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The leximin foundation of the equal-income Walras rule*

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Abstract

We introduce the leximin criterion studied in the literature of social welfare functionals into the pure exchange economy model. We show that Leximin Justification Possibility (LMJP) and Leximin All Unanimity (LMAU), both formulated on the idea of leximin fairness, fully characterize the equal-income Walras rule.

Keywords: the equal-income Walras rule; extended preference; leximin criterion; Suppes criterion;

1 Introduction

The notion of extended preference, developed primarily in the literature of Arrow's impossibility theorem (Arrow 1963), makes it possible to compare the welfare of different individuals in that problem.[1] The purpose of this study is to apply this approach to the resource allocation problem of exchange economies with a finite number of agents and goods.

The study consists of three parts. In the first part, we introduce extended preferences defined in the economic environment and discuss their properties.

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[1] d'Aspremont (1985), d'Aspremont and Gevers (2002), Sen (1970,1977,1986), and Suzumura (1983) survey this area. Fleurbaey and Hammond (2004) and Mongin and d'Aspremont (2004) survey the area from a broader perspective standing on ethics and utility theories. Blackorby et al. (1984) provide a diagrammatic introduction. The most recent contribution is Yamamura (2017).

Sen (1974a,b) and Deschamps and Gevers (1978) consider the income distribution problem in a single commodity economy.

The second part formulates the leximin criterion; it characterizes the set of leximin-equitable allocations.[2] We derive an axiomatization of the leximin rule defined on exchange economies (Theorem 1), interpreted as a counterpart of those (Hammond 1976, d'Aspremont and Gevers 1977, and Deschamps and Gevers 1978) studied in the abstract framework of social choice.

The third part, adding an assumption of smooth preferences with a type of boundary condition, addresses the central issue. We show that the equal-income Walras rule is the unique rule satisfying Leximin Justification Possibility (LMJP) and Leximin All Unanimity (LMAU), both defined using the idea of leximin fairness (Theorem 2).

A duality relationship holds between LMJP and LMAU. Take a feasible allocation z arbitrarily. The former requires that if a rule selects it, there must exist at least one extended preference that makes it leximin-equitable.[3] The latter says that if z is leximin-equitable for all extended preferences, the rule must select it. To be precise, it does not have to be "for all." It is sufficient that z is leximin-equitable for extended preferences meeting a desirable property, as will be shown later.

The implication of Theorem 2 for interpersonal comparisons of welfare, especially the relevance with invariance axioms studied in social welfare functionals, is also discussed in this part.

This study is the first to axiomatize the equal-income Walras rule with the idea of leximin fairness. Most of the literature has characterized it using the concept of horizontal equity such as envy-free or others, with and without stability conditions in the setting of a variable number of agents (Thomson 1988, Nagahisa and Suh 1995, Maniquet 1996, and Toda 2004).

[2] Here, a feasible allocation is leximin-equitable if there exists no other feasible one better than it from the leximin point of view.

[3] As we will see later, a preference profile creates multiple, sometimes infinite, extended preferences.

The remaining paper is organized as follows: We provide all notations and definitions in section 2; study the properties of leximin-equitable allocations in section 3; presents the main results in section 4; conclude the study in section 5; and, finally, we also discuss subordinate matters in the appendix.

2 Notation and Definitions

2.1 Exchange Economies

The notation for vector inequalities is $\leq$, $<$, and $\ll$. Let $\Delta^l := \{p \in R^l_+ : \sum_{i=1}^{l} p_i = 1\}$ and $int.\Delta^l := \{p \in R^l_{++} : \sum_{i=1}^{l} p_i = 1\}$.

We consider exchange economies with a finite number of agents and a finite number of private goods. Let $N = \{1, 2, ..., n\}$ and $L = \{1, 2, ..., l\}$ be the set of agents and the set of private goods respectively. All agents have the same consumption set R^l_+. Let $z_i = (z_{i1}, ..., z_{il}) \in R^l_+$ and $z = (z_1, ..., z_n) \in R^{nl}_+$ be agent i's consumption and an allocation respectively. Let $\Omega \in R^l_{++}$ be the total endowment of the economy, owned collectively, and fixed throughout the study. An allocation z is feasible if $\sum_{i \in N} z_i \leq \Omega$. Let Z be the set of feasible allocations.

Let $\succcurlyeq_i$ be agent i's preference on R^l_+, where $\succ_i$ and $\sim_i$ read as usual. We assume that $\succcurlyeq_i$ is continuous, convex, and monotonic on R^l_+. We say that $\succcurlyeq_i$ is convex if for any $x, y \in R^l_+$, $x \succ_i y$ implies $tx + (1-t)y \succ_i y$ for all $t \in (0, 1)$ and that $\succcurlyeq_i$ is monotonic if $x > y$ implies $x \succ_i y$. Let Q be the set of preferences satisfying all the conditions. A list of all agents' preferences, denoted $\succcurlyeq = (\succcurlyeq_i)_{i \in N}$, is called a profile. Let $Q^n = Q \times \cdots \times Q$, where Q appears n times, be the set of profiles.

Take a profile $\succcurlyeq$ arbitrarily. A feasible allocation z is Pareto optimal if and only if there are no other feasible allocations z' with $z'_i \succ_i z_i$ for all $i \in N$.[4] Let $PO(\succcurlyeq)$ be the set of Pareto optimal allocations. A feasible allocation z

[4] The preferences are continuous and monotonic on R^l_+. Thus, the strong Pareto optimality reduces to the weak one.

is an equal-income Walrasian allocation if there exists some $p \in int.\Delta^l$ such that $z_i \succcurlyeq_i x$ for all $x \in R^l_+$ with $px \leq p\left(\frac{\Omega}{n}\right)$. Let $EIW(\succcurlyeq)$ be the set of the equal-income Walrasian allocations.

2.2 Extended Preferences

The notion of *extended preferences* is based on the principle of extended-sympathy mentioned by Arrow (1963) and initiated by Suppes (1966) and Sen (1970). The basic idea is that a hypothetically existing ethical observer compares the welfare of different persons from a social point of view while respecting (or sympathizing with) their subjective preferences.

