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Arrovian Social Choice with Non -Welfare Attributes*

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Abstract

A social state is assumed to be characterized by welfare and non-welfare attributes. Welfare attributes are utility profiles and all the relevant information. Non-welfare attributes are intrinsic to social states irrespective of utilities. Modifying transitive rationality of social preference, we show that dictatorship still holds under this setting (Theorem 1). We also show that non-welfare information is used only if the dictator is indifferent between two social states (Theorems 2-4).

Keywords: Arrow's impossibility theorem, welfarism, non-welafre attribute, attribute set.

1 Introduction

In the past decades, the study of social choice and welfare economics has been based on the idea that actions, policies, and/or rules should be evaluated on the basis of their consequences that are in most cases postulated by individual welfare. This idea, called *welfarism*, a form of consequentialism, has significantly spread in the field of normative economics, despite numerous criticisms against

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it[1].

In this paper, we study Arrovian social choice with welfare and non-welfare attributes. We premise that social choice should not be undertaken with welfare information only and it must also use non-welfare information. The formalization of non-welfare information is the key to solve the problem.

We assume that a social state is characterized by *welfare* and *non-welfare attributes*. Welfare attributes are utility profiles and all the relevant information such as ordinal preferences and the Borda numbers, which are derived from and dependent on utilities. In contrast to this, non-welfare attributes are intrinsic to social states irrespective of utilities. Social states are classified into some groups with respect to non-welfare attributes. We call the groups *attribute sets*. Any social state in the same attribute set is thought of as identical and undistinguished from each other from the viewpoint of the attributes in question. For any two social states, if they do not belong to the same attribute set, then the difference in non-welfare attributes counts for determining the social preference between the two states. This idea requires us to abandon or correct the neutrality axiom and similar others such as indifference Pareto, which are often implicitly imposed on Arrovian rules. We also argue that transitivity of social preference should be modified; significant amount of literature demonstrates that neutrality and the similar should be responsible for blocking the possibility of non-welfarist social choice[2], but no literature points out that transitivity of social preference, implicitly assumed in the literature, should be equally responsible.

We show that in this setting, the Arrovian dictatorship theorem still survives when non-welfare information is available (Theorem 1), and that non-welfare information is utilized for determining the social preference between any two

[1] Refer to Sen (1979a, 1979b) for the comprehensive studies of welfarism. The former argues welfarism in the philosophical context, whereas the latter does it in the context of social choice and welfare economics.

[2] See d'Aspremont and Gevers (1976) and Sen (1970,1977).

social states only when the dictator is indifferent between the two (Theorems 2-4). Therefore, welfare and non-welfare information are used in a lexicographic order; welfare information first and then non-welfare information.

An example of Sen (1979a) very well illustrates the importance of our idea; it intends to point out a problem with welfarism, an inappropriate information constraint for social decision[3]. Let there be two persons r (rich) and p (poor), and let there be two states x and y with the only difference being that in x there is no redistribution taxation, whereas in y some money obtained by taxing r has been passed on to p, even though r remains richer than p. The table below shows the utilities of the two persons in the two states.

		states	
		x (*no tax*)	y (*redistribution tax*)
utilities	r	10	8
	p	4	7

Now we replace the above with the following. Let r be a romantic dreamer and p be a miserable policeman. In b, the policeman tortures the dreamer; in a, he does not. The table below shows the utilities of the two persons in the two states.

		states	
		a (*no torture*)	b (*torture of r by p*)
utilities	r	10	8
	p	4	7

If we stick to welfarism, the taxation and torture problems have no difference in utilities so that both should have the same resolution[4]. The redistribution from the rich to poor should be made (y is socially better than x in the taxation problem) if and only if torture is undertaken (b is socially better than a in the torture problem). Everybody would think that this is a stupid idea. The two problems should not be dealt with equally. Life, body, or physical freedom

[3] Refer also to Sen (1979b) that provides a similar example pinpointing the problem with welfarism.

[4] We make a brief remark on welfarism in section 3.

should be counted as one of the important non-welfare attributes[5]. From this viewpoint, no torture states a, x, y belong to the same attribute set, but b does not. By taking the non–welfare information into account, the social preference of x and y can be different from that of a and b.

The paper is organized as follows. Section 2 provides notation and definitions, where several examples illustrate our basic idea about non-welfare attributes. Axioms on rules are introduced in Section 3, where transitive rationality of social preferences, neutrality, and indifference Pareto, are replaced with modified versions. We provide some remarks on welfarism here. Section 4 states the main results. Subordinate matters of limited importance relative to the main results are relegated to Appendix A and B.

2 Notation and Definitions

Let $N = \{1, 2, ..., n\}$ be the finite set of persons with at least two. Let X be the finite set of social states with at least three. Each person i's preference is represented by a utility function u_i defined on X. Let $U(X)$ be the set of all utility functions. A (utility) profile u is the list of individual utility functions $u = (u_1, ..., u_n)$, so the set of profiles is $U(X)^n$. A social choice rule F, simply a rule, is a mapping that associates with each profile $u \in U(X)^n$ a social preference $\geq_{F(u)}$, a complete binary relation on X[6]. The strict and indifferent relations associated with $\geq_{F(u)}$ are denoted by $>_{F(u)}$ and $=_{F(u)}$ respectively.

A social state $x \in X$ is characterized by welfare and non-welfare attributes. Given a profile, the welfare attributes of x are the profile itself and all the concepts derived from it, such as ordinal preferences of x and the Borda number of x, which depend on profiles. In contrast, non-welfare attributes are intrinsic to x independently of profiles. Let non-welfare attributes be given. We assume

[5]Libertarian claims of individual rights have been thought of as a typical issue of non-welfare value in the literature. Refer to Sen (1969,1970,1976,1979b) for details.

