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Sources of Inequality and Business Cycles: Evidence from the US and Japan

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文部科学大臣認定　共同利用・共同研究拠点

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Abstract

We investigate (i) sources of inequality and business cycle fluctuations in the US and Japan and (ii) the effects of reducing inequality on business cycles. Developing a heterogeneous-agent business cycle model with unconstrained and hand-to-mouth households and various wedges to represent economic distortions, we estimate the model by the Bayesian methods. We find that, in the US, the labor market distortion specific to unconstrained households is the common factor that significantly impacts business cycles and consumption inequality, whereas there are no common factors in Japan. In both countries, the primary source of business cycles is distortions in aggregate productivity, and that of consumption inequality is household-specific labor market distortions. We assess labor market reforms and redistribution policy as means to reduce consumption inequality. Our findings imply that the effects of lowering inequality on business cycle volatility depend on the country and how it is done. We also find that labor market reform is welfare-improving in both countries while redistribution policy is not.

Keywords: Inequality; consumption inequality; business cycle accounting; wedges; distortions; hand-to-mouth

JEL codes: E25; E32; E37

1 Introduction

Inequality and business cycles are two important aspects of modern macroeconomics, each with significant implications for social welfare. However, the relation between these two phenomena is complex. While inequality is often a consequence of macroeconomic dynamics, it can also affect the business cycle through various channels.

This paper seeks to accomplish two main research objectives. First, we aim to identify the driving sources of inequality and business cycles in the US and Japan. In particular, we investigate whether there are common factors that significantly impact both inequality and business cycles. Second, we explore the effects of reducing inequality on business cycles. Our focus is on whether reducing inequality increases or decreases business cycle volatility.

To reach these research objectives, we develop a tractable heterogeneous-agent business cycle model with unconstrained (U) households and hand-to-mouth (HtM) households. While U households are the standard representative permanent income households, HtM households consume their entire disposable income in each period. Each household faces idiosyncratic risks of being HtM from U and those of being U from HtM, as employed by Bilbiie (2020, 2024), and Bilbiie, Primiceri and Tambalotti (2022).

We also introduce, into the model, time-varying "wedges," which resemble aggregate productivity, distortionary taxes on investment and labor income, and government consumption, following the business cycle accounting (BCA) approach proposed by Chari, Kehoe and McGrattan (2002, 2007). These wedges can be interpreted as distortions in the economy; the efficiency wedge is for distortion in aggregate productivity, the investment wedge is for distortion in investment activity, the labor wedge is for labor market distortions, and the government wedge is for distortion in the resource constraint. While our model is relatively simple and describes only the real side, introducing these wedges enables the model to cover various detailed models, including monetary ones. Because of the heterogeneity among households, we introduce three types of labor wedges: neutral to two types of households, U-specific and HtM-specific labor wedges.

In this paper, we focus on consumption inequality as inequality. This contrasts recent studies by Bilbiie et al. (2022) and Bayer, Born and Luetticke (2024), who focus on income inequality and wealth distributions in the US to investigate the relation between inequality and business cycles. As Attanasio and Pistaferri (2016) and Meyer and Sullivan (2023) point out, consumption inequality is a good indicator of economic well-being because social welfare is measured by consumption and leisure. Then, consumption inequality is an important topic, as well as income inequality and wealth distribution. We construct the data on consumption inequality by using the Consumer Expenditure Surveys of the Bureau of Labor Statistics for the US and the Family Income and Expenditure Survey (*Kakei Chosa*) of the Ministry of Internal Affairs and Communications for Japan.

By the Bayesian estimations of the model, we find that, in the US, the U-specific labor wedge significantly impacts both business cycles and consumption inequality, while the primary source of business cycles is the efficiency wedge, and that of consumption inequality is the HtM-specific labor wedge. On the other hand, we find that no common factors significantly impact both business cycles and consumption inequality in Japan. In Japan, the key for business cycles is the distortion in aggregate productivity, and that for consumption inequality is the labor market distortions specific to U and HtM households. The most significant contributor to consumption inequality is the HtM in the US, while it is the U in Japan.

To investigate the relation between inequality and business cycles, we conduct two counterfactual simulations and analyze the effects of reducing consumption inequality on output growth volatility. The first is the labor market reforms by removing the effects of the U-specific and HtM-specific labor wedges, which are the primary driving sources of consumption inequality in both countries. We find that it increases the output growth volatility in the US, whereas it reduces the output growth volatility in Japan. The second is the redistribution policy. We find that it reduces the output growth volatility in the US, whereas it increases in Japan. Then, the effects of reducing inequality on the business cycle depend on the country and how to do it.

