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Kenichi Tamegawa\textsuperscript{2}, Yasuharu Ukai\textsuperscript{3}, and Ryokichi Chida\textsuperscript{4}

Abstract
This paper analyzes the effect of the cloud computing system on the macro economy based on a dynamic stochastic general equilibrium model: the DSGE Model. Ukai (2013) statistically found that only 10 percent of Japanese firms listed on the Tokyo Stock Exchange, (TSE), architected a pure public cloud computing system in 2012. Ukai and Inagaki (2014) also found that the mean of the net assets at hybrid cloud computing firms was greater than the mean at non-cloud computing firms. However, this difference dramatically decreased after statistically controlling for the number of employees. Ukai and Inagaki also estimated a positive relationship between the increase in retained earnings and the increase in depreciation costs as well as the negative dummy variable effect of cloud computing to a positive regression coefficient. Based on this micro data evidence, we conducted a simulation analysis using the DSGE Model. In our model the firms were divided into cloud computing firms and non-cloud computing firms. Our simulation concluded that a 10 percent adoption rate of cloud computing among Japanese firms corresponded to a 10 percent upward shift in production function. In addition, after an investigation of impulse response functions, we concluded that the cloud computing system creates an upward economic growth path in the first stage, but that gradually the path becomes stable in the last stage.

JEL classification: O33, E27

Keywords: Cloud computing system, Dynamic stochastic general equilibrium model

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1. Introduction

This paper analyzes the effect of the cloud computing system on the macro economy based on the dynamic stochastic general equilibrium model: the DSGE Model.

Cloud computing is innovative computer architecture based on broadband access service. The users of the cloud computing service view their computer processing as a network service. The providers of the cloud computing service, on the other hand, establish a large-scale data center, construct a service system for software, and collect data through the Internet. The cloud computing service is classified into three types: public, private, and hybrid cloud computing. Public cloud computing is open to any computer-using company that pays a service charge to the providers. On the contrary, private cloud computing is used by member companies that have constructed a cloud computing system over a restricted company information infrastructure. Some companies use public and private cloud computing simultaneously, which we call hybrid cloud computing.

Ukai (2013) statistically found that only 10 percent of Japanese firms listed on the Tokyo Stock Exchange, (TSE) used a purely public cloud computing system. Ukai and Inagaki (2014) also found that the mean of the net assets at cloud computing firms was greater than the mean at non-cloud computing firms. However, this difference dramatically decreased after statistically controlling for the number of employees. Ukai and Inagaki also estimated a positive relationship between depreciation cost and the rate of cloud computing in firms to the total number of firms in the TSE. Based on micro data, what is the macroeconomic implication of these four statistical findings? In order to answer this question we conducted simulation analyses using the DSGE Model.

2. Related Literature

Bayrak, Conlay and Wilkie (2011) wrote one of the most comprehensive surveys of academic research on cloud computing in 2010. Most literature on cloud computing in 2010 appeared in academic journals of computer or management science. Only four economics papers were listed by Bayrak, Conlay, and Wilkie. Three of them were based on game theory or mechanism design.

In Table 1, we grouped seven economics papers by type of tool and the characteristics of the data. Takemura, Watanabe and Ukai, (2005); Ukai (2009); Ercolani (2013); Chard, Caton, Rana, and Bubendorfer (2010); Belalem, etc. (2011) used micro data for their microeconomic models. On the other hand, Takagi and Tanaka (2013), and Etro
(2009) used macro data for their macroeconomic model. Therefore, microeconomic analyses could not suggest the macroeconomic implications of the conclusion and macroeconomic analyses could not show the microeconomic evidence of the assumptions on production function.