[6]We say that $\geq_{F(u)}$ is complete on X if and only if for all $x, y \in X$, $x \geq_{F(u)} y$ or $y \geq_{F(u)} x$.

that all the social states are classified into subgroups in which each member is thought of as identical from the viewpoint of the non-welfare attributes. Thus X has a partition $\{X_\lambda\}_{\lambda\in\Lambda}$, where $X = \bigcup_{\lambda\in\Lambda} X_\lambda$ and $X_\lambda \cap X_{\lambda'} = \emptyset$ for all $\lambda \neq \lambda'$. If $x, y \in X_\lambda$, we cannot distinguish between x and y from the viewpoint of the non-welfare attributes. We call X_λ an attribute set. We assume that there exist at least two attribute sets. Let $X(x)$ stand for the attribute set containing x. We occasionally substitute $X(x)$ for X_λ in order to say $x \in X_\lambda$.

Remark 1 A weaker definition of attribute sets is that X has a covering, where $X = \bigcup_{\lambda\in\Lambda} X_\lambda$ holds, but not necessarily $X_\lambda \cap X_{\lambda'} = \emptyset$ for all $\lambda \neq \lambda'$. However, this definition is essentially the same as our definition as all the intersections are new attribute sets. Let R and B be the sets of red-colored objects and blue-colored objects respectively. If there exist some objects that look like they are red and blue, where the intersection $R \cap B$ is nonempty, we let $P = R \cap B$ be a new attribute set called the set of purple-colored objects.

The examples below illustrate the attribute sets.

Example 1 Underage individuals watching pornography or not.

Since the advent of the Internet, pornography has been readily available to anyone in any location. Let $X = \{G, U, O\}$, where G stands for grown-ups watching pornography, U stands for under-aged watching it, and O stands for nobody watching it. The non-welfare attribute is a type of morality. The attribute sets are $\{U\}$ and $\{G, O\}$[7].

Example 2 Mac or Windows.

$X = \{(m, m), (m, w), (w, m), (w, w)\}$. Two workers use computers. Let (m, w) stand for a situation where Worker 1 uses Mac and Worker 2 uses Win-

[7] This example originally comes from Sen (1969,1970)'s Lady Chatterley's Lover, which Sen addressed later as " ...may now appear puzzling." (Sen 1979, p480). Therefore, we have updated the example conforming to the Internet age.

dows. The other notation is also similar. The non-welfare attribute is corporation. The attribute sets are $\{(m,m),(w,w)\}$ and $\{(m,w),(w,m)\}$.

Example 3 Building a commercial complex (C) or protecting the natural environment (E).

$X = \{C, E\} \times \prod_{i=1}^{n} X_i$, where X_i is the set of person i's personal feature. The non-welfare attribute is the environment. The attribute sets are $\{C\} \times \prod_{i=1}^{n} X_i$ and $\{E\} \times \prod_{i=1}^{n} X_i$.

3 Axioms

Arrow's dictatorship theorem requires three axioms, transitive rationality of social preference, Arrow's independence, and Pareto condition when rules are defined on the set of preference profiles[8]. The theorem requires one more axiom, ordinal noncomparability[9], when the rules are defined on the set of utility profiles. We argue that all the axioms, except for transitive rationality, are used with no modification, but an extensive discussion is needed for the modification of transitive rationality.

A rule F satisfies Independence (I) if for any $u, u' \in U(X)^n$ and any $x, y \in X$, if $u_i(x) = u'_i(x), u_i(y) = u'_i(y)$ for all $i \in N$, then $\geq_{F(u)} \cap \{x,y\}^2 = \geq_{F(u')} \cap \{x,y\}^2$. Independence is assumed in all the results in this paper, implying that the social preference of x and y should be the same in two profiles if, person-wise, the individual utility of x and y is the same in those profiles. Also note that non-welfare attributes of x and y are intrinsic to themselves and do not change across those profiles.

A rule F satisfies Pareto (P) if for any $u \in U(X)^n$ and any $x, y \in X$, $u_i(x) > u_i(y)$ for all $i \in N$, then $x >_{F(u)} y$. We have no reluctance to acknowledge the

[8] The assumption that rules are defined on the set of preference profiles, not on the proper subset is occasionally thought of as one of the axioms, called universal domain.

[9] Refer to Sen (1977). d'Aspremont and Gevers (2002) surveys the related works.

existence of non-welfare value superior to individual welfare, which will be able to overturn unanimous decisions that Pareto requires. However this is beyond the scope of the subject we present. We deal with social choice problems, where welfare and non-welfare values are equally respected, and hence Pareto still remains as a requirement of the rules.

A rule F satisfies Ordinal Noncomparability (ON) if for any $u, u' \in U(X)^n$ and any $i \in N$, there exists a strictly increasing real valued function ψ_i such that $u'_i(x) = \psi_i(u_i(x))$ for all $x \in X$, then $\geq_{F(u)} = \geq_{F(u')}$[10]. As this transformation of utility functions has no influence on non-welfare attributes, ON still remains as an axiom on rules.

A rule F satisfies λ-Full Rationality (λFR) if for any $u \in U(X)^n$, any $X_\lambda, X_{\lambda'}, \lambda \neq \lambda'$ and any $\{x, y, z\} \subset X_\lambda \cup X_{\lambda'}$, $x \geq_{F(u)} y \geq_{F(u)} z$ implies $x \geq_{F(u)} z$. Note that transitivity of $\geq_{F(u)}$ does not always hold on $\{x, y, z\}$ if each of the three belongs to a different attribute set. It is easy to observe that a rule F satisfies λFR if and only if for any $X_\lambda, X_{\lambda'}, \lambda \neq \lambda'$ and any $\{x, y, z\} \subset X_\lambda \cup X_{\lambda'}$,(i) $x =_{F(u)} y =_{F(u)} z$ implies $x =_{F(u)} z$ and (ii) either $x >_{F(u)} y \geq_{F(u)} z$ or $x \geq_{F(u)} y >_{F(u)} z$ implies $x >_{F(u)} z$. There are four cases for λFR. (0) either $x, y, z \in X_\lambda$ or $x, y, z \in X_{\lambda'}$, (1) $x, y \in X_\lambda, z \in X_{\lambda'}$, (2) $x \in X_\lambda, y, z \in X_{\lambda'}$, and (3) $x, z \in X_\lambda, y \in X_{\lambda'}$. Case (0) is essentially equivalent to Full Rationality (FR) imposed on Arrovian rules. A rule F satisfies FR if for any $u \in U(X)^n$ and any $x, y, z \in X$, $x \geq_{F(u)} y \geq_{F(u)} z$ implies $x \geq_{F(u)} z$. An everyday example illustrates Case (1).

