conclusion

This paper studies the controversial issue about the relationship between uncertainty and incentives in linear contracts. To examine this relationship, we consider the model in which the agent is risk averse and has specific knowledge about productivity of each activity. Our model is similar to Baker and Jorgensen (2003) and Rantakari (2008) in the sense that a risk neutral principal offers a linear contract to a risk averse agent, while is similar to Raith (2008) in the sense that the agent has specific knowledge and the principal designs linear contracts that combine output-based pay and input-based pay.

Our results provide rationales for some previous empirical studies. For examples, the main result of this paper is that the agent's equilibrium effort level is unaffected not only by the amount of uncertainty, but also his risk attitude. This result is consistent with the prediction of Prendargast (2002), which is the "tenuous" relationship between uncertainty and incentives. In addition, another result of this paper (proposition 3) is consistent with Gibbs et al. (2009) in the sense that the principal controls the agent's risk and incentives by adjusting the weights on available performance measures.

Appendix

Proof of lemma 1

Note that CE_{ac} satisfies

$$u(CE_{ac}, 0, 0) = E[u(w, e_1, e_2)|\epsilon]. \tag{8}$$

LHS of (**??**) can be rewritten as

$$u(CE_{ac}, 0, 0) = -exp[-R(CE_{ac})]. \tag{9}$$

On the other hand, RHS of (**??**) is

$$\begin{aligned}
& E[u(w, e_1, e_2)|\epsilon] \\
= & \int -exp[-R(w - \frac{d}{2}\sum_{i=1}^{2} e_i^2)\frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\exp(-\frac{\epsilon^2}{2\sigma_\epsilon^2})d\epsilon \\
= & \int -\exp[-R\sum_{i=1}^{2}\{\alpha + \beta(\theta_i e_i + \epsilon) + \gamma e_i - \frac{d}{2}e_i^2\}]\frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\exp(-\frac{\epsilon^2}{2\sigma_\epsilon^2})d\epsilon \\
= & \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\int -\exp[-\frac{\epsilon^2}{2\sigma_\epsilon^2} - R\beta\epsilon - R\sum_{i=1}^{2}\{\alpha + \beta(\theta_i e_i) + \gamma e_i - \frac{d}{2}e_i^2\}]d\epsilon \\
= & \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\int -\exp[-(a\epsilon^2 + b\epsilon + c)]d\epsilon,
\end{aligned}$$

where

$$a = \frac{1}{2\sigma_\epsilon^2}, \quad b = R\beta, \quad c = R\sum_{i=1}^{2}\{\alpha + \beta(\theta_i e_i) + \gamma e_i - \frac{d}{2}e_i^2\}.$$

By using Gauss integral, it can be rewritten as

$$\begin{aligned}
&\frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}(-\exp[\frac{b^2}{4a}-c]\sqrt{\frac{\pi}{a}}) \\
= \;& \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} - \exp[\frac{(R\beta)^2}{4(\frac{1}{2\sigma_\epsilon^2})} - R\sum_{i=1}^{2}\{\alpha+\beta(\theta_i e_i)+\gamma e_i - \frac{d}{2}e_i^2\}]\sqrt{\frac{\pi}{\frac{1}{2\sigma_\epsilon^2}}} \\
= \;& -\exp[\frac{1}{2}R^2\beta^2\sigma_\epsilon^2 - R\sum_{i=1}^{2}\{\alpha+\beta(\theta_i e_i)+\gamma e_i - \frac{d}{2}e_i^2\}]] \\
= \;& -\exp[-R(\sum_{i=1}^{2}\{\alpha+\beta(\theta_i e_i)+\gamma e_i - \frac{d}{2}e_i^2\} - \frac{1}{2}R\beta^2\sigma_\epsilon^2)].
\end{aligned}$$

As a result, we obtain

$$CE_{ac} = \sum_{i=1}^{2}\{\alpha+\beta(\theta_i e_i)+\gamma e_i - \frac{d}{2}e_i^2\} - \frac{1}{2}R\beta^2\sigma_\epsilon^2.$$