We also assess the welfare implications of these policies. Labor market reform is welfare-improving in both countries by mitigating distortions. In contrast, redistribution policy is not. It lowers either the welfare level of U or HtM and reduces social welfare. Labor market reforms work well in Japan because they help reduce inequality, stabilize business cycles, and improve welfare. On the other hand, in the US, no such policy works well. Labor market reforms reduce inequality and improve welfare but destabilize business cycles. Redistribution policy reduces inequality and stabilizes business cycles but lowers welfare.

Related literature: Many researchers have studied the sources of business cycle fluctuations. In particular, King and Rebelo (1999) and Hayashi and Prescott (2002) emphasize the importance of aggregate productivity shock. Since the seminal work of Smets and Wouters (2003, 2007), estimations of medium-scale DSGE models using the Bayesian method have become popular in investigating the source of business cycles. Recent studies emphasize the importance of investment shocks as well as productivity shocks in business cycles, including Justiniano, Primiceri and Tambalotti (2010, 2011), and Kaihatsu and Kurozumi (2014a) for the US, and Sugo and Ueda (2008), Hirose and Kurozumi (2012), Kaihatsu and Kurozumi (2014b), and Inaba, Nutahara and Shirai (2022) for Japan.

The relation between inequality and business cycles by the estimation of a heterogeneous-agent New-Keynesian (HANK) model has been studied by Bayer et al. (2024) and Bilbiie et al. (2022). Bayer et al. (2024) emphasize that including data on inequality measures in observations is important for the estimation results. Bilbiie et al. (2022) is closely related to our paper since they also investigate the effects of reductions in consumption inequality on business cycles in their models. There are two main differences between their papers and ours. The first is on the models; they develop medium-scale HANK models with many frictions and shocks, while our model is relatively simple with time-varying wedges following the BCA approach. The second is that they use income inequality measures as an observable variable in estimations, while we use the data on consumption inequality.

Our paper is closely related to the paper by Bilbiie et al. (2022) because they also focus on the effects of reducing inequality on business cycles. They find that reducing inequality stabilizes the business cycles by estimating their model using the US data and considering redistribution policy to reduce inequality. We use Japan's data in addition to the US data and introduce labor market reform as an alternative policy to reduce inequality. One of our important findings is that the effects of reducing inequality depend on the country and how to reduce inequality.

The paper by Berger, Bocola and Dovis (2023) is also related to ours. They show that a class of HANK models can be equivalently represented as a representative-agent NK model with a time-varying discount factor wedge, as in the BCA approach. By measuring their wedges by micro data and estimating their model, they show the importance of imperfect risk-sharing among households in business cycles.

While we employ a DSGE model to investigate the relation between business cycles and inequality, some researchers conduct time-series analyses by VAR models. De Giorgi and Gambetti (2017) and Geiger, Mayer and Scharler (2020) investigate the effects of macro shocks on consumption inequality in the US. Inui, Sudo and Yamada (2017) investigate the impact of monetary policy shocks on inequality in Japan. Theophilopoulou (2022) studies the effects of uncertainty shocks on inequality in the UK.

We employ a kind of the BCA approach proposed by Chari et al. (2002, 2007). We extend the prototype model of BCA to a heterogenous-agent one and introduce additional wedges. BCA has been applied to different periods in various countries. The results obtained by BCA vary depending on the country and the sample period. The efficiency wedge is found as the most important driving force by Chari et al. (2002, 2007) for the US during the Great Depression, by Kobayashi and Inaba (2006) and Otsu (2011) for Japan during the 1990s, by Šustek (2011) for the US during 1959–2004, by Gerth and Otsu (2018) for the European Great Recession, by Chakraborty and Otsu (2013) for Brazil and Russia in the late 2000s, by Cho and Doblas-Madrid (2013) for the East Asian Crisis, and by Brinca, Chari, Kehoe and McGrattan (2016) for many OECD countries during the Great Recession. The labor wedge is found as the primary driving source of business cycles by Brinca et al. (2016) for France, the UK, Belgium, and New Zealand in the 1980s and the US during the Great Recession. Chakraborty and Otsu (2013) and Brinca et al. (2016) find that the investment wedge plays a dominant role in China and India in the late 2000s and in Spain, Ireland, and Iceland during the Great Recession.