Example 4 Let a non-welfare attribute be religion. Let x, y, z be social states as follows.

[10]Without ON, the utilitarian rule defined by $x >_{F(u)} y$ if and only if $\sum_{i \in N} u_i(x) > \sum_{i \in N} u_i(y)$ is a counter example against Arrow's dictator theorem in the utility framework.

For any rule satisfying I, ON is equivalent to cardinal noncomparability (CN), which says that for any $u, u' \in U(X)^n$ and any $i \in N$, if there exists some constants $a_i > 0$ and b_i such that $u'_i = a_i u_i + b_i$, then $\geq_{F(u)} = \geq_{F(u')}$. Refer to d'Aspremont and Gevers (1976), Theorem 2.

x :We are Christians with a piece of bread per day;

y :We are Christians with no bread per day; and

z :We are not religious with bread as much as we like per day.

Then it appears natural that $x \geq_{F(u)} y \geq_{F(u)} z$ implies $x \geq_{F(u)} z$. If we Christians like having one piece of bread better than no bread ($x \geq_{F(u)} y$) and if we like being a Christian with no bread better than being a rich person with no religious faith ($y \geq_{F(u)} z$), then we like being a Christian with one piece of bread better than a rich person with no religious faith ($x \geq_{F(u)} z$).

The formal meaning of Case (1) is as follows. Note that $\{x, z\}$ and $\{y, z\}$ have no difference in non-welfare attributes; $x \in X_\lambda$ and $z \in X_{\lambda'}$ whereas $y \in X_\lambda$ and $z \in X_{\lambda'}$. Thus, if we have $x <_{F(u)} z$ and $y \geq_{F(u)} z$, this implies that the welfare attributes of y are more highly praised in social preference than that of x. However, this is a contradiction because $x \geq_{F(u)} y$ was made only by welfare attributes. In this case, we can ignore non-welfare attributes as x and y have no difference in non-welfare attributes. Therefore $x \geq_{F(u)} z$ holds true. Case (2) can be justified as well.

A slight modification of the everyday example of Case (1) illustrates Case (3).

Example 5 Let x, y, z be social states as follows.

x :We are Christians with a piece of bread per day;

y :We are not religious with bread as much as we like per day; and

z :We are Christians with no bread per day.

If we like being a Christian with a piece of bread better than being a rich person with no religious faith ($x \geq_{F(u)} y$), but if we might as well discard the faith as starve to death ($y \geq_{F(u)} z$), then we Christians like having food better than no food ($x \geq_{F(u)} z$).

The formal meaning of Case (3) is explained as well as that in Case (1). Note that $\{x, y\}$ and $\{y, z\}$ have no difference in non-welfare attributes; $x \in X_\lambda$ and $y \in X_{\lambda'}$ whereas $y \in X_{\lambda'}$ and $z \in X_\lambda$. Thus, $x \geq_{F(u)} y \geq_{F(u)} z$ implies that the welfare attributes of x are not less praised in social preference than that of z. As there exists no difference in non-welfare attributes between x and z, the social preference on $\{x, z\}$ should be made only by the welfare attributes so that we conclude $x \geq_{F(u)} z$.

Example 6 Let $X = \{x, y, z\}$. The attribute sets are $\{x, y\}$ and $\{z\}$. The table below shows that (1)-(3) are independent of each other.

	(1)	(2)	(3)
$x =_{F(u)} z =_{F(u)} y >_{F(u)} x$	yes	yes	no
$z =_{F(u)} x =_{F(u)} y >_{F(u)} z$	yes	no	yes
$y =_{F(u)} x =_{F(u)} z >_{F(u)} y$	no	yes	yes

The social preference in the first row of the table satisfies (1) and (2), but not (3). The social preferences in the second and third rows read the same way.

As we noted before, transitivity of $\geq_{F(u)}$ does not always hold if three social states belong to different attribute sets. The example below illustrates this point.

Example 7 There exist three non-welfare attributes, religion, health and marriage. Let x, y, z be the social states as follows:

x :We are Christians and smokers, and same-sex marriage is not legalized;

y :We are non-Christians and nonsmokers, and same-sex marriage is not legalized; and

z :We are are non-Christians and smokers, and same-sex marriage is legalized.

The table below illustrates the three social states.

	x	y	z
religion (Christian)	yes	no	no
health (Nonsmoking)	no	yes	no
marriage (S.S. Marriage)	no	no	yes

We assume that welfare information is identical among the three states. The attribute sets are $\{x\}, \{y\}, \{z\}$. In this example $x \geq_{F(u)} y \geq_{F(u)} z$ does not imply $x \geq_{F(u)} z$. Note that the social decision for any two social states are made by two of the three non-welfare attributes; $x \geq_{F(u)} y$ is made by religion and health whereas $y \geq_{F(u)} z$ is made by health and marriage. Similarly $x \geq_{F(u)} z$ has to be made by marriage and religion. However, $x \geq_{F(u)} y$ and $y \geq_{F(u)} z$ provide us with no clue to this decision.

