ソシオネットワーク戦略ディスカッションペーパーシリーズ ISSN 1884-9946
第 68 号　2019 年 1 月
RISS Discussion Paper Series
No.68　January, 2019

VCG Mechanism for Multi-unit Auctions and Appearance of Information: An Experiment

Satoshi Takahashi, Yoichi Izunaga, Naoki Watanabe

文部科学大臣認定　共同利用・共同研究拠点

関西大学ソシオネットワーク戦略研究機構

The Research Institute for Socionetwork Strategies,
Kansai University
Joint Usage / Research Center, MEXT, Japan
Suita, Osaka, 564-8680, Japan
URL: http://www.kansai-u.ac.jp/riss/index.html
e-mail: riss@ml.kandai.jp
tel. 06-6368-1228
fax. 06-6330-3304

VCG Mechanism for Multi-unit Auctions and Appearance of Information: An Experiment

Satoshi Takahashi, Yoichi Izunaga, Naoki Watanabe

文部科学大臣認定　共同利用・共同研究拠点

関西大学ソシオネットワーク戦略研究機構

The Research Institute for Socionetwork Strategies,
Kansai University

Joint Usage / Research Center, MEXT, Japan

Suita, Osaka, 564-8680, Japan

URL: http://www.kansai-u.ac.jp/riss/index.html

e-mail: riss@ml.kandai.jp

tel. 06-6368-1228

fax. 06-6330-3304

VCG Mechanism for Multi-unit Auctions and Appearance of Information: An Experiment

Satoshi Takahashi* Yoichi Izunaga† Naoki Watanabe‡

January 19, 2019

Abstract

This paper investigates whether, in multi-unit auctions, different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue when VCG mechanism is applied to subject experiments of those auctions. We examine two types of appearance of information about bidders' valuations of the item given to them and their bids asked to submit; One type is unit valuations and unit bids themselves and the other type is unit valuations and unit bids multiplied by the units. We observed that there was no significant difference on average in allocative efficiency and in seller's revenue between those two types of appearance of information. Rather, in each appearance of information, there was a significant difference in subjects' bidding behavior between different displays of draws of unit valuations, but this behavioral difference did not significantly affect the difference in allocative efficiency.

Keywords: multi-unit auctions, VCG mechanism, experiment
JEL Classification: C92, D44, D82

*Graduate School of Informatics and Engineering, The University of Electro-Communications, Chofu, Tokyo, Japan. E-mail: `stakahashi@uec.ac.jp`

†Faculty of Business Sciences, University of Tsukuba, Otsuka Bunkyo, Tokyo, Japan. E-mail: `izunaga@gssm.otsuka.tsukuba.ac.jp`

‡Graduate School of Business Administration, Keio University, 4-1-1 Hiyoshi Kohoku, Yokohama, Kanagawa 223-8526, Japan. E-mail: `naoki50@keio.jp`.

1 Introduction

This paper investigates whether, in multi-unit auctions, different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue when Vickrey-Clarke-Groves (VCG) mechanism is applied to subject experiments of those auctions. We examine two types of appearance of information about bidders' valuations of the item given to them and their bids asked to submit; One type is unit valuations and unit bids themselves and the other type is valuations and bids, i.e., unit valuations and unit bids multiplied by the units.

In various auctions VCG mechanism attains allocative efficiency but suffers from its computational complexity, and thus approximation algorithms for VCG mechanism has been developed. Takahashi and Shigeno (2011), for instance, developed a greedy based approximation (GBA) algorithm for multi-unit auctions. Takahashi et al. (2018) reported that in their subject experiment VCG mechanism attained higher allocative efficiency than GBA algorithm, although there was no significant difference in seller's revenue between GBA algorithm and VCG mechanism; The average rate of efficiency was 97.37% in VCG and it was 93.65% in GBA.

In the experiment conducted by Takahashi et al. (2018), bidders submitted their unit bids after confirming their unit valuations on computer screens, because this type of appearance of information was considered as a key feature for the GBA to work well; In GBA, a bidder who submits the highest unit bid is given a priority to obtain some units of the item in the process of the item allocation. In many experiments, however, bidders submitted bids after confirming their valuations. Thus, we need to investigate whether different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue. In this paper, we examine the performance of VCG.