Introducing HtM households into the model has a long tradition in macroeconomics, as done by Campbell and Mankiw (1989), Galí, López-Salido and Vallés (2007), Bilbiie (2008), Kaplan and Violante (2014), and Bilbiie et al. (2022). On the empirical side, Kaplan, Violante and Weidner (2014) estimate the share of HtM households in the total households and analyze demographic characteristics and portfolio composition for the US, Canada, Australia, the UK, Germany, France, Italy, and Spain. Hara, Unayama and

Weidner (2016) estimate them for Japan.

Finally, our paper is related to the literature on measuring consumption inequality. As pointed out by Attanasio and Pistaferri (2016) and Meyer and Sullivan (2023), consumption inequality provides a better indicator of economic well-being because social welfare is measured by consumption and leisure. In the US, the measurement of consumption inequality and the relation between consumption inequality and income inequality have been studied by many researchers, including Krueger and Perri (2006), Attanasio, Battistin and Ichimura (2007), Blundell, Pistaferri and Preston (2008), Heathcote, Perri and Violante (2010), Attanasio and Pistaferri (2014), Attanasio, Hurst and Pistaferri (2014), Aguiar and Bils (2015), and Meyer and Sullivan (2023). In Japan, there is also an accumulation of empirical analyses of consumption inequality, including papers by Ohtake and Saito (1998), Abe and Yamada (2009), Yamada (2012), and Inui et al. (2017). Lise, Sudo, Suzuki, Yamada and Yamada (2014) and Higa (2019) show the widening of consumption inequality in recent years in Japan.

Structure of the rest of the paper: The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 explains the data for estimations and our estimation strategy. Section 4 explains the main estimation results. The driving sources of inequality and business cycles are shown. Section 5 examines the effects of reductions of consumption inequality on business cycle volatility through two types of counterfactual simulations. The welfare implications of these policies are also investigated. Finally, Section 6 presents the conclusion.

2 The Model

2.1 Households

There are two types of households: Unconstrained (U) and Hand-to-mouth (HtM). The total population is one, and the number of HtM households is θ.

The U households are the standard representative permanent income households. All U households pool their income and consume the same amount C_t^U. They supply labor L_t^U given the wage rate W_t. They also invest X_t^U, and their capital holding at the end of period t is K_t^U.

Each U households face an idiosyncratic risk of being HtM at the next period at probability $1-s$, as employed by Bilbiie (2020, 2024), and Bilbiie et al. (2022). This risk engages precautionary saving of U households. Then, the U households maximize the following utility V_t^U

$$V_t^U = u(C_t^U, L_t^U) + \beta E_t \left[s V_{t+1}^U + (1-s) V_{t+1}^H \right], \tag{1}$$

where $u(C_t^U, L_t^U)$ is the instantaneous utility, and V_t^H is the utility of HtM.

The ownership of the capital stock belongs to the U households as a whole, not to each member of the U households. Thus, being from the U household to the HtM household doesn't change capital stock held by the U households. In addition, we assume that an individual who was the U household in the period $t-1$ owns the right to receive the rental rate of capital R_t in the period t. Under this setting, the budget constraint of U households is

$$C_t^U + (1+\tau_t^X) X_t^U \leq s R_t K_{t-1}^U + (1-\tau_t^L - \tau_t^{LU}) W_t L_t^U + T_t^U, \tag{2}$$

where τ_t^X, τ_t^L, and τ_t^{LU} denotes the investment wedge, the neutral labor wedge, and the U-specific labor wedge, respectively.[1] T_t^U is the lump-sum tax to the U households. $sR_tK_{t-1}^U$ is the sum of the capital income of individuals who are the U households in both periods $t-1$ and t. As explained by Chari et al. (2007), these wedges, which resemble distortionary taxes on investment and labor supply activities, can be interpreted as reduced forms of distortions in the economy. We consider the neutral labor wedge, which affects both households and household-type specific ones since the model has two households.