An extended preference $\succcurlyeq_E$ created from $\succcurlyeq \in Q^n$ is a complete and transitive binary relation on $R^l_+ \times N$. We read $(x,i) \succcurlyeq_E (y,j)$ as "being agent i with consumption x is at least as well off as being agent j with consumption y."[5] We read $\succ_E$ and $\sim_E$ as usual.

We say that a set of utility functions $(u_i)_{i\in N}$ represents $\succcurlyeq_E$ if $(x,i) \succcurlyeq_E (y,j) \Longleftrightarrow u_i(x) \geq u_j(y)$ for all $i,j \in N$ and all $x,y \in R^l_+$. We call $(u_i)_{i\in N}$ a representation of $\succcurlyeq_E$.

We assume that extended preferences satisfy the following properties.

E.1. For any $i \in N$, $x \succcurlyeq_i y \Longleftrightarrow (x,i) \succcurlyeq_E (y,i)$ for all $x,y \in R^l_+$.

E.2. $(\frac{\Omega}{n}, i) \sim_E (\frac{\Omega}{n}, j)$ for all $i,j \in N$.

E.3. $\succcurlyeq_E$ has a representation.

E.1 is called the axiom of identity, assumed in almost all literature related to extended preferences with the abstract framework of social choice.[6] In contrast, E.2 is an assumption specific to the economic environment. It implies the existence of an agreement of the society that distributing endowments among agents equally in the physical sense is equitable. What the equal-income Walras rule is appealing to us is that the equal division of initial endowments reflects

[5] Note that we admit the comparability of individual welfare, but not the cardinality.
[6] Refer to Sen (1970) and d'Aspremont (1985) for more details.

the concept of equal opportunity. That is so, no matter what individual preferences are. E.2 crystallizes this idea. E.3 says that extended preferences should have their utility representations. Just as utility functions are necessary only for technical reasons, E.3 is so for the same ground as well.

Let $E(\succcurlyeq)$ be the set of extended preferences created from $\succcurlyeq$ that satisfy the three properties. Note that $E(\succcurlyeq)$ is nonempty: For each i, we define u_i representing $\succcurlyeq_i$ such that $u_i(\frac{\Omega}{n})$ is equal across all i. Then, comparing the utilities among agents makes an extended preference satisfying E.1-E.3.

If we need to refer to multiple extended preferences created from $\succcurlyeq$, we use the notation $\succcurlyeq_{E'}$, $\succcurlyeq_{E''}$ and so on. Notice the difference between $\succcurlyeq_{E'}$ and $\succcurlyeq'_E$.

2.3 Two criteria of fairness

Let $\succcurlyeq_E$ be taken arbitrarily and fixed throughout this subsection.

2.3.1 The leximin criterion

The leximin criterion, a lexicographic extension of the difference principle of Rawls (1971), is defined as follows. Given an allocation z, we arrange all (z_i, i) in ascending order such that $(z_{i_1}, i_1) \preccurlyeq_E (z_{i_2}, i_2) \preccurlyeq_E \cdots \preccurlyeq_E (z_{i_{n-1}}, i_{n-1}) \preccurlyeq_E (z_{i_n}, i_n)$, where tie is broken arbitrarily. The agent i_k $(k = 1, .., n)$ is the kth worst off agent in z. We denote i_k by $i_k(z)$. A lexicographic order $\geq_{L(E)}$ on the set of allocations is defined as follows:

$$
\begin{aligned}
z >_{L(E)} z' \quad &\Longleftrightarrow \quad \begin{array}{c} \exists k \in \{1, ..., n-1) \text{ s. t.} \\ (z_{i_\tau(z)}, i_\tau(z)) \sim_E (z'_{i_\tau(z')}, i_\tau(z')) \forall \tau \in \{1, ..., k-1\} \\ \& \\ (z_{i_k(z)}, i_\tau(z)) \succ_E (z'_{i_k(z')}, i_\tau(z')). \end{array} \\
z =_{L(E)} z' \quad &\Longleftrightarrow \quad (z_{i_k(z)}, i_k(z)) \sim_E (z'_{i_k(z')}, i_k(z')) \forall k = 1, ..., n. \\
z \geq_{L(E)} z' \quad &\Longleftrightarrow \quad z >_{L(E)} z' \vee z =_{L(E)} z'.
\end{aligned}
$$

Given z and z', z is at least as leximin-just as z' if $z \geq_{L(E)} z'$. If this holds with $z >_{L(E)} z'$, z is more leximin-just than z'. In contrast, if that does with

$z =_{L(E)} z'$, z is equally as leximin-just as z'. Note that all are well defined and transitive, and $\geq_{L(E)}$ is complete. A feasible allocation is leximin-equitable if there is no other feasible one that is more leximin-just than it. Let $LME(\succcurlyeq_E)$ be the set of those allocations. We will discuss the non-emptiness of $LME(\succcurlyeq_E)$ in the next section.

2.3.2 Suppes criterion

Given an allocation z and a permutation π on N, let z^π be an allocation such that $z_i^\pi = z_{\pi(i)}$ for every $i \in N$. Let Π be the set of permutations. The three relations below constitute the *Suppes criterion* (Suppes 1966), occasionally called the grading principle, interpreted as an interpersonal extension of the Pareto criterion.

Given allocations z, z', z is at least as Suppes-just as z' if there is some $\pi \in \Pi$ such that $(z_i, i) \succcurlyeq_E (z'_{\pi(i)}, \pi(i))$ for all $i \in N$. If this holds with $\succ_E$ for at least one member i, z is more Suppes-just than z'. In contrast, if that does with $\sim_E$ for all i, z is equally as Suppes-just as z'. All are well defined and transitive. The relation of "equally as Suppes-just as" is symmetric, whereas that of "more Suppes-just than" is asymmetric. A feasible allocation is Suppes-equitable if and only if there is no other feasible one that is more Suppes-just than it. Let $SE(\succcurlyeq_E)$ be the set of those allocations. Note that $LME(\succcurlyeq_E) \subset SE(\succcurlyeq_E) \subset PO(\succcurlyeq)$ holds.[7].