Concerning the DSGE model, Takagi and Tanaka (2013), and Etro (2009) treated the cloud computing system as a technical level parameter of production function in only one representation industry. This treatment had no evidence from microeconomic data. In order to overcome this statistical weakness we assume two types of production functions. In one type firms use public cloud computing, and in the other type, firms do not use public cloud computing. Ukai (2013) has presented microeconomic evidence for the parameters of two types of production functions.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Micro</th>
<th>Macro</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ercole (2013)</td>
<td></td>
</tr>
<tr>
<td>Auction Model</td>
<td>Chard, Caton, Rana &amp; Bubendorfer (2010)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Belalem, etc. (2011)</td>
<td></td>
</tr>
<tr>
<td>DSGE Model</td>
<td></td>
<td>Etro (2009)</td>
</tr>
</tbody>
</table>

3. The Model

In our model, we introduced firms, households, and government analysis. Under an assumption of flexible price, our model is simply a variant of Real Business Cycle (RBC) models. However, in contrast to standard RBC models, firms in our model can choose to adopt cloud computing or not.
3.1 Firms
There are two types of firms: firms with or without cloud computing systems. Firm $i$ can choose whether or not to adopt a cloud computing system.

We assume that if firm $i$ adopted cloud computing, then the production function would be as follows:

$$Y_{c,i,t} = z_t h^i v (K_{c,i,t})^{\alpha_c} (l_{c,i,t})^{1-\alpha_c}$$

where $0 \leq h^i \leq 1$

where $1 \leq v$.

$$Y_{c,i,t} = z_t h^i v (K_{c,i,t})^{\alpha_c} (l_{c,i,t})^{1-\alpha_c}$$

where $Y_{c,i,t}$ denotes the output for the firms using a cloud computing system; $K_{c,i,t}$ denotes capital stock; $l_{c,i,t}$ hours worked, and $z_t$ productivity over all firms including the firms not using a cloud computing system. Symbol $v$ represents cloud technology. Symbol $h^i$ denotes perishable human capital due to cloud computing technology. Otherwise, we could also interpret $h^i$ as the probability that adopting a cloud computing system would suit an individual firm. In this view, cloud computing is interpreted as follows: while $v$ represents the general advantage of cloud computing, $h^i$ represents the idiosyncratic security risk of an information system. In this case, $hv$ is the net profitability of cloud computing.

On the other hand, if the firms do not adopt cloud computing then they would have the following production function:

$$Y_{nC,i,t} = z_t (K_{nC,i,t})^{\alpha_{nC}} (l_{nC,i,t})^{1-\alpha_{nC}}.$$ (2)

Symbol $Y_{nC,i,t}$ denotes the output for the firms without cloud computing; $K_{C,i,t}$ capital stock, and $l_{c,i,t}$ hours worked. We normalize fixed human capital for the firms without cloud computing to 1.

Furthermore, we assume the adjustment costs needed to install new capital as follows: for $i = C, nC$

$$\Phi_i \left( \frac{l_{i,t}}{K_{i,t}} \right) = \frac{\chi_i}{2} \left( \frac{l_{i,t}}{K_{i,t}} - \delta_i \right)^2.$$ (3)

Symbol $\delta_i$ denotes the depreciation rate for capital stock. It is natural to assume

5 Bayrak, Conlay and Wilkie (2011) insisted that data security was of great concern in a cloud computing system for both consumers and producers of computer services (p.14). In addition to these authors, Takemura and Komatsu (2013) statistically found that information security has no significant relation with enterprise size. Therefore, information security risks exist in all sizes of enterprises.