The evolution of the capital stock is given by

$$K_t^U = (1-\delta) K_{t-1}^U + X_t^U. \tag{3}$$

HtM households consume their entire disposable income. As in U households, all HtM households pool their income and consume the same amount C_t^H. They supply labor L_t^H given the wage rate W_t. Unlike U households, HtM households cannot invest. Then,

[1] In an earlier version of this paper, we checked the robustness of our main results in the case without the neutral labor wedges.

the budget constraint of HtM households is

$$C_t^H = (1-s)\frac{1-\theta}{\theta}R_t K_{t-1}^U + (1-\tau_t^L - \tau_t^{LH})W_t L_t^H + T_t^H, \tag{4}$$

where τ_t^{LH} is the HtM-specific labor wedge, and T_t^H is the lump-sum tax to HtM households. $(1-s)\frac{1-\theta}{\theta}R_t K_{t-1}^U$ is the sum of the capital income of individuals who were the U households in the period $t-1$. $\frac{1-\theta}{\theta}$ is to adjust the populations of U (θ) and HtM ($1-\theta$).

The first-order necessary conditions of two households are given by

$$\begin{aligned}(1+\tau_t^X)\Lambda_t^U = \beta E_t \big\{ & s\Lambda_{t+1}^U \left[(1+\tau_{t+1}^X)(1-\delta) + sR_{t+1}\right] \\ & +(1-s)\Lambda_{t+1}^H (1-s)\frac{1-\theta}{\theta}R_{t+1}\big\},\end{aligned} \tag{5}$$

$$u_L(C_t^U, L_t^U) = \Lambda_t^U (1-\tau_t^L - \tau_t^{LU})W_t, \tag{6}$$

$$u_L(C_t^H, L_t^H) = \Lambda_t^H (1-\tau_t^L - \tau_t^{LH})W_t, \tag{7}$$

where Λ_t^U and Λ_t^H are the marginal utilities. The instantaneous utility function is specified as

$$u(C_t^i, L_t^i) = \log(C_t^i) + \xi \log(1-L_t^i),$$

for $i = U$ and H.

2.2 Firms

Firms are perfectly competitive. Firms produce output Y_t using labor input L_t and capital service K_t. The production function is given by

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}, \tag{8}$$

where $\alpha \in (0,1)$ is the cost share of capital, and Z_t is the efficiency wedge, which resembles aggregate productivity.

The marginal productivity conditions are given by

$$R_t = \alpha \frac{Y_t}{K_t}, \tag{9}$$

$$W_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{10}$$

2.3 Fiscal policy

The government spends G_t using lump-sum taxes of two households: T_t^U and T_t^H. Then, the government budget constraint is given by

$$G_t = (1-\theta)T_t^U + \theta T_t^H. \tag{11}$$

The government spending G_t is given by

$$G_t = Z_t g_t, \tag{12}$$

where g_t is the government wedge. In the model, G_t is the government spending, but to fit the data, we consider G_t also includes the net exports in the empirical part following Chari et al. (2007).

The lump-sum taxes are exogenous for households, and they are given by

$$T_t^U = \tau_t^X X_t^U + (\tau_t^L + \tau_t^{LU})W_t L_t^U - \frac{1}{1-\theta}G_t, \tag{13}$$

$$T_t^H = (\tau_t^L + \tau_t^{LH})W_t L_t^H. \tag{14}$$

In this setting, the government spending G_t is financed by the lump-sum tax to the U household. This setting works as a redistribution policy from the U (non-poor) to HtM (poor).

2.4 Aggregations and the market clearing conditions:

In our model, only U households invest and hold capital. Then, the aggregate investment X_t and the aggregate capital, which is equal to capital service K_t at equilibrium, are given by

$$X_t = (1-\theta)X_t^U, \tag{15}$$

$$K_t = (1-\theta)K_{t-1}^U. \tag{16}$$

The aggregate consumption C_t and the aggregate labor L_t are the weighted sum of those of U and HtM. Then, they are given by

$$C_t = (1-\theta)C_t^U + \theta C_t^H, \tag{17}$$

$$L_t = (1-\theta)L_t^U + \theta L_t^H. \tag{18}$$

Finally, the resource constraint is given by

$$Y_t = C_t + X_t + G_t. \tag{19}$$

2.5 Exogenous shocks and equilibrium conditions

There are six exogenous wedges in the model. They follow the AR(1) processes:[2]

$$z_t = (1-\rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z, \tag{20}$$

$$\tau_t^X = (1-\rho_X)\tau^X + \rho_X \tau_{t-1}^X + \varepsilon_t^X, \tag{21}$$

$$\tau_t^L = (1-\rho_L)\tau^L + \rho_L \tau_{t-1}^L + \varepsilon_t^L, \tag{22}$$