The examples in the previous sections suggest that using non-welfare attributes as an informational basis of social choice implies violating either (i) transitivity of social preference, (ii) Strong Neutrality, or (iii) Indifference Pareto, a special form of Strong Neutrality. The last two axioms are well known in the literature of Arrovian social choice. A rule F satisfies Indifference Pareto (IP) if for any $u \in U(X)^n$ and any $x, y \in X$, if $u_i(x) = u_i(y)$ for all $i \in N$ then $x =_{F(u)} y$. A rule F satisfies Strong Neutrality (SN) if for any $u, u' \in U(X)^n$ and any $x, y, z, w \in X$, if $u_i(x) = u'_i(z)$ and $u_i(y) = u'_i(w)$ for any $i \in N$, then $x \geq_{F(u)} y \iff z \geq_{F(u)} w$. Sen (1977) shows that for any rule with Full Rationality and Independence, Strong Neutrality is equivalent to Indifference Pareto[11]. d'Aspremont and Gevers (1976, Lemma 3) goes further, which says that if a rule satisfies SN, social preferences are determined only by comparisons of utility levels, irrespective of what social states bring about them[12]. These results support our observation.

Unlike Pareto, Indifference Pareto is counterintuitive because it is against

[11] Refer to Theorem 6 in Sen (1977).

[12] This is called welfarism theorem. Refer also to Sen (1979b), which proves the counterpart of welfarism theorem in the traditional ordinal framework.

the idea that the social preference of x and y should be made by the difference of non-welfare value if there exists no difference with respect to individual welfare. Therefore it should be replaced with λ-Indifference Pareto, a modified version, which says that a rule F satisfies λ-Indifference Pareto (λIP) if for any $u \in U(X)^n$ and any $x, y \in X$, if $u_i(x) = u_i(y)$ for all $i \in N$ and $x, y \in X_\lambda$ for some λ then $x =_{F(u)} y$. The same reason holds for Strong Neutrality, which should be replaced with λ-Strong Neutrality, a modified version. A rule F satisfies λ-Strong Neutrality (λSN) if for any $u, u' \in U(X)^n$ and any $x, y, z, w \in X$, if $u_i(x) = u'_i(z)$ and $u_i(y) = u'_i(w)$ for any $i \in N$, and $x, z \in X_\lambda$, $y, w \in X_{\lambda'}$ for some λ, λ' ($\lambda = \lambda'$ is possible) imply $x \geq_{F(u)} y \Longleftrightarrow z \geq_{F(u')} w$. We show that λIP and λSN are identical if a rule satisfies λFR and I (Theorem 3).

We are now in a position to grasp the definition of using non-welfare attributes. We say that a rule F satisfying Independence uses non-welfare attributes if there exist

(i) some $x, y \in X$ and some $u \in U(X)^n$ such that $X(x) \neq X(y)$ and (i-a) $u_i(x) = u_i(y)$ for all i and $x \neq_{F(u)} y$ or (i-b) $u_i(x) > u_i(y)$ for all i and $x \leq_{F(u)} y$; or

(ii) some $x, y, z, w \in X$ and some $u, u' \in U(X)^n$ such that

(ii-a) $u_i(x) = u'_i(z)$ and $u_i(y) = u'_i(w)$ for all $i \in N$;

(ii-b) $z \notin X(x) \cup X(y)$ or $w \notin X(x) \cup X(y)$; and

(ii-c) $x \geq_{F(u)} y \Longleftrightarrow z \geq_{F(u')} w$ does not hold.

Cases (i) and (ii) provide an evidence for using non-welfare attributes as an informational basis of social choice. Case (i-b) is excluded if a rule satisfies Pareto. Case (i) and (ii) do not overlap with each other, as two alternatives suffice for Case (i) whereas at least three alternatives suffice for Case (ii). Obviously, if a rule satisfying Independence and Pareto uses non-welfare attributes, it violates either Indifference Pareto or Neutrality, but it does not always violate their λ-mates. We say that a rule satisfies the Use of Non-Welfare Attribute

(UNWA) if it uses non-welfare attributes. Hence, the discussion so far is summarized as follows: (1) Using non-welfare information as a basis of social decision is equivalent to abandoning Strong Neutrality; (2) according to welfarism theorem, Strong Neutrality is equivalent to the combination of Independence, Indifference Pareto, and transitivity of social preference; and (3) the last two axioms of which are responsible for impeding non-welfarist social choice, and should be replaced with the modified versions.

A rule F is the Pareto extension rule if and only if for all $u \in U(X)^n$ and all $x, y \in X$, $x \geq_{F(u)} y \iff \neg\left(u_i(y) > u_i(x)\ \forall i \in N\right)$. A person i is decisive for (x, y) if for any $u \in U(X)^n$, $u_i(x) > u_i(y)$ implies $x >_{F(u)} y$. A person i is dictator on $Y \subset X$ if he is decisive for any pair in $Y \times Y$. A person i is dictator if he is dictator on X[13].

According to Sen (1979a), welfarism requires that "The judgement of the relative goodness of alternative states of affairs must be based exclusively on the respective collections of individual utilities in these states.[14]" The states of affairs correspond to social states in this paper. We reject welfarism on the same ground as Sen (1979a). However, the conditional application of welfarism represented by the family of λ-axioms, that is, λFR, λSN, and λIP, appears to be immune to criticism against welfarism. It can be said that welfarism is applicable only in the case of there being no difference in non-welfare information. We would like to call this view *conditional welfarism*, distinguishing it from welfarism.

4 Results

We deal with the class of rules satisfying λFR, I, P, ON, λIP and UNWA, where λIP is identical to λSN as will be shown later. The theorem below is a

[13] We can say that i is dictator if he is decisive for all pairs in X.

[14] In philosophical contexts, welfarism is defined a bit more demandingly, that is, taken as an increasing function of individual utilities. Refer to Sen (1977,1979a) for details.

counterpart of Arrow's impossibility theorem (Arrow 1951,63).