Our main observation is that there was no significant difference on average in allocative efficiency and in seller's revenue between those two types of appearance of information. Rather, in each appearance of information, there was a significant difference in subjects' bidding behavior between different displays of draws of unit valuations, but this behavioral difference did not significantly affect the difference in allocative efficiency.

The rest of this paper is organized as follows. Section 2 explains VCG mechanism for multiunit auctions for a single item. Section 3 describes the experimental design, and Section 4 shows the results. Section 5 closes this paper with some remarks for further research.

2 VCG Mechanism

We deal with an auction, where a seller wishes to sell M units of a single item and solicits bids from n buyers each of whom can purchase up to M units of the item. Let $N = \{1, ..., n\}$ be the set of buyers (bidders). For each bidder $i \in N$, denote his or her anchor values on the quantity by $\{d_i^k \mid k = 0, ..., \ell_i\}$, where $d_i^{k-1} < d_i^k$ for all k with $1 \leq k \leq \ell_i$, and denote his or her unit bids by $\{b_i^k \mid k = 1, ..., \ell_i\}$, where b_i^k is a buyer price in half-open range $(d_i^{k-1}, d_i^k]$ for $k = 1, ..., \ell_i$. It is assumed that $d_i^0 = 0$ and $d_i^{\ell_i} \leq M$ for every bidder $i \in N$. Each bidder i has a list of his or her anchor values and unit bids, i.e., $\{d_i^k \mid k = 0, ..., \ell_i\}$ and $\{b_i^k \mid k = 1, ..., \ell_i\}$. Let $\ell = \sum_{i \in N} \ell_i$.

Define a function $B_i : \mathbb{R}_+ \to \mathbb{R}$ for each $i \in N$ by

$$B_i(y) = \begin{cases} b_i^k \cdot y & (d_i^{k-1} < y \leq d_i^k, k = 1, ..., \ell_i), \\ 0 & (y = d_i^0, y > d_i^{\ell_i}). \end{cases} \tag{1}$$

The unit bids represent the gradients of this function and the anchor values stand for its discontinuous points. For each bidder $i \in N$, denote his or her unit valuations by $\{v_i^k \mid k = 1, ..., \ell_i\}$ and define another function $V_i : \mathbb{R}_+ \to \mathbb{R}$ by

$$V_i(y) = \begin{cases} v_i^k \cdot y & (d_i^{k-1} < y \leq d_i^k, k = 1, ..., \ell_i), \\ 0 & (y = d_i^0, y > d_i^{\ell_i}). \end{cases} \tag{2}$$

A vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ that satisfies $\sum_{i \in N} x_i \leq M$ and $x_i \geq 0$ for any $i \in N$ is called an allocation, where x_i is the units of the item assigned to bidder $i \in N$ in the allocation. An item allocation problem $(AP)_B$ is to find allocations that maximize the total amount of bids is formulated by

$$(AP)_B \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in N} B_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in N} x_i \leq M \\ & x_i \geq 0 \ \ (\forall i \in N). \end{array} \right.$$

Another problem $(AP)_V$ is formulated in the same way by

$$(AP)_V \left| \begin{array}{ll} \text{maximize} & \sum_{i \in N} V_i(x_i) \\ \text{subject to} & \sum_{i \in N} x_i \leq M \\ & x_i \geq 0 \ \ (\forall i \in N), \end{array} \right.$$

in order to find efficient allocations that maximize the total amount of valuations.

The payment scheme is as follows. Denote by $\boldsymbol{x}^*$ an optimal solution of $(AP)_B$. Let $\boldsymbol{x}^{-j}$ be an optimal solution of the following restricted item allocation problem $(AP)_B^{-j}$ with the set of bidders $N^{-j} = N \setminus \{j\}$.

$$(AP)_B^{-j} \left| \begin{array}{ll} \text{maximize} & \sum_{i \in N^{-j}} B_i(x_i) \\ \text{subject to} & \sum_{i \in N^{-j}} x_i \leq M \\ & x_i \geq 0 \ \ (\forall i \in N^{-j}). \end{array} \right.$$

In VCG mechanism, bidder j's payment p_j is determined by

$$p_j = \sum_{i \in N^{-j}} B_i(x_i^{-j}) - \sum_{i \in N^{-j}} B_i(x_i^*). \tag{3}$$

Under this payment scheme, it is the dominant strategy for each bidder to truthfully bid his or her unit valuations; Thus, the solutions of $(AP)_B$ maximize the total sum of valuations in $(AP)_V$ as well, which leads the allocative efficiency.