$$\tau_t^{LU} = (1-\rho_{LU})\tau^{LU} + \rho_{LU} \tau_{t-1}^{LU} + \varepsilon_t^{LU}, \tag{23}$$

$$\tau_t^{LH} = (1-\rho_{LH})\tau^{LH} + \rho_{LH} \tau_{t-1}^{LH} + \varepsilon_t^{LH}, \tag{24}$$

$$g_t = (1-\rho_g)g + \rho_g g_{t-1} + \varepsilon_t^g, \tag{25}$$

where $z_t = Z_t/Z_{t-1}$, and z, τ^X, τ^L, τ^{LU}, τ^{LH}, and g are the steady-state values. For $j = z$, X, L, LU, LH, and g, ρ_j is the persistent parameter and ε_t^j is an i.i.d. structural shock to a wedge.

The model is detrended by the efficiency wedge Z_t. The detrended equilibrium conditions are log-linearized around a balanced growth path. Appendix A shows the log-linearized equilibrium system.

2.6 Interpretations of wedges

Our model is a tractable two-agent business cycle model with various exogenous wedges. However, our simple model can cover various detailed models by adjusting wedges.

The efficiency wedge resembles aggregate productivity in the production function. Then, the efficiency wedge would cover distortions from misallocation among sectors.

[2]In the business cycle accounting literature, it is often that the wedges follow the following VAR(1) process. In this paper, we assume AR(1) process for simplicity of analysis. An extension to VAR(1) specification of wedges is considered in Appendix E. In addition, Chari et al. (2007) define the efficiency wedge is on the level of aggregate productivity, while we define the efficiency wedge as the growth rate of aggregate productivity. This setting eliminates arbitrariness in taking the trend and increases the fit of the data.

In addition, Chari et al. (2007) show that equilibrium allocation generated by an input-financing friction model employed by Kiyotaki and Moore (1997) is covered by their prototype model with the efficiency wedge.

The labor wedges would cover the distortions of labor market behaviors and distortionary labor income taxations. As in Equations (6) and (7), the labor wedge captures the distortions on the consumption–leisure decision. Chari et al. (2007) show that their prototype model with the labor wedge covers a sticky-wage model à la Erceg, Henderson and Levin (2000). In sticky-price models, the real marginal cost of firms fluctuates, generating the gap between the wage and the marginal product of labor. As shown by Šustek (2011), the labor wedge would capture this gap. If these distortions are common among two types of households, the neutral labor wedge works. If the distortions are specific to U or HtM, U-specific or HtM-specific labor wedge works. In other words, our model is real, but it can capture monetary models by adjusting the labor wedge. Wage markup shock is supposed as an important source in business cycles as shown by Smets and Wouters (2007). Since the wage markup generates the deviation of the real wage rate from the marginal product of labor, the labor wedge can capture its effect.

The investment wedge covers the distortions of investment activities in addition to distortionary investment taxation. As in Equation (5), the investment wedge captures the distortions on the intertemporal decision. Chari et al. (2007) show that their prototype model with the investment wedge can cover the models with financial frictions à la Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). In sticky-price models, the real marginal cost of firms fluctuates, generating the gap between the rental rate of capital and the marginal product of capital. Then, the investment wedge would capture this gap, as shown by Šustek (2011). Bilbiie et al. (2022) emphasize the importance of risk to switch from the U to the HtM $(1 - s)$ as countercyclical in business cycles. This risk distorts intertemporal decisions, then the investment wedge would be able to capture it in our model.

Finally, as shown by Chari et al. (2007), the government wedge covers an open economy model with sudden stops in addition to wasteful government consumption. Then, while our model is a closed economy, our model can cover open economy models.

3 Data and Estimation Strategy

Data: We use five observable variables. Four of them are quarterly macro series: growth rates of real GDP per capita Y_t, real consumption per capita C_t, real investment per capita X_t, and labor supply L_t. The last one is inequality, which is annual. As a measure of inequality, we focus on consumption inequality between HtM and aggregate consumption: C_t^H/C_t. A smaller value of C_t^H/C_t means severe consumption inequality. If amounts of consumption are identical among households, then C_t^H/C_t is one.

According to the empirical facts, the fraction of HtM households is about 20%, and their income is lower than non-HtM. Then, as a proxy of C_t^H/C_t, we calculate the ratio of consumption of the lowest 20 % to that of all consumer units in the "Quintiles of income before taxes" of the Consumer Expenditure Surveys (CEX) of BLS for the US. For Japan, we use the Family Income and Expenditure Survey (*Kakei Chosa*) of the Ministry of Internal Affairs and Communications. This data is annual.