3 An axiomatization of the leximin rule

Take $\succcurlyeq_E$ arbitrarily. Let $EQ(\succcurlyeq_E)$ be defined as follows.

$$EQ(\succcurlyeq_E) = \{z \in Z : (z_i, i) \sim_E (z_j, j)\ \forall i, j \in N\}.$$

The lemma below demonstrates that $LME(\succcurlyeq_E)$ is nonempty and consists

[7] $LME(\succcurlyeq_E) \subset SE(\succcurlyeq_E)$ is by definition. $SE(\succcurlyeq_E) \subset PO(\succcurlyeq)$ is due to E.1 (the axiom of identity).

of the intersection of $EQ(\succsim_E)$ and $SE(\succsim_E)$, which is equivalent to $EQ(\succsim_E) \cap PO(\succsim)$.

Lemma 1 $\emptyset \neq LME(\succsim_E) = EQ(\succsim_E) \cap SE(\succsim_E) = EQ(\succsim_E) \cap PO(\succsim)$.

Proof. Let $u = (u_i)_{i \in N}$ be a representation of $\succsim_E$, fixed throughout the proof. Note that E.3 assures the existence. (i)-(iv) below completes the proof.

(i) $LME(\succsim_E) \subset EQ(\succsim_E) \cap SE(\succsim_E)$: As $LME(\succsim_E) \subset SE(\succsim_E)$, it suffices to show $LME(\succsim_E) \subset EQ(\succsim_E)$. Suppose not. Then, there exists some $z \in LME(\succsim_E)$ with $z \notin EQ(\succsim_E)$.

Let us arrange the pairs of agent and consumption at z in ascending order. We find that $<$ appears at some $k \in \{1, ..., n-1\}$ such that

$$u_{i_1}(z_{i_1}) \leq \cdots \leq u_{i_{k-1}}(z_{i_{k-1}}) \leq u_{i_k}(z_{i_k}) < u_{i_{k+1}}(z_{i_{k+1}}) \leq u_{i_{k+2}}(z_{i_{k+2}}) \leq \cdots \leq u_{i_n}(z_{i_n}).$$

Note that E.2 and $z \in LME(\succsim_E)$ imply $u_{i_{k+1}}(z_{i_{k+1}}) > u_{i_{k+1}}(\frac{\Omega}{n})$. As the preference is monotonic, we have $z_{i_{k+1}} \neq 0$. Take $\varepsilon \in R^l_+/\{0\}$ sufficiently small such that the transfer ε of goods from agent i_{k+1} to agent i_k makes the ascending order unchanged. Note that this transfer is possible as u_{i_k} and $u_{i_{k+1}}$ are continuous functions. Let z' be the feasible allocation made from z by that transfer. Then, we have $z' >_{L(E)} z$, which contradicts $z \in LME(\succsim_E)$.

(ii) $EQ(\succsim_E) \cap SE(\succsim_E) \subset EQ(\succsim_E) \cap PO(\succsim)$: This directly follows from $SE(\succsim_E) \subset PO(\succsim)$.

(iii) $EQ(\succsim_E) \cap PO(\succsim) \subset LME(\succsim_E)$: Suppose not. Then, there exists some $z \in EQ(\succsim_E) \cap PO(\succsim)$ with $z \notin LME(\succsim_E)$. Then, there exists some feasible allocation z' that is more leximin-just than z. The diagram below illustrates the comparison of utilities in z and z'.

$$\begin{array}{ccccccccccccc}
u_1(z_1) & = & u_2(z_2) & = & \cdots & = & u_{k-1}(z_{k-1}) & = & u_k(z_k) & = & \cdots & = & u_n(z_n) \\
\shortparallel & & \shortparallel & & & & \shortparallel & & \wedge & & & & \\
u_{i_1}(z'_{i_1}) & \leq & u_{i_2}(z'_{i_2}) & \leq & \cdots & \leq & u_{i_{k-1}}(z'_{i_{k-1}}) & \leq & u_{i_k}(z'_{i_k}) & \leq & \cdots & \leq & u_{i_n}(z'_{i_n})
\end{array}$$

As z' is more leximin-just than z, there exists k shown in the diagram. The diagram reveals $u_i(z'_i) \geq u_i(z_i)$ for all i, with inequality for $i_k, ..., i_n$, which contradicts $z \in PO(\succsim)$.

(iv) $\emptyset \neq LME(\succsim_E)$: The proof reduces to that of $EQ(u) \cap PO(u) \neq \emptyset$, where $EQ(u) = \{z \in Z : u_i(z_i) = u_j(z_j) \ \forall i, j\}$. Let U be a subset in utility space such that $U = \{(u_i(z_i))_{i \in N} : z = (z_i)_{i \in N} \in Z\}$. Imagine a 45-degree line on R^n passing through the origin. That line intersects with U at a point $(u_i(\frac{\Omega}{n}))_{i \in N}$, as $u_i(\frac{\Omega}{n})$ is equal across all agents thanks to E.2. As U is closed and upper bounded, the line intersects with the upper boundary of U, which is in $EQ(u) \cap PO(u)$. Refer to the appendix for the details of the proof of the last part. ■

Lemma 1 provides an axiomatization of the leximin rule. Let $\mathcal{E} = \{ \succsim_E \in E(\succsim) : \succsim \in Q^n\}$. Let $\mathcal{D}$ be a nonempty subset of $\mathcal{E}$. A social choice rule f, a rule hereafter, associates with each $\succsim_E \in \mathcal{D}$ a nonempty subset of feasible allocations. The leximin rule f_{LM} associates $LME(\succsim_E)$ with each $\succsim_E \in \mathcal{D}$. A rule f satisfies (i) Suppes Equity (SE) if for any $\succsim_E \in \mathcal{D}$, $f(\succsim_E) \subset SE(\succsim_E)$, (ii) Complete Equity (CE) if for any $\succsim_E \in \mathcal{D}$, $f(\succsim_E) \subset EQ(\succsim_E)$, and (iii) Suppes Non-Discrimination (SND) if for any $\succsim_E \in \mathcal{D}$, and any $z, z' \in Z$, if z is equally as Suppes-just as z', then $z \in f(\succsim_E) \iff z' \in f(\succsim_E)$. The theorem below is a direct consequence of Lemma 1.