Theorem 1 *Suppose that there exists at least one attribute set with at least two elements.*

(1) Suppose that there exist at least two attribute sets that are singleton. Then if a rule F satisfies λFR, I, P and ON, there exists a person i who is decisive for any pair (x, y) except for all the pairs such that $\{x\} = X_\lambda$ and $\{y\} = X_{\lambda'}$.

(2) Suppose that there exists at most one attribute set that is singleton. Then if a rule F satisfies λFR, I, P and ON, there exists dictator.

Proof. (1). Take X_λ containing at least two elements and $X_{\lambda'}(\lambda' \neq \lambda)$ arbitrarily. First we show that there exists dictator on $X_\lambda \cup X_{\lambda'}$. Take any two elements x and y in X_λ and any z in $X_{\lambda'}$ arbitrarily. Thanks to λFR, $\geq_{F(u)}$ is complete and transitive on $\{x, y, z\}$ and hence a modified version of Arrow's Theorem due to Sen (Theorem 1, 1977) is applied. Thus there exists a dictator i on $\{x, y, z\}$. This further implies that i is dictator on X_λ and decisive for any pairs in $(X_\lambda \times X_{\lambda'}) \cup (X_{\lambda'} \times X_\lambda)$. The only remaining thing to prove is that i is dictator on $X_{\lambda'}$ if $X_{\lambda'}$ contains at least two elements. Let $z, w \in X_{\lambda'}$ and $u_i(z) > u_i(w)$. We can let $u_i(z) > u_i(x) > u_i(w)$ and $x \in X_\lambda$. As i is decisive on $\{x, z\}$ and $\{x, w\}$, we have $z >_{F(u)} x >_{F(u)} w$. By λFR, we have $z >_{F(u)} w$, the desired result. By noting that this holds for any $X_{\lambda'}$, this completes the proof of (1).

(2). (1) completes the proof. ■

If a rule uses non-welfare attributes, the following holds.

Theorem 2 *Let F be a rule satisfying λFR, I, and UNWA. The followings are true.*

(1) F violates either FR or IP.

(2) If there exist only two attribute sets, then F satisfies FR and violates IP.

Proof. (1) It is easy to observe that any rule F satisfying FR, I and IP satisfies SN, which cannot be compatible with UNWA.

(2) Noting that FR is reduced to λFR for this case, we know that (2) follows from (1). ■

The theorem below is easily proved.

Theorem 3 *For any rule satisfying λFR and I, it satisfies λSN if and only if it satisfies λIP.*

If IP is imposed on rules, the theorem below elaborates the decision power structure in (2) of Theorem 1.

Theorem 4 *Let F satisfy λFR, I, IP, ON, and P. If there exists at most one attribute set that is singleton, then F is a dictatorial rule with a decision hierarchy, and hence it satisfies FR and violates UNWA.*

See Appendix A for the proof and definition of decision hierarchy. Theorem 4 says that if there exists at most one attribute set that is singleton, there exists no rule satisfying λFR, I, P, IP and UNWA.

For rules satisfying λFR, I, P, ON, λIP(=λSN), and UNWA, there are eight cases that are logically possible. Table 1 lists the cases.

Table 1

	at least two singleton attribute sets	at most one singleton attribute sets
only two attribute sets FR is satisfied IP is violated	Case 1	Case 2
three or more attribute sets FR is violated IP is satisfied	Case 3	Case 4
three or more attribute sets FR is satisfied IP is violated	Case 5	Case 6
three or more attribute sets FR is violated IP is violated	Case 7	Case 8

Essentially, Case 1 is impossible. Case 4 is also impossible due to Theorem 4. Note that there exists dictator in Case 5. See Appendix B for more detailed argument on the remaining cases.

We show independence of the axioms, λFR, I, P, ON, λIP and UNWA. Each attribute set is indexed by X_τ $(\tau = 1, ..., t)$. For any $x \in X$, let $\tau(x) \in \{1, ..., t\}$ be such that $x \in X_{\tau(x)}$.

Example 8 (The simple majority rule weighted by non-welfare value) Let $N(x, y, u) = \#\{i \in N : u_i(x) > u_i(y)\}$. Let a rule F be defined as follows: For any $u \in U(X)^n$ and any $x, y \in X$,

$x >_{F(u)} y \Longleftrightarrow N(x, y, u) > N(y, x, u)$ or $[N(x, y, u) = N(y, x, u)$ and $\tau(x) > \tau(y)]$

$x =_{F(u)} y \Longleftrightarrow N(x, y, u) = N(y, x, u)$ and $\tau(x) = \tau(y)$.

This rule has no dictator and satisfies all the axioms except for λFR.

Example 9 (The Borda rule weighted by non-welfare value) Let $\beta(x, u) = \sum_{i=1}^{n} \#\{z \in X : u_i(x) \geq u_i(z)\}$. Let $k > 0$ be such that $n + k > kt$. Let a rule F be defined as follows: For any $u \in U(X)^n$ and any $x, y \in X$,

$$x \geq_{F(u)} y \Longleftrightarrow \beta(x, u) + k\tau(x) \geq \beta(y, u) + k\tau(y).$$

This rule has no dictator and satisfies all the axioms except for I. Note that P is assured by the condition $n + k > kt$.

Example 10 (The non-welfare value first rule) Let a rule F be defined as follows: For any $u \in U(X)^n$ and any $x, y \in X$,

$$x >_{F(u)} y \Longleftrightarrow [\tau(x) > \tau(y)] \text{ or } [\tau(x) = \tau(y) \& u_1(x) > u_1(y)]$$

$$x =_{F(u)} y \Longleftrightarrow \tau(x) = \tau(y) \& u_1(x) = u_1(y).$$

This rule has no dictator and satisfies all the axioms except for P.