6 Please refer Table 6-11 in Ukai and Inagaki (2014).
\( x_c \leq x_{nc} \) because of the nature of cloud computing. Capital stock is accumulated as follows:

\[
K_{t+1} = (1 - \delta_t)K_{t} + I_{l,t}.
\] (4)

Taking the above assumption into consideration, we state the profit maximization problem for firms as follows

\[
\max E_0 \sum_{t=0}^{\infty} \Lambda_t \left( Y_{l,t} - w_t I_{l,t} - I_{t,t} - \Phi_t \left( \frac{I_{l,t}}{K_{t,t}} \right) K_{t,t} \right)
\]

\[
K_{t+1} = (1 - \delta_t)K_{t} + I_{l,t}
\]

\[
\Lambda_t = \prod_{j=0}^{t} \frac{1}{R_j}
\]

\( R_0 = 1 \). Both type of firms choose the time path for capital stock and hours worked according to (5). The first-order conditions are as follows: for \( i = C, nc \)

\[-1 - \chi_i \left( \frac{I_{l,t}}{K_{l,t}} - \delta_i \right) + q_{l,t} = 0 \] (6)

\[-q_{l,t} + E_t \left[ \frac{1}{R_t} \left( \alpha_t Y_{l,t+1} + q_{l,t+1}(1 - \delta_t) \right) \right] = 0 \] (7)

\[ w_t = (1 - \alpha_t) \frac{Y_{l,t}}{h_{l,t}} \] (8)

Next, we consider the problem that addresses how a certain firm decides to adopt cloud computing. To do this, we define the value function as \( V_0^i(h) \) for \( i = C, nc \) that is associated with the problem (5). This function depends on which number \( h \) takes.

Suppose that at time zero, the firms plan their action along with the problem (5). Next, they compare the present value of profit with cloud computing with the value of profit without cloud computing. If \( V_0^C(h) > V_0^{nc}(h) \), then the firms adopt cloud computing. Therefore, the threshold value denoted by \( h^* \) is such that \( V_0^C(h^*) = V_0^{nc}(h^*) \).

In the following, for analytical simplicity, we assume that \( h \) takes binary values \( \bar{h} \) and \( \underline{h} \) with the probability \( 1 - h^* \) and \( h^* \), respectively. These values are related as \( \bar{h} > h^* > \underline{h} \). In these assumptions, the firms that have \( \bar{h} \) adopt cloud computing.

3.2 Households

Identical households that possess financial assets \( R_t \) supply work force \( I_t \) and earn real

---

7 In detail, for \( i = C, nc \), \( V_0^i(h) = \max \left\{ E_0 \sum_{t=0}^{\infty} \Lambda_t \left( \frac{Y_{l,t}}{h_{l,t}} - w_t I_{l,t} - I_{t,t} - \Phi_t \left( \frac{I_{l,t}}{K_{t,t}} \right) K_{t,t} \right) \right\} \] constraints

8 The median of net profit at public cloud firms was higher than the median of net profit at non-cloud firms. Please refer to Table 10 in Ukai and Inagaki (2014).
wages, \( w_t \). Furthermore, they receive interest payments \( A_t R_t \) and profits \( \Pi_t \). These profits are made from adjustment costs since we assume that there are firms that make capital stock effective. After paying a lump-sum tax \( T_t \), these firms decide their consumption and work plan. Then, the budget constraint for households is expressed as follows:

\[
A_{t+1} = A_t R_t + \Pi_t + w_t l_t - C_t - T_t .
\]  

We assume the following temporal utility function with consumption good \( C \) and leisure time \( 1 - l_t \).

\[
\ln(C_t) + \theta \ln(1 - l_t)
\]  

Households maximize inter-temporal utility under the discount rate \( \beta \). This maximization yields the following first order conditions

\[
E_t \left[ \beta R_t \frac{C_t}{C_{t+1}} \right] = 1.
\]

\[
\frac{\theta}{1 - l_t} = \frac{w_t}{C_t}.
\]