Theorem 1 *The leximin rule is the only rule satisfying SE, CE, and SND.*

The perfect divisibility of goods makes the leximin rule select feasible allocations with no utility gap,[8] which is the reason that the leximin rule satisfies CE. In contrast, the literature concerning social welfare functionals that studied the axiomatization of the leximin rule (Hammond 1976, d'Aspremont and Gevers 1977, and Deschamps and Gevers 1978) lacks that assumption.[9] In those axiomatizations, a requirement of equity, Hammond equity or its variants, substitutes

[8] The continuity of utility functions also contributes to it.
[9] All assume the set of alternatives to be finite.

for CE. This requirement, in collaboration with other well-known axioms, improves the inequalities of utilities as much as possible. One of the remaining problems is refining Theorem 1 to allow comparison with the axiomatizations of the leximin rule in the literature. Among others, the axiomatization of Hammond is the closest to ours. Though dressed in different formulas, SE and SND appear in the axiomatization. Clarifying the relevance of CE with Hammond equity is the key to solve the problem.

Taking production into account is a problem to yet be considered. We conjecture that, in such cases, the leximin rule does not attain egalitarian allocations.

A feasible allocation z meets the equal-division lower bound if $z_i \succcurlyeq_i \frac{\Omega}{n}$ for all i. This criterion, advocated by many researchers[10], says that each agent should find his bundle at least as desirable as equal division. Lemma 2 below shows that leximin-equitable allocations meet that criterion.

Lemma 2 *For any $z \in LME(\succcurlyeq_E)$, $z_i \succcurlyeq_i \frac{\Omega}{n}$ for all i.*

Proof. Notice that the relation below holds.

$$(z_1, 1) \sim_E (z_2, 2) \sim_E \cdots \sim_E (z_n, n) \succcurlyeq_E (\frac{\Omega}{n}, 1) \sim_E (\frac{\Omega}{n}, 2) \sim_E \cdots \sim_E (\frac{\Omega}{n}, n),$$

where $(z_1, 1) \sim_E (z_2, 2) \sim_E \cdots \sim_E (z_n, n)$ follows from Lemma 1, $(\frac{\Omega}{n}, 1) \sim_E (\frac{\Omega}{n}, 2) \sim_E \cdots \sim_E (\frac{\Omega}{n}, n)$ from E.2, and $\succcurlyeq_E$ from that z is at least as leximin-just as $\left(\frac{\Omega}{n}, ..., \frac{\Omega}{n}\right)$. This reveals $(z_i, i) \succcurlyeq_E (\frac{\Omega}{n}, i)$ for all i. Invoking the axiom of identity, we have $z_i \succcurlyeq_i \frac{\Omega}{n}$ for all i. ■

We use Lemma 2 as well as Lemma 1 for the proof of the axiomatization of the equal-income Walras rule.

Last but not least, we point out the relevance of the Suppes criterion with egalitarian equivalence (Pazner and Schmeidler 1978). An allocation z is egalitarian-equivalent if there exists some consumption x with $z_i \sim_i x$ for all i. By definition, $\left(\frac{\Omega}{n}, ..., \frac{\Omega}{n}\right)$ is egalitarian-equivalent. Moreover, the following holds.

[10] Refer to Kolm (1973), and in particular, Pazner (1977), for instance.

Proposition 1 *If an allocation $z \in Z$ is equally as Suppes-just as $\left(\frac{\Omega}{n}, ..., \frac{\Omega}{n}\right)$, then z is egalitarian-equivalent.*

Proof. By supposition, there exists a permutation π such that $(z_i, i) \sim_E (\frac{\Omega}{n}, \pi(i))$ for all i. E.2 implies $(\frac{\Omega}{n}, \pi(i)) \sim_E (\frac{\Omega}{n}, i)$, and hence $(z_i, i) \sim_E (\frac{\Omega}{n}, i)$. Thus, E.1 means $z_i \sim_i \frac{\Omega}{n}$. As this holds for all i, z is egalitarian-equivalent. ■

4 An axiomatization of the equal-income Walras rule

4.1 Domains and Rules

Later discussions require a few additional assumptions about preferences and the set of profiles.

Let $\widehat{Q} \subset Q$ be the set of preferences represented by a continuously differentiable utility function u with $\left\{x \in R^l_+ : u(x) \geq u\left(\frac{\Omega}{n}\right)\right\} \subset R^l_{++}$, a type of boundary condition. Let $D = \widehat{Q} \times \widehat{Q} \times \cdots \times \widehat{Q}$, where $\widehat{Q}$ appears n times, be called the domain.

We return the definition of rules to the standard one. Let $F : D \longrightarrow Z$ be a *rule*, which associates with each profile a nonempty subset of feasible allocations. A rule F decides $F(\succcurlyeq)$ through interpersonal comparisons of welfare. It uses *some* extended preferences created from $\succcurlyeq$, but not necessarily *all*.

Given a profile $\succcurlyeq \in D$ and a feasible allocation $z \in Z$, let $D(\succcurlyeq, z)$ be a nonempty subset of $E(\succcurlyeq)$, interpreted as the set of extended preferences used for deciding whether $z \in F(\succcurlyeq)$. We call $D(\succcurlyeq, z)$ the decision set of $(\succcurlyeq, z)$, or the decision set briefly.