3 Experimental Design

This laboratory experiment is a computerized one whose software (cgi script) is coded with Python. This experiment has 4 sessions and each session consists of 20 rounds in total. In each round, 5 units of a virtual item are auctioned off to 3 bidders, where for every bidder i, the number of anchor values is set as $\ell_i = 5$, and thus his or her anchor values are $d_i^0 = 0$, $d_i^1 = 1$, ..., $d_i^5 = 5$. For each bidder $i \in N$, his or her unit valuations, $\{v_i^k \mid k = 1, ..., \ell_i\}$, are independently and uniformly distributed over integers between 1 and 200. Bids are made by non-negative integers.

In 2 out of 4 sessions , at the beginning of each round, each bidder $i \in N$ is given his or her unit valuations $\{v_i^k \mid k = 1, ..., \ell_i\}$ by the experimenter, which are privately shown only in his or her computer screen. Then, each bidder i submits his or her unit bids $\{b_i^k \mid k = 1, ..., \ell_i\}$ privately to the experimenter. The computer determines the allocation of the item and bidders' payments according to $(AP)_B$ and (3). When k units of the item is allocated to bidder i, he or she receives the points that amounts $v_i^k \cdot k$ minus his or her payment. In the other 2 sessions, each bidder i is given his or her valuations $\{v_i^k * k \mid k = 1, ..., \ell_i\}$ and submits his or her bids $\{b_i^k * k \mid k = 1, ..., \ell_i\}$. Table 1 shows the difference in appearance of the information given to bidder i in the case of 3 units, as an example, in Appearance 1 of which $*k$ $(k = 1, 2, 3)$ is put automatically in his or her computer screen.

Table 1: Different appearance of information.

Appearance 1	v_i^k shown; b_i^k bid			
# of units		1	2	3
bidder i	valuation	80*1	60*2	55*3
	bid	70*1	55*2	50*3
Appearance 2	$v_i^k * k$ shown; $b_i^k * k$ bid			
# of units		1	2	3
bidder i	valuation	80	120	165
	bid	70	110	150

In each round, there is a 120-second time limit for submitting bids. If no bidder bids within the time limit, all three bidders then obtain zero point at that round. When some allocations attain the maximum total amount of bids, an allocation is chosen at random. The units assigned to a bidder and his or her payment are shown to the bidder in 5 seconds at the end of each round. The cumulative points of bidders are not shown to them. Subjects were not allowed to take notes throughout the session.

For each appearance of information, 2 sessions are paired in this experiment; In one session, each bidder's unit valuation of the item is drawn at random for each unit and given to him or her as it is in the first 10 rounds, while in the second 10 rounds the values drawn at random are realigned in the monotone decreasing (non-increasing) order from $k = 1$ to $k = 5$ and given to each bidder as his or her unit valuations in that order. The displays of draws are changed between the first and second 10 rounds in the other paired session. Every subject thus bids under both displays in the same session. In analysis, however, the data should be merged in order to cancel the effect of the order of the displays on the results.

The instruction is given at the beginning of each session, where how VCG mechanism works is demonstrated with an example. (See the Appendix.) Subjects are informed that they will be paid according to the total points they obtain in 6 rounds (3 from the first 10 rounds and 3 from the subsequent 10 rounds) randomly selected by a computer at the end of the session they participated in, with the pre-determined exchange rate in addition to the show-up fee. The exchange rate was 1 point = 1 JPY and the show-up fee was 1500 JPY. Before proceeding to the experiment, subjects play 1 round for practice to familiarize themselves with the software.