Example 11 (The utilitarian rule weighted by non-welfare value) Let a rule F be defined as follows: For any $u \in U(X)^n$ and any $x, y \in X$,

$$x >_{F(u)} y \Longleftrightarrow \sum_{i=1}^{n} u_i(x) > \sum_{i=1}^{n} u_i(y) \text{ or } \left[\sum_{i=1}^{n} u_i(x) = \sum_{i=1}^{n} u_i(y) \& \tau(x) > \tau(y)\right]$$

$$x =_{F(u)} y \Longleftrightarrow \sum_{i=1}^{n} u_i(x) = \sum_{i=1}^{n} u_i(y) \& \tau(x) = \tau(y).$$

This rule has no dictator and satisfies all the axioms except for ON.

Example 12 (The hierarchical dictatorial rule weighted by non-welfare value) Let each alternatives be indexed, $1, 2, ..., q$, where $\#X = q$. Let $\mu(x) \in \{1, 2, ..., q\}$ be the number of x. Let a rule F be defined as follows: For any $u \in U(X)^n$ and any $x, y \in X$,

$$x >_{F(u)} y \Longleftrightarrow \begin{cases} \exists k \in \{1, ..., n\} \text{ s.t. } u_i(x) = u_i(y) \ \forall i \leq k-1 \ \& u_k(x) > u_k(y) \\ \text{or} \\ u_i(x) = u_i(y) \ \forall i \ \& \ \mu(x) > \mu(y). \end{cases}$$

No $=_{F(u)}$ exists by definition. This is a dictatorial rule with an extended hierarchy. The hierarchy is extended in the sense that non-welfare value is personified and participates in the hierarchy. This rule satisfies all the axioms except for λIP.

Example 13 (The complete dictatorial rule) Let a rule F be such that there exists some $i \in N$, called complete dictator, such that $x \geq_{F(u)} y \Longleftrightarrow u_i(x) \geq u_i(y)$ for any $u \in U(X)^n$ and any $x, y \in X$. This rule satisfies all the axioms except for UNWA.

5 Conclusion

The study of non-welfarist social choice has not progressed sufficiently to answer Sen (1979a)'s argument that focused its importance in the context of the criticism against welfarism[15]. We have offered a formal framework to treat social choice with non-welfare values. Although most of the results show impossibility, some are considered to be possible. The majority rule, Borda rule, and utilitarian rule weighted by non-welfare value (Examples 8, 9 and 11) appear strange at first glance. However they look attractive against the dictatorial theorems shown in the paper. They teach us that non-welfare value should be respected in some sophisticated form in order to escape from dictatorship. An extensive study of the rules is recommended[16].

6 Appendix A

We obtain a refinement of the Arrow's impossibility theorem when we impose IP on rules. Let D be a nonempty subset of $U(X)^n$. We say that person i is dictator, complete dictator, converse dictator, and complete converse dictator for D if the followings hold respectively: For any $u \in D$ and any $x, y \in X$, $u_i(x) > u_i(y) \Longrightarrow x >_{F(u)} y$ (dictator); $u_i(x) \geq u_i(y) \Longleftrightarrow x \geq_{F(u)} y$ (complete dictator); $u_i(x) > u_i(y) \Longrightarrow x <_{F(u)} y$ (converse dictator); and $u_i(x) \geq u_i(y) \Longleftrightarrow x \leq_{F(u)} y$ (complete converse dictator).

A rule F is a dictatorial rule with a decision hierarchy if there exist persons $i_1, i_2, ..., i_{k-1}, i_k$ $(1 \leq k \leq n)$ such that i_1 is dictator, i_2 is dictator or converse dictator for $D_{i_1} = \{u \in U(X)^n : u_{i_1}(x) = u_{i_1}(y)$ for all $x, y \in X\}$, i_3 is

[15] An exception is Fleurbaey (2003) that offers a new framework, data filter approach, to make non-welfare social choice possible. Another framework for investigating this issue is the capability approach initiated by Sen (1979c), and the most formal elaboration of the approach is Sen (1985). Alexander (2008) provides an introduction to the approach and a comprehensive survey.

[16] Lexmin rule, a lexcographic extension of Rawls' difference principle, should also be studied. Axiomatic characterizations of the rule were given by d'Aspremont and Gevers (1977), Hammond (1976), and Strasnick (1976).

dictator or converse dictator for $D_{i_1 i_2} = \{u \in U(X)^n : u_{i_1}(x) = u_{i_1}(y)$ and $u_{i_2}(x) = u_{i_2}(y)$ for all $x, y \in X\}$,..., i_{k-1} is dictator or converse dictator for $D_{i_1 i_2 \cdots i_{k-2}} = \{u \in U(X)^n : u_{i_1}(x) = u_{i_1}(y), u_{i_2}(x) = u_{i_2}(y), ..., u_{i_{k-2}}(x) = u_{i_{k-2}}(y)$ for all $x, y \in X\}$ and i_k is complete dictator or complete converse dictator for $D_{i_1 i_2 \cdots i_{k-1}} = \{u \in U(X)^n : u_{i_1}(x) = u_{i_1}(y), u_{i_2}(x) = u_{i_2}(y), ...,$ $u_{i_{k-1}}(x) = u_{i_{k-1}}(y)$ for all $x, y \in X\}$[17].

There is a vast variety of decision hierarchies. The shortest decision hierarchy consists of only one person i_1 who is complete dictator whereas all the persons take part in the longest decision hierarchy. Owing to Independence, social preferences induced from a dictatorial rule with a decision hierarchy are lexicographic ordering; for any $u \in U(X)^n$ and any $x, y \in X$,

$x >_{F(u)} y \iff u_{i_1}(x) > u_{i_1}(y)$ or $[\exists k' \leq k$ s.t. $u_{i_1}(x) = u_{i_1}(y), u_{i_2}(x) = u_{i_2}(y), ..., u_{i_{k'-1}}(x) = u_{i_{k'-1}}(y)$ & $(u_{i_{k'}}(x) > u_{i_{k'}}(y)$ or $u_{i_{k'}}(x) < u_{i_{k'}}(y))]$.