A rule F decides $z \in F(\succcurlyeq)$ based only on extended preferences in $D(\succcurlyeq, z)$, not on extended preferences excluded from $D(\succcurlyeq, z)$. Let us call $\bigcup_{(\succcurlyeq, z) \in D \times Z} D(\succcurlyeq, z)$ the *extended-domain*. To simplify the notation, we occasionally write $\mathcal{D} =$

$\bigcup_{(\succsim, z) \in D \times Z} D(\succsim, z)$.[11] We impose an assumption on $\mathcal{D}$.

D.1. Let $z \in PO(\succsim) \cap R^{nl}_{++}$ and p be the supporting price vector associated with z. For any $\succsim_E \in D(\succsim, z)$, there exists a representation $(u_i)_{i \in N}$ of $\succsim_E$ such that $u_i(x) = px + \varepsilon_i(x)$ for all $x \in R^l_{++}$, where ε_i is a higher-order error term with $\varepsilon_i(z_i) = 0$ and $\frac{\varepsilon_i(x)}{||x - z_i||} \longrightarrow 0$ as $x \longrightarrow z_i$.

We discuss the existence of $\mathcal{D}$ later. The justification of D.1 begins with a similar idea as that of Local Independence (LI) of Nagahisa (1991), a local version of Arrow's IIA, which states that the decision whether $z \in F(\succsim)$ depends only on the preferences around z.[12]

D.1 applies this idea to extended preferences. When we focus on only around z_i, we can see that $\succsim_i$ is approximately identical to that represented by a linear utility function $u_i(x) = px$. Let $\succsim^p$ be the profile made with the linear utility function. The following logic justifies D.1.

(1) As mentioned above, the material in creating extended preferences used for deciding $z \in F(\succsim)$ is preferences only around z.

(2) We can make the same dish with the same ingredients. This metaphor holds for $D(\succsim, z)$ and $D(\succsim^p, z)$ as well. They are almost identical because both are thought of as made from the same material.

(3) As for $\succsim^p$, there is no compelling reason to dismiss the idea that only that preference is the extended preference because all agents are identical in the preference point of view.[13]

Those reasons lead to the acceptance of D.1. Thus, as for around z, all

[11] $\mathcal{D}$ appeared in the previous section, so abuse of notation is permitted here.

[12] Refer to Nagahisa (1991) for more details on LI. Yoshihara (1998), Fleurbaey et al. (2005), and Miyagishima (2015) contribute generalizations and related axioms of LI. Sakai (2009) provides an ordering version of LI. Urai and Murakami (2015) use LI for economies with money.

LI, together with well-known axioms, characterizes the Walras rule (Nagahisa 1991, Nagahisa and Suh 1995). Gevers (1986) and Hurwicz (1979) initiated the study of the axiomatic analysis of the Walras rule. Hammond (2010) is a comprehensive survey of the field.

[13] Note that $\succsim^p$ does not belong to the domain. But, the reference to a hypothetical profile in creating extended preferences is not unreasonable.

$\succsim_E \in D(\succsim, z)$ should be almost the same as $(x,i) \succsim_E (y,j) \Leftrightarrow px \geq py$.

D.1 means that $D(\succsim, z)$ has about the same amount of information as that of the identical linear profile, which is generalized to the following definition. We say that $D(\succsim, z)$ and $D(\succsim', z)$ are informationally equivalent around z if (i) for any $\succsim'_E \in D(\succsim', z)$, there exists some $\succsim_E \in D(\succsim, z)$ such that $u'_i(x) = u_i(x) + \varepsilon_i(x)$ for all $x \in R^l_+$, and all $i \in N$, where $(u'_i)_{i \in N}$ and $(u_i)_{i \in N}$ are representations of $\succsim'_E$ and $\succsim_E$ respectively, and $\varepsilon_i(z_i) = 0$, $\frac{\varepsilon_i(x)}{||x - z_i||} \longrightarrow 0$ as $x \longrightarrow z_i$, and (ii) the same as (i) holds with interchanging the role of $D(\succsim, z)$ and $D(\succsim', z)$. The lemma below provides a necessary and sufficient condition for $D(\succsim, z)$ and $D(\succsim', z)$ to be informationally equivalent. The utilities and marginal utilities at z_i are the keys to it.

Lemma 3 *Let u_i and u'_i be continuously differentiable utility functions. The followings are equivalent.*

1. $u'_i(x) = u_i(x) + \varepsilon_i(x)$ for all $x \in R^l_{++}$, and all $i \in N$, where $\varepsilon_i(z_i) = 0$, $\frac{\varepsilon_i(x)}{||x - z_i||} \longrightarrow 0$ as $x \longrightarrow z_i$.

2. The marginal utilities at z_i stay unchanged across u_i and u'_i, and $u_i(z_i) = u'_i(z_i)$.

Proof. As 1 implies 2, we show the converse.

The continuous differentiability of u'_i implies

$u'_i(x) = u'_i(z_i) + \sum_{h \in L} \frac{\partial u'_i(z_i)}{\partial x_h}(x_h - z_{ih}) + \delta'_i(x)$, where $\delta'_i(z_i) = 0$, $\frac{\delta'_i(x)}{||x - z_i||} \longrightarrow 0$ as $x \longrightarrow z_i$.

As 2. holds, we have

3. $u'_i(x) = u_i(z_i) + \sum_{h \in L} \frac{\partial u_i(z_i)}{\partial x_h}(x_h - z_{ih}) + \delta'_i(x)$.

The continuous differentiability of u_i implies

$u_i(x) = u_i(z_i) + \sum_{h \in L} \frac{\partial u_i(z_i)}{\partial x_h}(x_h - z_{ih}) + \delta_i(x)$, where $\delta_i(z_i) = 0$, $\frac{\delta_i(x)}{||x - z_i||} \longrightarrow 0$ as $x \longrightarrow z_i$.

A deformation of this equation is $u_i(z_i) + \sum_{h\in L} \frac{\partial u_i(z_i)}{\partial x_h}(x_h - z_{ih}) = u_i(x) - \delta_i(x)$. Substituting this into 3., we have $u_i'(x) = u_i(x) + \delta_i'(x) - \delta_i(x)$, which completes the proof by considering $\delta_i'(x) - \delta_i(x)$ as $\varepsilon_i(x)$. ■

The example below illustrates an extended-domain satisfying D.1.