### 3.3 Aggregation and Market Equilibrium

Two types of firms appear in our model: firms with cloud computing and firms without cloud computing. As stated above, whether firms install cloud computing depends on \( h \). If \( h = \bar{h} \), then firms choose to introduce cloud computing. The probability that \( h = \bar{h} \) occurs is \( 1 - h^* \). We define the share of the firms with cloud computing as \( \omega \). By the law of large numbers, \( \omega = 1 - h^* \). Moreover, we define the aggregate variables of the firms with cloud computing as \( 1 \) with the subscript \( C \). Note that since \( h^l \) is a binary value, \( Y_{C,t} = Y_{C,l,t} \) for all cloud-adopting firms. Similarly, the variables related to the firms without cloud computing are identified by the subscript \( nC \). In this case, noting that for all firms with cloud computing, their behavior is identical, aggregate output can be

\[
Y_t = \omega Y_{C,t} + (1 - \omega) Y_{nC,t}
\]  

where \( \omega = 1 - h^* \).

The aggregated labor market is described as the following equations:

\[
w_t l_t = (1 - \alpha_C)\omega Y_{C,t} + (1 - \alpha_{nC})(1 - \omega) Y_{nC,t} .
\]

\[
l_t = \omega l_{C,t} + (1 - \omega) l_{nC,t} .
\]
The resource constraint is implied by the asset market equilibrium condition:

\[ A_{t+1} = \omega K_{c,t+1} + (1 - \omega)K_{nc,t+1} \Rightarrow Y_t = C_t + I_{c,t} + I_{nc,t} + G_t. \]  

(16)

4. Simulation Analysis

In this section, after using parameter settings that are suited to the Japanese economy, we conduct several simulation analyses using Dynare 4.3.3 and MATLAB 8.1(R2013a), from MathWorks, Inc. First, limiting the discussion to the steady state, we numerically analyze our economy. Next, we show how the share of cloud-adopted firms depends on cloud technology. Finally, if the cloud technology level is enhanced, we calculate the transition rate of the main economic variables from an old steady state to a new steady state.

4.1 Steady-state Analysis

Assume that the economy is in the non-stochastic steady state. Then, we consider how \( h^* \) is determined. To do this, in the steady state, the net present value of profits with or without cloud computing is as follows:

\[ V^C(h) = \frac{zh v(K_c)^{\alpha_c}(l_c)^{1-\alpha_c} - w l_c - l_c}{1 - 1/R}. \]  

(17)

\[ V^NC(h) = \frac{z(K_{nc})^{\alpha_{nc}}(l_{nc})^{1-\alpha_{nc}} - w l_{nc} - l_{nc}}{1 - 1/R}. \]  

(18)

Therefore, the expression of \( h^* \) that equalizes \( V^C(h) \) with \( V^NC(h) \) is written as:

\[ h^* = \frac{z(K_{nc})^{\alpha_{nc}}(l_{nc})^{1-\alpha_{nc}} - w(l_{nc} - l_c) - (l_{nc} - l_c)}{zv(K_c)^{\alpha_c}(l_c)^{1-\alpha_c}}. \]  

(19)

9 Please refer to http://www.dynare.org.

10 Please refer to Table 2 (p.59), Ukai (2003).
Table 2. Exogenous parameters

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.976</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>0.3</td>
<td>Capital share of C-firms</td>
</tr>
<tr>
<td>$\alpha_{NC}$</td>
<td>0.362</td>
<td>Capital share of nC-firms</td>
</tr>
<tr>
<td>$\delta_C$</td>
<td>0.089</td>
<td>Depreciation rate of C-firms</td>
</tr>
<tr>
<td>$\delta_{NC}$</td>
<td>0.12</td>
<td>Depreciation rate of nC-firms</td>
</tr>
<tr>
<td>$K_{NC}$</td>
<td>1</td>
<td>Capital stock of nC-firms</td>
</tr>
<tr>
<td>$l_C$</td>
<td>0.35</td>
<td>Hours worked for C-firms</td>
</tr>
<tr>
<td>$l_{NC}$</td>
<td>0.3</td>
<td>Hours worked for nC-firms</td>
</tr>
<tr>
<td>$v$</td>
<td>1.10</td>
<td>Technology level of cloud computing system</td>
</tr>
<tr>
<td>$z$</td>
<td>1</td>
<td>Common technology level</td>
</tr>
</tbody>
</table>