4 Results

This experiment was conducted at the University of Tsukuba in Japan; 2 sessions in February 2015 and 2 sessions in January 2017, respectively. Each session involves 8 groups of 3 subjects. At the beginning of each round, all subjects were randomly re-grouped into 8 groups by a computer. Subjects are not informed of who are in the same group. Subjects were recruited from all over the campus, and undergraduate students whose major is engineering were most populous among them. Once a subject participated in a session, he or she was prohibited to participate in any other sessions for this experiment.

Upon arrival, subjects were provided with a written instruction, and then the experimenter read it around. (The instruction is available upon request.) Subjects could ask questions regarding the instruction by raising their hand and the experimenter gave the answers to those questions privately. Any communication among subjects were strictly prohibited; Thus, their interactions were only through the information they enter in their computer screens. Each session lasted about 100 minutes including the instruction. There was no observation of bidding made after the time limit. Features of the experimental sessions are summarized in Table 2.

Let $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$ be an observed allocation. The rate of efficiency is

Table 2: Features of the experimental sessions.

session no.	appearance of info.	show-up fee (JPY)	point-to-JPY ratio	# of subj.	session date	avg. point per subject
1	1	1500	1.0	24	Feb.14, 2015	475.05
2	1	1500	1.0	24	Feb.14, 2015	691.29
3	2	1500	1.0	24	Jan.26, 2017	469.42
4	2	1500	1.0	24	Jan.26, 2017	674.05

defined by

$$\frac{\sum_{i\in N} V_i(\hat{x}_i)}{\text{the optimal value of } (AP)_V}. \tag{4}$$

The rate of seller's revenue (profit) is defined by

$$\frac{\text{the total amount of observed payments}}{\text{the total amount of optimal payments}}, \tag{5}$$

where the total amount of optimal payments is represented by $\sum_{j\in N} p_j$ and p_j is calculated with (3) for each bidder $j \in N$ under the assumption that every bidder truthfully bids his or her (unit) valuations.

In what follows, we analyze the data taken from the last 5 out of 10 rounds in each display of draws to allow subjects the opportunity to learn better bidding behavior. There was no case where some allocations attained the same maximum total amount of bids. The data were merged for each display of draws in order to cancel the effect of the order of displays on the results. Tables 3 and 4 show the average rates of efficiency and seller's revenue, respectively, as well as their standard deviations. The sample size is 80 (5 rounds, 8 groups, 2 sessions) for each rate. The p-values for the two-sided permutation test (perm.) are also reported in those tables, where in each display of draws the null hypothesis is that there is no difference in those averages between Appearance 1 and Appearance 2. For both rates of efficiency and seller's revenue, as is seen in Tables 3 and 4, the null hypothesis was not rejected at the 5% significance level in each displays of draws. Our main observation is thus stated as follows.

Table 3: The rates of efficiency.

display of draws	at random		decreasing	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
mean	0.9306	0.9378	0.9172	0.9337
st.dev.	0.0704	0.0393	0.0621	0.0296
p-value (perm.)	0.8293		0.5567	

Table 4: The rates seller's revenue.

display of draws	at random		decreasing	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
mean	0.9477	0.9641	1.0564	0.8979
st.dev.	0.0932	0.0374	0.2581	0.1418
p-value (perm.)	0.6596		0.1120	

Observation 1 *In each display of draws, there was no difference on average in allocative efficiency and in seller's revenue between Appearance 1 and Appearance 2, respectively.*

Note that for both appearance of information, the standard deviation of seller's revenue observed in the display with the monotone decreasing order of unit valuations is much larger than the one observed in the other display of draws. This is the reason why Takahashi et al. (2018) chose the display of draws in which values drawn at random are put as unit valuations as they are.