$x =_{F(u)} y \iff u_{i_1}(x) = u_{i_1}(y), u_{i_2}(x) = u_{i_2}(y), ..., u_{i_{k-1}}(x) = u_{i_{k-1}}(y)$ and $u_{i_k}(x) = u_{i_k}(y)$.

Theorem 5 *If a rule F satisfies FR, I, P, ON, and IP, it is a dictatorial rule with a decision hierarchy.*

Proof. Let i_1 be dictator. Let $x, y \in X$ and $u \in U(X)^n$ such that $u_{i_1}(x) = u_{i_1}(y)$ and $u_i(x) > u_i(y)$ for all $i \neq i_1$ be given. Then we have $x >_{F(u)} y$, $x <_{F(u)} y$ or $x =_{F(u)} y$. Owing to FR, I and IP, these hold for *all* $x, y \in X$. That is, for any $x, y \in X$ and any $u \in U(X)^n$, if $u_{i_1}(x) = u_{i_1}(y)$ and $u_i(x) > u_i(y)$ for all $i \neq i_1$, then $x >_{F(u)} y$ (Case1), $x <_{F(u)} y$ (Case 2) or $x =_{F(u)} y$ (Case 3).

Case 1: If $n \geq 3$, then letting D_{i_1} be the new domain with the society of $n-1$ persons except for i_1, we can apply Arrow's impossibility theorem and

[17] Dictators i_2 on D_{i_1}, ..., and i_{k-1} on $D_{i_1 i_2 \cdots i_{k-2}}$ are called quasi dictators according to the language of Bordes and Salles (1978). Aleskerov and Vladimirov (1986) provides a more comprehensive study of decision hierarchy.

show the existence of i_2 who is dictator for D_{i_1}. If $n = 2$, IP shows that i_2 is complete dictator for D_{i_1}, which completes the proof.

Case 2: If $n \geq 3$, then letting D_{i_1} be the new domain with the society of $n-1$ persons except for i_1, we can apply Arrow's impossibility theorem without Pareto (See Wilson (1972), Binmore (1976), and Fountain and Suzumura (1982)[18]) and show the existence of i_2 who is converse dictator for D_{i_1}. If $n = 2$, IP shows that i_2 is complete converse dictator for D_{i_1}, which completes the proof.

Case 3: We show that i_1 is complete dictator, which completes the proof. Take three alternatives a, b, c and $u \in U(X)^n$ such that $u_{i_1}(a) = u_{i_1}(b) = u_{i_1}(c)$, and $u_i(a) > u_i(b), u_i(a) > u_i(c)$ for all $i \neq i_1$. Case 3 implies $b =_{F(u)} a =_{F(u)} c$, which by FR further implies $b =_{F(u)} c$. Noting that for any $i \neq i_1$, no preference between b and c is specified, we have the desired result.

If either Case 1 or Case 2 holds, the same proof is repeated by letting $D_{i_1 i_2}$ be the new domain with the society of $n-2$ persons except for i_1 and i_2. If either Case 1 or Case 2 hold again here, the proof is also repeated again by letting $D_{i_1 i_2 i_3}$ be the new domain with the society of $n-3$ persons except for i_1, i_2 and i_3. The proof completes when it finds person i_k $(k \leq n)$ who is complete dictator. Aso note that IP is applied when $k = n$. ■

Note that if a rule is a dictatorial rule with a decision hierarchy, it does not use non-welfare attributes. Also note that if we impose strong Pareto (SP), which says that $\forall u \in U(X)^n, \forall\ x, y \in X$, $[u_i(x) \geq u_i(y)\ \forall i\ \&\ u_i(x) > u_i(y)\ \exists i \Longrightarrow x >_{F(u)} y]$, then the decision hierarchy is uniquely characterized, where someone is dictator and each of the rest plays as dictator at each stage in the hierarchy.

Proof of Theorem 4. : For any distinct $x, y \in X$, let A be a decision

[18] It is easy to observe that their results are reestablished on rules defined on the set of utility profiles, by adding ON as a new axom.

hierarchy, i.e., a sequence of persons $i_1, i_2, \cdots, i_k (k \leq n)$, where i_1 is dictator on $\{x, y\}$, $i_2, ..., i_{k-1}$ are dictator or converse dictator on $D_{i_1}, ..., D_{i_1 \cdots i_{k-2}}$, and i_k is complete dictator or complete converse dictator on $D_{i_1 \cdots i_{k-1}}$. For any $x, y \in X$, $x \neq y$, we write $x \overset{A}{\longleftrightarrow} y$ if $\geq_{F(u)} \cap \{x, y\}^2$ coincides with the ordering determined by A for any $u \in U(X)^n$. That is, we write $x \overset{A}{\longleftrightarrow} y$ if the decision hierarchy A determines the social preference on $\{x, y\}$ for any profile.

Then we have the following:

(1) for any $x, y \in X, x \neq y$, there exists some decision hierarchy A such that $x \overset{A}{\longleftrightarrow} y$.

Proof of (1): If x and y belongs to the same attribute set, there exists some z that does not belong to the attribute set. As λFR reduces to FR on $\{x, y, z\}$, Theorem 5 is applied and hence we have (1).

Otherwise, we can take z such that x and z belong to the same attribute set without loss of generality. Since λFR reduces to FR on $\{x, y, z\}$, Theorem 5 is applied and hence we have (1).

Second we show that

(2) for any $x, y, z \in X$, where x, y, z are distinct to each other, $x \overset{A}{\longleftrightarrow} y$ implies $x \overset{A}{\longleftrightarrow} z$.