Given $\beta$, $\alpha_C$, $\alpha_{NC}$, $\delta_C$, $\delta_{NC}$, $K_{NC}$, $l_C$, $l_{NC}$, $v$ and $z$, the other variables in the steady state can be calculated.\(^{11}\) We set $\alpha_{NC}$ and $\delta_{NC}$ according to Hayashi and Prescott (2002). We set $l_{NC} = 0.3$, which we calculate from the Monthly Labor Survey, Japan, 2012, which is $0.3 = (\text{hours} \times 19.1 \text{ days}/24 \text{ hours})$. On the other hand, $l_C$ is set to 0.35, which implies that $Y_C/Y_{NC}$ is 1.062, which is nearly equal to 1.1 of $v$. $\delta_{NC}$ is set to 0.12. Given the steady-state values as shown in Table 2, we can obtain Table 3, which addresses endogenous variables.

Table 3. Endogenous parameters

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^C$</td>
<td>0.493</td>
<td>Output of C-firms</td>
</tr>
<tr>
<td>$y^{NC}$</td>
<td>0.464</td>
<td>Output of nC-firms</td>
</tr>
<tr>
<td>$K^C$</td>
<td>1.12</td>
<td>Capital stock of C-firms</td>
</tr>
<tr>
<td>$I^C$</td>
<td>0.100</td>
<td>Investment of C-firms</td>
</tr>
<tr>
<td>$I^{NC}$</td>
<td>0.12</td>
<td>Investment of nC-firms</td>
</tr>
<tr>
<td>$h^*$</td>
<td>0.90</td>
<td>Threshold value of information system safety</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>0.90</td>
<td>Actual information system safety</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.10</td>
<td>The share of C-firms</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.467</td>
<td>Total output (weighted average)</td>
</tr>
</tbody>
</table>

\(^{11}\) In detail, see Appendix B.
It is worth noting that \( \nu \) is chosen such that \( \omega = 0.1 \), which is the estimated value in the Japanese economy. Therefore, we can infer that in the Japanese economy, adopting cloud computing raises productivity by approximately 11\%.

It is interesting to investigate how \( \omega \) is related to other parameters, especially the productivity of cloud computing \( \nu \). Figure 1 depicts this relationship.

As is clear in the expression for \( h^* \), the threshold value \( h^* \) is negatively related to \( \nu \). As the productivity of cloud computing \( \nu \) increases, many firms recognize the productivity level of cloud computing.\(^{12}\) This observation means that the higher the productivity level yields a higher value of \( \omega \), which is defined as \( 1 - h^* \). Therefore, the number of the firms with cloud computing rises and the output also increases.

\(^{12}\) Note that \( \nu \to \infty \) implies \( \omega \to 1 \). This is clear from Equation (19).
Figure 2 describes how $v$ relates to total output. In Figure 2, output is normalized such that $Y$ is equal to 1 at $v = 1$. Considering that output with cloud computing is larger than that without cloud computing, we can easily understand that as the share of cloud-adopting firms rises, total output is also increased.
4.2 Short-run dynamics

In this section, we investigate the properties of our model, using impulse response functions to productivity shock. We assume that this shock is common to all firms (cloud + non cloud). To conduct this investigation, we linearize the model around the steady state. Figure 3 depicts the percentage deviation from the steady-state values under the parameters used in 4.2, when 1 percent productivity shock is given. The parameters $\chi_c$ and $\chi_{nc}$ that represent adjustment costs are set to 30 and 60.

First, note that in the short run, the advantage of adopting cloud computing is captured in adjustment costs; the firms with cloud computing can easily adjust to capital stock because their adjustment costs are lower.

As shown Figure 1, productivity shock raises consumption through an increase in permanent income. This dampens savings and therefore, real interest rates rise. Thus, investment will decrease as a result. The firms with cloud computing largely decrease their investment because of lower adjustment costs.