Proof of lemma 2

CE_{bc} satisfies

$$u(CE_{bc},0,0) = E[u(w,e_1^*,e_2^*)|\epsilon,\theta]. \tag{10}$$

LHS of (**??**) can be rewritten as

$$u(CE_{bc},0,0) = -exp[-R(CE_{bc})]. \tag{11}$$

On the other hand, RHS of (**??**) is

$$\begin{aligned}
&E[u(w,e_1^*,e_2^*)|\epsilon,\theta] \\
= \;& E[\int -\exp[-R\sum_{i=1}^{2}\{\alpha+\beta(\theta_i e_i^*)+\gamma_i e_i^* - \frac{d}{2}(e_i^*)^2\}]\frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\exp(-\frac{\epsilon^2}{2\sigma_\epsilon^2})d\epsilon|\theta]
\end{aligned}$$

Note that $Prob(\theta_1 = \theta^H and\ \theta_2 = \theta^L) = Prob(\theta_1 = \theta^L and \theta_2 = \theta^H) = \frac{1}{2}$. Then, it can be rewritten by

$$\begin{aligned}
&\frac{1}{2}\{\int -\exp[-R(\alpha+\beta(\theta^H e_1^*(\theta^H)+\theta^L e_2^*(\theta^L))+\gamma_1 e_1^*(\theta^H)+\gamma_2 e_2^*(\theta^L) \\
&-\frac{d}{2}((e_1^*(\theta^H))^2+(e_2^*(\theta^L)^2\})]\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{\epsilon^2}{2\sigma_\epsilon^2})d\epsilon \\
&+\frac{1}{2}\{\int -\exp[-R(\alpha+\beta(\theta^L e_1^*(\theta^L)+\theta^H e_2^*(\theta^H))+\gamma_1 e_1^*(\theta^L)+\gamma_2 e_2^*(\theta^H) \\
&-\frac{d}{2}((e_1^*(\theta^L))^2+(e_2^*(\theta^H)^2\})]\frac{1}{\sqrt{2\pi\sigma_\epsilon^2}}\exp(-\frac{\epsilon^2}{2\sigma_\epsilon^2})d\epsilon.
\end{aligned}$$

Since $e_1^*(\theta^H) = e_2^*(\theta^H)$ and $e_1^*(\theta^H) = e_2^*(\theta^H)$, then

$$\int - \exp[-R \sum_{k\in\{H,L\}} \{\alpha + \beta\theta^k e^*(\theta^k) + \gamma e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k))^2\})] \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp(-\frac{\epsilon^2}{2\sigma_\epsilon^2}) d\epsilon$$

Furthermore, by using Gauss integral, it can be rewritten as

$$\begin{aligned} & -\exp[-R(\alpha - \frac{1}{2}R\beta^2\sigma_\epsilon^2 + \sum_{k\in\{H,L\}} \{\beta\theta^k e^*(\theta^k) + \gamma e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k))^2\}] \\ = \; & u(\alpha - \frac{1}{2}R\beta^2\sigma_\epsilon^2 + \sum_{k\in\{H,L\}} \{\beta\theta^k e_j^*(\theta^k) + \gamma^k e_j^*(\theta^k) - \frac{d}{2}(e_j^*(\theta^k))^2\}, 0, 0) \end{aligned}$$

As a result, we obtain

$$CE_{bc} = \alpha - \frac{1}{2}R\beta^2\sigma_\epsilon^2 + \sum_{k\in\{H,L\}} \{\beta\theta^k e^*(\theta^k) + \gamma e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k))^2\}$$

Proof of proposition 1

Note that $e^*(\theta_i) = \frac{\beta\theta_i + \gamma}{d}$. Then, the principal's problem that is given by (**??**) can be rewritten as follows.

$$\begin{aligned} & \max_{\beta,\gamma} \sum_{k\in\{H,L\}} \{\theta^k e^*(\theta^k) - \frac{d}{2}(e^*(\theta^k)^2\} - \frac{1}{2}R\beta^2\sigma_\epsilon^2 \\ = \; & \max_{\beta,\gamma} \frac{\beta(\theta^H)^2 + \gamma}{d} + \frac{\beta(\theta^L)^2 + \gamma}{d} - \frac{d}{2}\{(\frac{\beta\theta^H + \gamma}{d})^2 + (\frac{\beta\theta^L + \gamma}{d})^2\} - \frac{1}{2}R\beta^2\sigma_\epsilon^2 \\ = \; & \max_{\beta,\gamma} \frac{1}{2d}[\beta\{(2-\beta)((\theta^H)^2 + (\theta^L)^2) - dR\beta\sigma_\epsilon^2\} + 2(1-\beta)\gamma(\theta^H + \theta^L)] \quad (12) \end{aligned}$$