As noted at the end of Section 2, VCG mechanism, in theory, induces allocative efficiency by providing every bidder with an incentive to submit his or her true valuations for each unit. In order to examine this feature, we counted the number of bids that are approximately truth-telling and the number of allocations that are approximately efficient. We say that a bid for a unit of the item is approximately truth-telling when it satisfies

$$\frac{|\text{unit valuation} - \text{unit bid}|}{\text{unit valuation}} \leq 0.05 \tag{6}$$

and that an allocation is approximately efficient when it satisfies

$$\text{the rate of efficiency} \geq 0.95. \tag{7}$$

Table 5 shows the observed numbers of approximately truth-telling bids and approximately efficient allocations. For each appearance of information, the sample size is 1200 (5 rounds, 24 bidders, 5 units, 2 sessions) for approximately truth-telling bids and it is 80 for approximately efficient allocations in each display of draws. The p-values for the two-sided Fisher exact test (Fisher) are also reported, where for each appearance of information the null hypothesis is that there is no difference in number of approximately truth-telling bids (approximately efficient allocations) between displays of draws. For the numbers of the approximately truth-telling bids, the null hypothesis was rejected at the 5% significance level in each appearance of information, whereas for the numbers of approximately efficient allocations the null hypothesis was not rejected at the same significance level in each appearance of information. Our next observation is thus stated as follows.

Table 5: Numbers of approximately truth-telling bids and approximately efficient allocations.

	truth-telling		efficiency	
display of draws	at random	decreasing	at random	decreasing
Appearance 1	517	450	60	57
p-value (Fisher)	0.0141		0.1216	
Appearance 2	493	381	60	54
p-value (Fisher)	< 0.0001		0.3826	

Observation 2 *In each appearance of information, there was a significant difference in numbers of approximately truth-telling bids between displays of draws, but this behavioral difference did not significantly affect the difference in numbers of approximately efficient allocations between displays of draws.*

For both appearance of information in both display of draws, the numbers of approximately truth-telling bids were less than a half of 1200 samples, as is seen Table 5. We thus report more results on the subjects' bidding behavior. In this experiment, each unit valuation was drawn independently of the other unit valuations, and thus we here analyze the data unit by unit. If the absolute value of a unit valuation minus a unit bid falls within 5% of all those absolute values, we then dropped the data as an outlier for our regression analysis.

Tables 6 and 7 show the regression results for Appearance 1 and Appearance 2, respectively. Figures 1 to 4 plot unit valuations and unit bids observed for Appearance 1 and Appearance 2, respectively. The coefficients on valuations were less than one and they are statistically significant when values drawn at random and shown to them as they were, regardless of appearance of information. Some coefficients on valuations in the other display of draws were, however, more than one and they are statistically significant. Our last observation is thus stated as follows.

Observation 3 *For both Appearance 1 and Appearance 2, subjects underbid when unit valuations were drawn at random and shown to them as they were, whereas they did not necessarily do so when values drawn at random were realigned in the monotone decreasing order and given to them as their unit valuations.*

Note that Chen and Takeuchi (2010) reported subjects' underbidding when VCG was applied, although they studied combinatorial auctions. In a single unit auction, however, many researchers have reported that subjects overbid in the second-price auction (VCG mechanism). (See, e.g., Kagel and Levin (2016), which is a comprehensive survey of results in experiments of various auctions.) We observed that subjects overbid also in multi-unit auctions when VCG was applied in the display of draws of unit valuations that were aligned in the monotone decreasing order.

Observations 2 and 3 jointly imply that in each appearance of information, there was a significant difference in subjects' bidding behavior between displays of draws, but this behavioral difference did not significantly affect the difference in numbers of approximately efficient allocations.

Table 6: Regression results for Appearance 1.

	at random				
# of units	1	2	3	4	5
Constant	-5.0432	-0.1143	2.3098	-1.6640	2.2452
p-value	0.3150	0.8416	0.6750	0.7530	0.6910
Valuation	0.8925	0.9850	0.9392	0.9688	0.9634
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.6370	0.6480	0.6050	0.6580	0.6300
	descending				
# of units	1	2	3	4	5
Constant	-11.7126	-2.5721	-5.3490	1.0001	2.9950
p-value	0.3900	0.7360	0.2930	0.8530	0.3300
Valuation	0.9395	0.9528	1.0247	0.9359	0.7826
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.3660	0.5590	0.6610	0.4250	0.3250

Table 7: Regression results for Appearance 2.