Proof of (2): If x and y belong to the same attribute set, we can prove $x \overset{A}{\longleftrightarrow} z$ along the same line as (1). Next suppose not. We can take w such that x and w belong to the same attribute set without loss of generality. As λFR reduces to FR on $\{x, y, w\}$ and $\{x, z, w\}$, Theorem 5 and $x \overset{A}{\longleftrightarrow} y$ imply $x \overset{A}{\longleftrightarrow} z$, which is the desired result.

With (1) and (2), we know that F is a dictatorial rule with the decision hierarchy A, and hence F satisfies FR and violates UNWA. ■

7 Appendix B

Cases 2, 5 and 6: Given $u \in U(X)^n$, we define a lexicographic ordering $\geq_{L(u)}$ as follows.

For any $x, y \in X$, the asymmetric part of $\geq_L$ is defined by

$$x >_{L(u)} y \Longleftrightarrow \begin{cases} \exists k \in \{1, ..., n\} \text{ s.t. } u_i(x) = u_i(y) \ \forall i \leq k-1 \ \& u_k(x) > u_k(y) \\ \text{or} \\ u_i(x) = u_i(y) \ \forall i \ \& \ \tau(x) > \tau(y). \end{cases}$$

The symmetric part is defined by $x =_{L(u)} y \Longleftrightarrow u_i(x) = u_i(y) \ \forall i \ \& \ \tau(x) = \tau(y)$.

Let a rule F be such that $x \geq_{F(u)} y \Longleftrightarrow x \geq_{L(u)} y$ for any $u \in U(X)^n$ and any $x, y \in X$. Person 1 is dictator for F. This rule illustrates Cases 2, 5 and 6. It is obvious that F satisfies FR, I, P, ON, λIP and UNWA, and violates IP.

Case 3: Let a rule F be such that for any $u \in U(X)^n$ and any $x, y \in X$,

$$x \geq_{F(u)} y \Longleftrightarrow \begin{cases} u_1(x) \geq u_1(y) & \text{if } x, y \in X_\lambda \cup X_{\lambda'} \text{ with } \# X_\lambda \geq 2 \text{ or } \# X_{\lambda'} \geq 2 \\ \text{or} & \\ \neg \left(u_i(y) > u_i(x) \ \forall i \in N \right) & \text{otherwise.} \end{cases}$$

This rule illustrates Case 3. This rule satisfies λFR, I, P, ON, UNWA and IP (and hence λIP), and violates FR. Note that Person 1 is decisive for any pair (x, y) except for $\{x\} = X(x)$ and $\{y\} = X(y)$. Let $X = \{x, y, z, w\}$ where the attribute sets are $\{x, y\}$, $\{z\}$ and $\{w\}$. According the rule, 1 is complete dictator[19] on $\{x, y, z\}$ and $\{x, y, w\}$ and the Pareto extension rule govern $\{z, w\}$. Non-welfare attributes are used as $u_1(x) > u_1(y)$ implies $x >_{F(u)} y$ whereas $u_1(z) > u_1(w)$ does not always imply $z >_{F(u)} w$. FR is also violated since $z >_{F(u)} x >_{F(u)} w$ does not always imply $z >_{F(u)} w$.

Case 7: Let a rule F be defined as follows. For any $x, y \in X$ with $x \neq y$, if $\# (X(x) \cup X(y)) \geq 3$, then $x \geq_{F(u)} y \Longleftrightarrow u_1(x) \geq u_1(y)$, and if $\# (X(x) \cup X(y)) < 3$[20], then $x >_{F(u)} y \Longleftrightarrow [u_i(x) > u_i(y) \forall i]$ or $[\tau(x) > \tau(y)$ and $\neg (u_i(y) > u_i(x) \ \forall i)]$. We also define $x =_{F(u)} y$ if $x = y$.

[19] See Example 13 for the definition.

[20] Thus $\# (X(x) \cup X(y)) = 2$, and $X(x)$ and $X(y)$ are singleton.

Note that $x =_{F(u)} y$ never occurs if $x \neq y$ and $X(x)$ and $X(y)$ are singleton. This rule illustrates Case 7, satisfying λFR, I, P, ON, λIP and UNWA, and violating FR and IP. Person 1 is decisive for any pair (x, y) except for $\{x\} = X(x)$ and $\{y\} = X(y)$, but not dictator.

Case 8: Let A, B, C be such that $A = \{x : \tau(x) = 1\}$, $B = \{x : \tau(x) = 2\}$ and $C = \{x : \tau(x) \geq 3\}$. A binary relation $\geq_T$ is defined by its asymmetric parts $>_T$ and symmetric parts $=_T$ as follows:

$$
\begin{array}{lcc}
x >_T y & \Longleftrightarrow & [x \in A \& y \in B] \vee [x \in B \& y \in C] \vee [x \in C \& y \in A] \\
x =_T y & \Longleftrightarrow & [x, y \in A] \vee [x, y \in B] \vee [x, y \in C]
\end{array}
$$

Note that $\geq_T$ is complete but not transitive; $>_T$ has cycles such that $x >_T y >_T z >_T x$ where $x \in A$, $y \in B$ and $z \in C$. Let F be such that for any $u \in U(X)^n$ and any $x, y \in X$,

$$
\begin{array}{lcc}
x >_{F(u)} y & \Longleftrightarrow & u_1(x) > u_1(y) \text{ or } [u_1(x) = u_1(y) \text{ and } x >_T y], \\
x =_{F(u)} y & \Longleftrightarrow & [u_1(x) = u_1(y) \text{ and } x =_T y].
\end{array}
$$

This rule illustrates Case 8. This is a dictatorial rule satisfying λFR, I, P, ON, λIP and UNWA and violating FR and IP. Letting $x \in A$, $y \in B$, $z \in C$, $u_1(x) = u_1(y) = u_1(z)$, we have $x >_{F(u)} y >_{F(u)} z >_{F(u)} x$. This shows that F violates FR and uses non-welfare attributes. It is easy to check that this rule satisfies λFR.

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