By differentiating (**??**) with respect to γ, we obtain the first-oder condition for γ. That is, γ^* satisfies the following condition.

$$\frac{1}{d}[(1-\beta)(\theta^H + \theta^L) - 2\gamma^*] = 0 \quad (13)$$

By solving (**??**) with respect to γ, we obtain

$$\gamma^* = \frac{1}{2}(1-\beta)(\theta^H + \theta^L) \quad (14)$$

On the other hand, by differentiating (**??**) with respect to β, and then we obtain the first-order condition for β. That is, β^* satisfies the following condition.

$$\frac{1}{d}[(1-\beta)(\theta^H)^2 + (\theta^L)^2 - \gamma(\theta^H + \theta^L) + dR\beta\sigma_\epsilon^2] = 0 \quad (15)$$

By solving (**??**) with respect to β, we obtain

$$\beta^* = \frac{(\theta^H)^2 + (\theta^L)^2 - \gamma(\theta^H + \theta^L)}{(\theta^H)^2 + (\theta^L)^2 + dR\sigma_\epsilon^2} \tag{16}$$

From (**??**) and (**??**)

$$\beta^* = \frac{(\theta^H - \theta^L)^2}{(\theta^H - \theta^L)^2 + dR\sigma_\epsilon^2}, \quad \gamma^* = \frac{d(\theta^H + \theta^L)R\sigma_\epsilon^2}{(\theta^H - \theta^L)^2 + dR\sigma_\epsilon^2}. \tag{17}$$

Note that $\frac{1}{2}(\theta^H + \theta^L) = M$ and $4(\theta^H - \theta^L)^2 = \sigma_\theta^2$. Then, we can rewrite as follows.

$$\beta^* = \frac{2\sigma_\theta^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2}, \quad \gamma^* = \frac{dMR\sigma_\epsilon^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2}. \tag{18}$$

Furthermore,

$$\beta^* + \frac{1}{M}\gamma^* = \frac{2\sigma_\theta^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2} + \frac{1}{M}\frac{dMR\sigma_\epsilon^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2} = 1.$$

Proof of proposition 2

First, we prove that (**??**) must be hold.

By differentiating β^* with respect to σ_ϵ, then we have

$$\frac{\partial \beta^*}{\partial \sigma_\epsilon^2} = -\frac{2dR\sigma_\theta^2}{(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2}.$$

Note that we assume that d, R, σ_ϵ and σ_θ^2 are strictry positive. It follows that $2dR\sigma_\theta^2 > 0$ and $(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2 > 0$.

As a result, we otbain

$$\frac{\partial \beta^*}{\partial \sigma_\epsilon^2} < 0.$$

Similarly,

$$\frac{\partial \beta^*}{\partial \sigma_\theta^2} = \frac{2dR\sigma_\epsilon^2}{(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2} > 0.$$

Furthermore,

$$\frac{\partial \beta^*}{\partial R} = -\frac{2d\sigma_\epsilon^2\sigma_\theta^2}{(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2} < 0.$$

Second, we prove that (**??**) must be hold. Note that we assume M also strictry positive. Then, we have

$$\frac{\partial \gamma^*}{\partial \sigma_\epsilon^2} = \frac{2dMR\sigma_\epsilon^2}{(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2} > 0,$$

$$\frac{\partial\gamma^*}{\partial\sigma_\theta^2} = -\frac{2dMR\sigma_\epsilon^2}{(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2} < 0,$$

and

$$\frac{\partial\gamma^*}{\partial R} = \frac{2dMR\sigma_\epsilon^2\sigma_\theta^2)^2}{(dR\sigma_\epsilon^2 + 2\sigma_\theta^2)^2} > 0,$$

Proof of proposition 3

Note that $E[e_1^*(\theta_1) + e_2^*(\theta_2)|\theta] = e^*(\theta^H) + e^*(\theta^L)$. Then,

$$\begin{aligned} e^*(\theta^H) + e^*(\theta^L) &= \frac{\beta\theta^H + \gamma}{d} + \frac{\beta\theta^L + \gamma}{d} \\ &= \frac{2(M\beta + \gamma)}{d} \\ &= \frac{2}{d}(\frac{2M\sigma_\theta^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2} + \frac{dMR\sigma_\epsilon^2}{dR\sigma_\epsilon^2 + 2\sigma_\theta^2}) \\ &= \frac{2M}{d} \end{aligned}$$

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