Labor supply decreases because for the same reason as consumption, that is, income effects. Total output rises though it is gradually diminished. Wages also rise due to the upward-shift of the labor supply curve.

A new feature that we can add to business cycle literature from our model is that if firms have access to the cloud computing system, more sensitive investment movements to exogenous shocks can be allowed. Therefore, we conjecture that if firms with the cloud computing system increase, fluctuations of investment could rise.

5. Conclusion and Remarks

As a result of our dynamic simulation, we see that the change in the rate of cloud computing will create an upward path. However, this kind of tendency gradually decreases with the negative effects of cloud computing; in other words, there is an increasing risk of information security. Some parameters in Table 2 might be weakly plausible from the standpoint of econometrics. We need more evidence of the security risks in the cloud computing system. In this paper the cloud effects on utility function (10) were neglected for the simple simulation. A diversification of consumer goods C could be a fruitful extension of our model.
Appendix A.

The full model:

\[
Y_{c,t} = z_t h v(K_{c,t})^{\alpha_c}(l_{c,t})^{1-\alpha_c} \\
Y_{nc,t} = z_t(K_{nc,t})^{\alpha_{nc}}(l_{nc,t})^{1-\alpha_{nc}} \\
q_{c,t} = 1 + \chi_c \left( \frac{l_{c,t}}{K_{c,t}} - \delta_c \right) \\
q_{nc,t} = 1 + \chi_{nc} \left( \frac{l_{nc,t}}{K_{nc,t}} - \delta_{nc} \right)
\]

\[
q_{c,t} = E_t \left[ \frac{1}{R_t} \left( \alpha_c \frac{Y_{c,t+1}}{K_{c,t+1}} + q_{c,t+1}(1 - \delta_c) \right) \right] \\
q_{nc,t} = E_t \left[ \frac{1}{R_t} \left( \alpha_{nc} \frac{Y_{nc,t+1}}{K_{nc,t+1}} + q_{nc,t+1}(1 - \delta_{nc}) \right) \right] \\
K_{c,t+1} = (1 - \delta_c)K_{c,t} + l_{c,t} \\
K_{nc,t+1} = (1 - \delta_{nc})K_{nc,t} + l_{nc,t} \\
w_t = (1 - \alpha_c) \frac{Y_{c,t}}{h_{c,t}} \\
w_t = (1 - \alpha_{nc}) \frac{Y_{nc,t}}{h_{nc,t}} \\
h_t = \omega h_{c,t} + (1 - \omega)h_{nc,t} \\
Y_t = \omega Y_{c,t} + (1 - \omega)Y_{nc,t} \\
E_t \left[ \beta R_t \frac{C_t}{C_{t+1}} \right] = 1 \\
\frac{\theta}{1 - h_t} = \frac{w_t}{C_t} \\
Y_t = C_t + l_{c,t} + l_{nc,t} + G_t
\]
Appendix B

Calculation of steady-state valued
We set exogenously \( l_{nc}, \sigma_{nc}, \sigma_c, z, K_{nc}, v, \beta, \delta_{nc}, \delta_c, \) and \( \frac{c}{\gamma} \). Using these values, we can calculate the steady-state as follows.

\[
\frac{Y_c}{Y_{nc}} = \frac{1 - \alpha_{nc}}{1 - \alpha_c} \times \frac{l_c}{l_{nc}}
\]

\[
1 = \frac{R}{R} = \beta,
\]

\[
q_c = q_{nc} = 1,
\]

\[
K_c = \frac{\alpha_c [R - (1 - \delta_{nc})]}{\alpha_{nc} [R - (1 - \delta_c)]} \frac{Y_c}{Y_{nc}} K_{nc},
\]

\[
l_c = \delta_c K_c,
\]