	at random				
# of units	1	2	3	4	5
Constant	5.1200	-4.0728	3.0490	-1.0193	-0.5544
p-value	0.3620	0.3720	0.4750	0.8110	0.8900
Valuation	0.7741	0.9252	0.8723	0.9551	0.9505
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.4980	0.6760	0.690	0.7380	0.7480
	descending				
# of units	1	2	3	4	5
Constant	-28.4380	-31.3011	27.1133	-39.5828	-1.4110
p-value	0.6820	0.522	0.0780	0.1370	0.9080
Valuation	1.2723	1.3799	0.7797	1.8544	1.4011
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.0390	0.0620	0.104	0.1100	0.1140

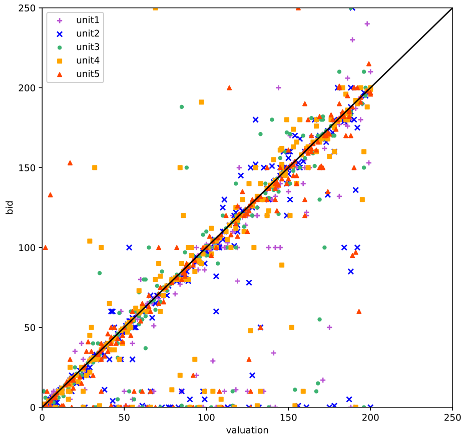

Figure 1: At random, Appearance 1.

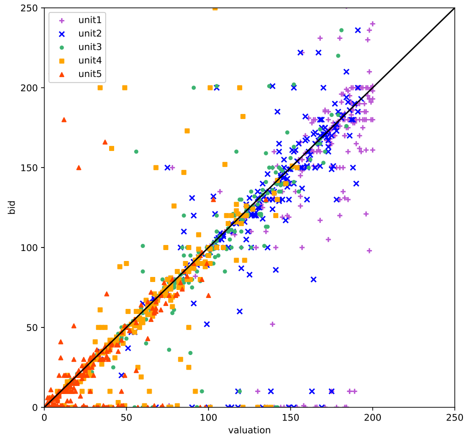

Figure 2: Descending, Appearance 1.

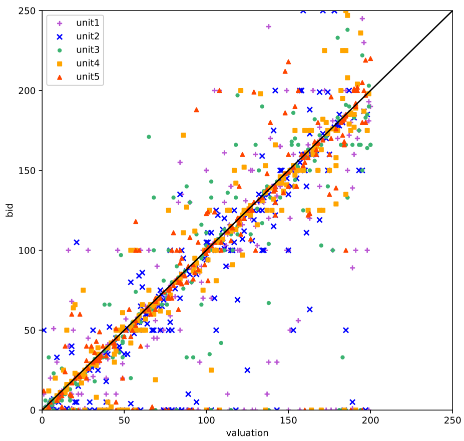

Figure 3: At random, Appearance 2.

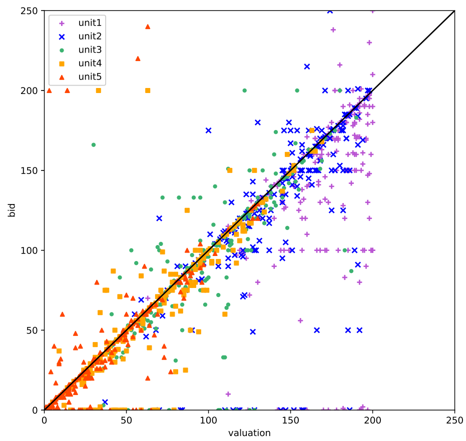

Figure 4: Descending, Appearance 2.

5 Final Remarks

As compared to combinatorial or multi-object auctions, there is few literature on experiments for multi-unit auctions for a single item. Kagel and Levin (2001) studied subjects' bidding behavior in multi-unit auctions, but they imposed a uniform price on all units of the item in their experiment. Allowing different prices for different units, we observed that there was no significant difference in both average rates of efficiency and seller's revenue between those two types of appearance of information. In each appearance of information, there was a significant difference in subjects' bidding behavior between different displays of draws, but this behavioral difference did not significantly affect the difference in allocative efficiency.

As noted in the Introduction, Takahashi et al. (2018) examined the performance of GBA in the display of draws of unit valuations which were given to subjects as they were, because this type of appearance of information was considered as a key feature for the GBA to work well. In this paper, we investigated the performance of VCG in different displays of draws. Also for GBA, in another paper, we need to investigate whether different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue.