Example 1 The construction of the domain goes on with two steps.

Step 1. Take $\succsim \in D$ and $z \in PO(\succsim) \cap R^{nl}_{++}$ arbitrarily. Let p be the price vector associated with z. Let w_i be a continuous differentiable utility function representing $\succsim_i$, expressed as follows around z_i: for all $x_i \in R^l_+$,

$w_i(x_i) = w_i(z_i) + \sum_{h\in L} \frac{\partial w_i(z_i)}{\partial x_{ih}}(x_{ih} - z_{ih}) + \varepsilon_i(x_i)$, where $\varepsilon_i(z_i) = 0$, $\frac{\varepsilon_i(x_i)}{||x_i - z_i||} \longrightarrow 0$ as $x_i \longrightarrow z_i$.[14]

As $\frac{\partial w_i(z_i)}{\partial x_{ih}} = \alpha_i p_h$ for all $h \in L$, where $\alpha_i > 0$ is the Lagrange multiplier, we have $w_i(x_i) = \alpha_i p x_i + w_i(z_i) - \alpha_i p z_i + \varepsilon_i(x_i)$.

Let γ_i $(i = 1, ..., n)$ be such that $\frac{w_i(\frac{\Omega}{n}) - (w_i(z_i) - \alpha_i p z_i + \gamma_i)}{\alpha_i}$ is equal across all agents. Let w_i' be the utility function with $w_i' = w_i + \gamma_i$. Then, we have

$w_i'(x_i) = \alpha_i p x_i + (w_i(z_i) - \alpha_i p z_i + \gamma_i) + \varepsilon_i(x_i)$, which is transformed to

$$\frac{w_i'(x_i) - (w_i(z_i) - \alpha_i p z_i + \gamma_i)}{\alpha_i} = p x_i + \frac{\varepsilon_i(x_i)}{\alpha_i}.$$

Obviously, the left term of this equation is interpreted as a continuously differentiable utility function representing $\succsim_i$, taken of the form of $p x_i + \frac{\varepsilon_i(x_i)}{\alpha_i}$. To simplify the notation, we denote it by v_i.

Step 2. Let $\succsim_{E^v}$ be the extended preference whose representation is $(v_i)_{i\in N}$. Note that $\succsim_{E^v}$ satisfy E.2 as $\frac{w_i'(\frac{\Omega}{n}) - (w_i(z_i) - \alpha_i p z_i + \gamma_i)}{\alpha_i}$ is equal across all agents. The extended-domain $\mathcal{D}$ consists of the following decision sets.

$$D(\succsim, z) = \{\succsim_{E^v}\} \text{ if } z \in PO(\succsim) \cap R^{nl}_{++}.$$

[14] This formula usually given on the interior of R^l_+ holds for the whole of R^l_+ by choosing $\varepsilon_i(x_i)$ appropriately.

We need not specify $D(\succcurlyeq, z)$ in the case of $z \notin PO(\succcurlyeq) \cap R^{nl}_{++}$.

4.2 Axioms

A rule F satisfies Leximin Justification Possibility (LMJP) if, for any $z \in F(\succcurlyeq)$, $z \in LME(\succcurlyeq_E)$ holds for some $\succcurlyeq_E \in D(\succcurlyeq, z)$. A rule F satisfies Leximin All Unanimity (LMAU) if for any $\succcurlyeq \in D$ and any $z \in Z$, $z \in \bigcap_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E)$ implies $z \in F(\succcurlyeq)$.

Both are defined using the concept of leximin fairness. Take a feasible allocation z arbitrarily. LMJP requires that if a rule selects it, there must exist at least one extended preference in the decision set that makes it leximin-equitable. In contrast, LMAU says that if all those extended preferences in that decision set make z leximin-equitable, the rule must select it. Thus, LMJP and LMAU have a duality relationship.

4.3 Results

Take $\succcurlyeq \in D$ arbitrarily. Let $LME^{\exists}(\succcurlyeq)$ and $LME^{\forall}(\succcurlyeq)$ be defined as follows.

$$LME^{\exists}(\succcurlyeq) = \left\{ z \in Z : z \in \bigcup_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E) \right\}$$

and

$$LME^{\forall}(\succcurlyeq) = \left\{ z \in Z : z \in \bigcap_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E) \right\}.$$

Lemma 4 *For any $\succcurlyeq \in D$, we have*

$LME^{\exists}(\succcurlyeq) = EIW(\succcurlyeq) = LME^{\forall}(\succcurlyeq).$

Proof. The following (i)-(iii) completes the proof.

(i) $LME^{\exists}(\succcurlyeq) \subset EIW(\succcurlyeq)$.

Take $z \in LME^{\exists}(\succcurlyeq)$ arbitrarily. By definition, there exists some $\succcurlyeq_E \in D(\succcurlyeq, z)$ such that $z \in LME(\succcurlyeq_E)$. It is easy to see $z \in PO(\succcurlyeq) \cap R^{nl}_{++}$. Lemma

1 implies $z \in PO(\succsim)$. As for $z \in R^{nl}_{++}$, this is a direct consequence of the combination of Lemma 2 and the boundary condition.

Thus, D.1 implies that $\succsim_E$ is represented by $(u_i)_{i \in N}$ such that $u_i(x) = px + \varepsilon_i(x)$ for all $x \in R^l_+$, where p associates with z. The relation below shows $pz_i = pz_j$ for all i, j.

$$pz_i = u_i(z_i) = u_j(z_j) = pz_j \text{ for all } i, j,$$

where $pz_i = u_i(z_i)$ and $u_j(z_j) = pz_j$ are by definition of $(u_i)_{i \in N}$, and Lemma 1 shows $u_i(z_i) = u_j(z_j)$.