\[
l_{nc} = \delta_{nc} K_{nc},
\]

\[
l = \omega l_c + (1 - \omega) l_{nc},
\]

\[
w = (1 - \alpha_{nc}) \frac{Y_{nc}}{l_{nc}},
\]

\[
\theta = \frac{w}{c} (1 - \lambda),
\]

\[
h^* = \frac{z(K_{nc})^{\alpha_{nc}} l_{nc}^{1 - \alpha_{nc}} - w(l_{nc} - l_c) - (l_{nc} - l_c)}{z v(K_c)^{\alpha_c} l_c^{1 - \alpha_c}},
\]

\[
\omega = 1 - h^*,
\]

\[
\bar{h} = \left\{ h \in \text{unit interval} \left| \frac{Y_c}{Y_{nc}} = \frac{z h v(K_c)^{\alpha_c} l_c^{1 - \alpha_c}}{z (K_{nc})^{\alpha_{nc}} l_{nc}^{1 - \alpha_{nc}}} \right. \right\},
\]

\[
Y = \omega Y_c + (1 - \omega) Y_{nc},
\]

\[
G = \frac{c + I_c + I_{nc}}{Y}.
\]
Appendix C

The log-linearized model

In the following, “^” denotes the percentage deviation from the steady state.

\[
\begin{align*}
\hat{y}_{c,t} &= \log z_t + \alpha_c R_{c,t} + (1 - \alpha_c) l_{c,t} \\
\hat{y}_{nc,t} &= \log z_t + \alpha_{nc} R_{nc,t} + (1 - \alpha_{nc}) l_{nc,t} \\
\hat{q}_{c,t} &= \chi_c \delta_c (l_{c,t} - R_{c,t}) \\
\hat{q}_{nc,t} &= \chi_{nc} \delta_{nc} (l_{nc,t} - R_{nc,t}) \\
q_{c,t} - \tilde{R}_t R_t &= E_t \left[ \alpha_c (\hat{y}_{c,t+1} - \tilde{R}_{c,t+1}) \frac{Y_c}{K_c} + \hat{q}_{c,t+1} (1 - \delta_c) \right] \\
\hat{q}_{nc,t} - \tilde{R}_t R_t &= E_t \left[ \alpha_{nc} (\hat{y}_{nc,t+1} - \tilde{R}_{nc,t+1}) \frac{Y_{nc}}{K_{nc}} + \hat{q}_{nc,t+1} (1 - \delta_{nc}) \right] \\
R_{c,t+1} &= (1 - \delta_c) R_{c,t} + \delta_c l_{c,t} \\
R_{nc,t+1} &= (1 - \delta_{nc}) R_{nc,t} + \delta_{nc} l_{nc,t} \\
\tilde{\omega}_t &= \hat{y}_{c,t} - l_{c,t} \\
\tilde{\omega}_t &= \hat{y}_{nc,t} - l_{nc,t} \\
\hat{l}_t &= \omega l_{c,t} + (1 - \omega) l_{nc,t} \\
\hat{y}_t &= \omega \hat{y}_{c,t} \frac{Y_c}{Y} + (1 - \omega) \hat{y}_{nc,t} \frac{Y_{nc}}{Y} \\
E_t [\tilde{R}_t + \hat{c}_t - \hat{c}_{t+1}] &= 0 \\
\hat{c}_t - \tilde{\omega}_t &= -\hat{l}_t \frac{l}{1 - l} \\
\hat{y}_t &= \frac{\hat{c}_t}{Y} + \frac{l_{c,t}}{Y} l_{c,t} \frac{l_{nc,t}}{Y} + \frac{\hat{c}_t}{Y} G \\
\end{align*}
\]
Figure 3. Impulse responses to the productivity shock

Note: Solid lines depict percentage deviation from the steady state.
References

6. EDINET (Electronic Disclosure for Investors' Network) http://disclosure.edinet-fsa.go.jp/