Kagel et al. (2001) conducted an experiment in which a human bidder with flat demand for two units competes against machine bidders each demanding a single unit, and they reported overbidding of each human bidder for both units. It is not appropriate to compare to their result, but our regression analysis showed that subjects overbid for some units in the display of draws of unit valuations which were realigned in the monotone decreasing order. On the other hand, Chen and Takeuchi (2010) reported subjects' underbidding when VCG was applied in combinatorial auctions. We also observed that subjects underbid in the display of draws of unit valuations which were given to subjects as they were. What factors induce subjects to overbid or underbid? This is still an open question for further research.

Notes

Acknowledgement

The authors wish to thank Toru Suzuki and Hiroshi Tanaka for their excellent research assistance.

Compliance with Ethical Standards

This research was supported by JSPS Grant-in-Aid for Young Scientists (B) 26870200 and Grant-in-Aid for Scientific Research (B) 15H02972 (Takahashi), and Japan Center for Economic Research, ORA-Plus research project "BEAM", and JSPS Grant-in-Aid for Challenging Research (Pioneering) 17H06190 (Watanabe).

Conflict of Interest

The Authors declare that they have no conflict of interest.

Ethical Approval

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. This article does not contain any studies with animals performed by any of the authors.

References

Chen, Y., Takeuchi, K., 2010. Multi-object auctions with package bidding: An experimental comparison of iBEA and Vickrey. *Games and Economic Behavior* 68, 557-569.

Kagel, J. H., Levin, D., 2001. Behavior in multi-unit demand auctions: Experiments with uniform price and dynamic Vickrey auctions. *Econometrica* 69, 413-451.

Kagel, J. H., Kinross, S., and Levin, D.. 2001. Comparing efficient multi-object auction institutions. mimeo., Ohio State University.

Kagel, J. H., Levin, D., 2016. Auctions: A survey of experimental research. In *Handbook of Experimental Economics* vol. II, Kagel, J. H., Roth, A. E. (eds.) Princeton University Press, 563-637.

Takahashi, S., Izunaga, Y., Watanabe, N., 2018. An approximation algorithm for multi-unit auctions: Numerical and subject experiments. *Operations Research and Decisions* 28, 75-95.

Takahashi, S., Shigeno, M., 2011. Approximation algorithms for a winner determination problem of single-item multi-unit auctions. *JSIAM Letters* 3, 29-32.

Appendix: Examples in the Instruction

In the instruction, we explained VCG mechanism with the following example.

Table 8: Different appearance of Information in sessions conducted in 2015 and 2017.

	2015: values by unit			
# of units		1	2	3
Bidder 1	valuation	80*1	60*2	55*3
	bid	70*1	55*2	50*3
Bidder 2	valuation	40*1	70*2	65*3
	bid	40*1	60*2	65*3
	2017: by values multiplied by units			
# of units		1	2	3
Bidder 1	valuation	80	120	165
	bid	70	110	150
Bidder 2	valuation	40	140	195
	bid	40	120	195

Item Allocation

Find an allocation that maximizes the total amount of bids among all possible allocations. In what follows, k_i units of the item are assigned to bidder $i = 1, 2$ in allocation (k_1, k_2), and the total amount of bids follows the allocation; (0, 0): 0, (1, 1): 70*1+40*1=110, (1, 0): 70*1=70, (2, 0): 55*2=110, (3, 0): 50*3=150, (0, 1): 40*1=40, (0, 2): 60*2=120, (0, 3): 65*3=195, (1, 2): 70*1+60*2=190, (2,1): 55*2+40*1 =150. Thus, VCG mechanism allocates 3 units to bidder 2. The total amount of bids is 195.

Payment determination

payment of bidder i (winner) =

(total amount of bids in the auction that excludes bidder i:
65*3 for bidder 1, 50*3 for bidder 2)

− (total amount of bids in the auction)

+ (bidder i's bid for the unit assigned to i)

- payment of bidder 1 $= (65 * 3) - 195 + 0 = 0$
- payment of bidder 2 $= (50 * 3) - 195 + (65 * 3) = 150$