Therefore, there exist only three cases: (i) $pz_i = p\left(\frac{\Omega}{n}\right)$ for all i, (ii) $pz_i < p\left(\frac{\Omega}{n}\right)$ for all i, and (iii) $pz_i > p\left(\frac{\Omega}{n}\right)$ for all i. As (ii) and (iii) contradict $\sum_{i \in N} z_i = \Omega$ resulting from $z \in PO(\succsim)$, (i) holds. By noting that p associates with z, this implies $z \in EIW(\succsim)$.

(ii) $EIW(\succsim) \subset LME^{\forall}(\succsim)$.

Take $z \in EIW(\succsim)$ arbitrarily. By definition, we have $z \in PO(\succsim) \cap R^{nl}_{++}$. Thus, D.1. implies that for any $\succsim_E \in D(\succsim, z)$, there exists a representation $(u_i)_{i \in N}$ such that $u_i(x) = px + \varepsilon_i(x)$ for all $x \in R^l_{++}$, where p associates with z. As $z \in EIW(\succsim)$ implies $pz_i = pz_j$ for all i, j, we have $u_i(z_i) = u_j(z_j)$ for all i, j, i.e., $z_i \sim_E z_j$ for all i, j. Note that this holds for any $\succsim_E \in D(\succsim, z)$. Thus, Lemma 1 shows $z \in LME(\succsim_E)$ for all $\succsim_E \in D(\succsim, z)$, which implies $z \in LME^{\forall}(\succsim)$.

(iii) $LME^{\forall}(\succsim) \subset LME^{\exists}(\succsim)$.

Obvious. ■

Theorem 2 *(1) A rule F satisfies LMJP if and only if $F(\succsim) \subset EIW(\succsim)$ for all $\succsim \in D$;*

(2) A rule F satisfies LMAU if and only if $F(\succsim) \supset EIW(\succsim)$ for all $\succsim \in D$; and

(3) The equal-income Walras rule is the only rule satisfying LMJP and LMAU.

Proof. It is a direct consequence of Lemma 4. ■

We see the inclusion relation reversed in (1) and (2), which shows that LMJP and LMAU are assigned a completely different role in the axiomatization of the equal-income Walras rule.

The examples below illustrate the independence of axioms. In both cases, the extended-domain $\mathcal{D}$ is arbitrary, with E.1, E.2, E.3, and D.1 imposed.

Example 2 Let a rule F be as follows.

$$F(\succcurlyeq) = \begin{cases} \left\{\left(\frac{\Omega}{n}, ..., \frac{\Omega}{n}\right)\right\} & \text{if } \left(\frac{\Omega}{n}, ..., \frac{\Omega}{n}\right) \in EIW(\succcurlyeq) \\ EIW(\succcurlyeq) & \text{otherwise.} \end{cases}$$

F satisfies LMJP, but not LMAU.

Example 3 Let x^0 be the allocation where no one receives anything. Let a rule F be such that $F(\succcurlyeq) = EIW(\succcurlyeq) \cup \{x^0\}$. F satisfies LMAU, but not LMJP.

Before closing this section, two more remarks remain.

(1) We consider the relevance between the equal-income Walras rule and the leximin rule from the viewpoint of invariance axioms studied in the social welfare functionals. The table below, a simplification of that of d'Aspremont and Gevers (1976), illustrates four invariance axioms.[15]

		Comparability: Nonexistent		Full
Measurability	Ordinal	$ON(\phi_i)$	$\longrightarrow$	$CO(\phi)$
		$\downarrow$		$\downarrow$
	Cardinal	$CN(\alpha_i, \beta_i)$	$\longrightarrow$	$CC(\alpha, \beta)$

[15] Refer to d'Aspremont (1985), d'Aspremont and Gevers (2002), Fleurbaey (2003), Sen (1977, 1979, 1986) for the details of invariance axioms.

The arrows indicate implication relations. Here, $ON(\phi_i)$ stands for Ordinality and Noncomparability (ON), which says that for any representations $u = (u_i)_{i\in N}$, $u' = (u'_i)_{i\in N} \in U$, if for every $i \in N$, there exists some strictly increasing numerical function ϕ_i such that $u'_i = \phi_i(u_i)$, then $f(u) = f(u')$.[16] Moreover, if ϕ_i is common for all i, ON reduces to Co-Ordinality, denoted by $CO(\phi)$. On the other hand, $CN(\alpha_i, \beta_i)$ stands for Cardinality and Noncomparability (CN), which says that for any representations $u = (u_i)_{i\in N}$, $u' = (u'_i)_{i\in N} \in U$, if for every $i \in N$, there exists some constants $\alpha_i > 0$ and β_i such that $u'_i = \alpha_i u_i + \beta_i$, then $f(u) = f(u')$. Moreover, if α_i and β_i are common for all i, CN reduces to Co-Cardinality, denoted by $CC(\alpha, \beta)$.

The equal-income Walras rule satisfies ON, whereas the leximin only satisfies CO. What puzzles us here is why LMJP and LMAU, despite both defined only with the leximin criterion, can axiomatize the equal-income Walras rule. Why do not the differences that appear when viewed from the invariance axioms affect the axiomatization? The proposition below replies to this question.

Proposition 2 *Take $z \in Z \cap R^{nl}_{++}$ arbitrarily. Suppose that each i's marginal rates of substitutions (MRS) at z stay unchanged for $\succcurlyeq$ and $\succcurlyeq'$. Then, $z \in LME(\succcurlyeq_E)$ implies $z \in LME(\succcurlyeq'_E)$.*

Proof. As $z \in PO(\succcurlyeq)$ follows, D.1 implies that there exists some $p \in int\Delta^l$ such that $u_i(x) = px + \varepsilon_i(x)$ for all i, where $(u_i)_{i\in N}$ is a representation of $\succcurlyeq_E$, and ε_i is a higher-order error term with $\varepsilon_i(z_i) = 0$. Lemma1 and $z \in LME(\succcurlyeq_E)$ implies pz_i is equal across all i.

The condition concerning MRS implies $z \in PO(\succcurlyeq')$ with p being the supporting price vector. This together with D.1 implies $u'_i(x) = px + \varepsilon'_i(x)$, where $(u'_i)_{i\in N}$ is a representation of $\succcurlyeq'_E$ and $\varepsilon'_i(z_i) = 0$. As pz_i was equal across all agents, so is $u'_i(z_i)$ as well. Thus, Lemma 1 implies $z \in LME(\succcurlyeq'_E)$. ■

[16] The definition of rules returns to f again, defined on U, the set of representations.

The MRS condition is equivalent to the following: For each i, there exist $\alpha_i > 0$ and β_i such that $u'_i(x) = \alpha_i u_i(x) + \beta_i + \varepsilon_i(x)$, where ε_i is a higher-order error term with $\varepsilon_i(z_i) = 0$. We see u'_i approximate to a positive affine transformation of u_i around z_i. If $z \in LME(\succsim_E) \implies z \in LME(\succsim'_E)$ was true for this case, we could say that the leximin rule satisfies a local version of CN. However, this relation cannot hold in general, as the coefficients α_i and β_i do not necessarily take the same for all i. That is where D.1 comes in, which assures $\alpha_i = 1$ and $\beta_i = 0$ for all i, declaring any other values void. Thus, the leximin rule can meet a local version of CC, specified $\alpha = 1$, and $\beta = 0$, which is the reason Proposition 2 holds.

Whether invariance axioms hold depends on the size of the domain. Restricting it to meet D.1 makes CN and CC equivalent. Thus, the leximin rule can satisfy CN, and we resolve the discrepancy.

(2) LMJP implies Pareto Optimality (PO) and Individual Rationality (IR) in the case that the initial endowments are distributed equally among agents.[17] This observation invites us to compare the two axiomatizations of the equal-income Walras rule, one of which is ours, LMJP+LMAU, and the other is Nagahisa and Suh (1995)'s, IR+PO+LI.[18] We cannot find any counterpart relation between the two axiomatic systems. For example, LMJP=IR+PO is false. Let F be a rule such that for every profile, it selects only one allocation that is Pareto optimal and individually rational. This rule satisfies IR and PO. However, Theorem 2 demonstrates that it violates LMJP. The possibility of LMJP=IR and LMJP=PO also disappears for the same ground. See the appendix for the full discussion.

[17] PO is by definition, IR follows from Lemma 2.

[18] The domain of Nagahisa and Suh differs slightly from that of ours. Thus, it is uncertain that their axiomatization still holds for the present study, while we believe probably correct. The following discussion assumes that the axiomatization is correct.

5 Conclusion

This paper combined two studies advanced independently in social choice; the study of interpersonal comparisons of welfare and that of axiomatic analysis of resource allocation problems.[19] We demonstrated a new axiomatization of the equal-income Walras rule, which displays its close relationship with the concept of leximin fairness. A significant number of studies have proved the advantages in normative aspects of the equal-income Walras rule.[20] This paper also belongs to that stream of research.

We conclude with one more remark, concerned with a policy implication. The claim that we make use of the equal-income Walras rule to solve the problem of resource allocations may sound unrealistic. It is unlikely that most people with already having private properties agree with it. But for issues with no ownership yet established, such as carbon emission trading, the polar and space developments, this proposal deserves consideration. The axiomatic approach suggests that the best way to address these issues is to refrain from competing for ownership as much as possible, to distribute the initial endowments equally, and to trust market transactions. We may not be able to eliminate the inequalities of today but can do those of tomorrow.

6 Appendix

The last part of the proof of Lemma 1:

Proof. Let T be such that $\{t : (t, ..., t) \in U\}$. T is nonempty because of $(\frac{\Omega}{n}, ..., \frac{\Omega}{n}) \in U$. As U is closed and upper bounded, so is T. Let t^* be the upper limit of T. Let $z^* \in Z$ be such that $t^* = u_i(z_i^*)$ for all i. Suppose that z^* is not Pareto optimal. By noting the monotonicity of preferences, there

[19] Chamber and Hayashi (2017) proposed an alternative axiomatic approach of the Walras rule that can cope with income distribution problems.

[20] Thomson (2007) includes a comprehensive survey of this direction.

exists some $z' \in Z$ such that $u_i(z'_i) > t^* = u_i(z^*_i)$ for all i. Without loss of generality, we assume that agent n has the lowest utility in z'. Thus, for any $i \neq n$, $u_i(z'_i) \geq u_n(z'_n) > t^* = u_i(z^*_i)$. We can choose $\lambda_i \in [0,1]$ such that $u_i(\lambda_i z'_i + (1-\lambda_i) z^*_i) = u_n(z'_n)$ for each i. Letting $z^\lambda \in Z$ be such that $\lambda_i z'_i + (1-\lambda_i) z^*_i$ for all i, we have $\left(u_i\left(z^\lambda_i\right)\right)_{i\in N} \in T$ and $u_i\left(z^\lambda_i\right) > t^*$, which contradicts that t^* is the upper limit, which completes the proof. ■

The comparison of the two axiomatizations:

Proof. As mentioned in the main text, LMJP=IR+PO, LMJP=IR, and LMJP=PO do not hold. As for LMAU, things are the same as well. The table below illustrates the remaining possibilities, where $\times$ shows that the independence of axioms lacks.

	LMJP=LI	LMJP=LI+IR	LMJP=LI+PO
LMAU=LI	$\times$	$\times$	$\times$
LMAU=LI+IR	$\times$	$\times$	Case 1
LMAU=LI+PO	$\times$	Case 2	$\times$

Thus, Cases 1 and 2 only remain. Let F be a rule such that it only selects $\left(\frac{\Omega}{n}, ..., \frac{\Omega}{n}\right)$ for any profile. This rule satisfies IR and LI. But, Theorem 2 shows LMJP=LI+IR is a contradiction. LMAU=LI+IR is so as well. Thus, the possibility of Case 1 and 2 disappears. ■

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