The sample period for the US is from 1984:Q1 to 2019:Q4, and that for Japan is from 1980:Q2 to 2019:Q4. The availability of GDP and consumption inequality data determines the sample period. Since our model is log-linearized, we stopped the sample period at 2019:Q4 to avoid the significant fluctuation caused by Covid-19. The data construction details are in Appendix B.

Estimation strategy: The observation equations for macro variables (Y_t, C_t, X_t ,and L_t) are

$$
\begin{aligned}
100\Delta \log Y_t &= 100(z-1) + (\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t),\\
100\Delta \log C_t &= 100(z-1) + (\hat{c}_t - \hat{c}_{t-1} + \hat{z}_t),\\
100\Delta \log X_t &= 100(z-1) + (\hat{x}_t^U - \hat{x}_{t-1}^U + \hat{z}_t),\\
100 \log L_t &= 100 \log L + \hat{L}_t,
\end{aligned}
$$

where z and L denote steady-state values, and $\hat{v}_t$ is the log-deviation from its steady-state value: $\hat{v}_t = \log(v_t) - \log(v)$ for $v = c$, z, y, x^U, and L. The data of the consumption inequality C_t^H/C_t is annual. We follow Pfeifer (2021) to model the observation equation

Table 1: Fixed Parameter Values

	Parameter	Value
β	Discount factor	0.99
α	Cost-share of capital	0.37
δ	Depreciation rate of capital	0.025
θ	Share of HtM HH	0.2
$1-s$	Transition rate from U to HtM	$1-0.987$
$\frac{G}{Y}$	Steady-state value of G_t/Y_t	0.1645 (US)
		0.2439 (JPN)

of consumption inequality with this mixed frequency. It is given by

$$100\log\left(\frac{C_t^H}{C_t}\right)_{\text{annual}} = 100\log\left(\frac{C^H}{C}\right) + \frac{1}{1+z+z^2+z^3}\left[(\hat{c}_{t-3}^H-\hat{c}_{t-3})+z(\hat{c}_{t-2}^H-\hat{c}_{t-2})+z^2(\hat{c}_{t-1}^H-\hat{c}_{t-1})+z^3(\hat{c}_t^H-\hat{c}_t)\right],$$

where C^H/C is the steady-state consumption inequality.[3]

Table 1 shows the fixed parameter values. The model is specified to be quarterly. The discount factor β is 0.99. The cost share of capital α is 0.37. The depreciation rate of capital δ is 0.025. These values are standard in the literature. The share of HtM households θ is 0.2. The transition rate from U to HtM $1-s$ is $1-0.987$. These two values are from the paper by Bilbiie et al. (2022). Finally, the steady-state share of government consumption in GDP G/Y is set as the data mean: 0.1645 for the US and 0.2439 for Japan.

The Bayesian estimation is employed by using Dynare. The prior distributions of the parameters are presented in Tables 2. For the steady-state investment wedge τ^X, growth rate $100(z-1)$, labor supply L, consumption inequality C^H/C, the prior distribution is the gamma distribution. The means of the prior distribution except for τ^X are set at the sample mean. Since there is no prior information on τ^X, we set the mean of τ^X as 0.5 so that the shape of the posterior mode is close to the log-likelihood kernel in the mode check plots in Dynare. For the persistence parameters of structural shocks, the prior distribution is the beta distribution with a mean of 0.5 and a standard deviation of 0.2.

[3] Appendix C explains the derivation of this observation equation.

Table 2: Prior Distribution

Parameter		Distribution	Mean	SD
τ^X	Steady-state investment wedge	Gamma	0.5	0.2
$100(z-1)$	Steady-state output growth	Gamma	0.4017 (US)	0.2
		Gamma	0.3112 (JPN)	0.2
$\frac{C^H}{C}$	Steady-state consumption inequality	Gamma	0.5788 (US)	0.05
		Gamma	0.7080 (JPN)	0.05
L	Steady-state total labor supply	Gamma	0.1973 (US)	0.05
		Gamma	0.2129 (JPN)	0.05
ρ_z	Persistence of efficiency wedge	Beta	0.5	0.2
ρ_x	Persistence of investment wedge	Beta	0.5	0.2
ρ_L	Persistence of neutral labor wedge	Beta	0.5	0.2
ρ_{LU}	Persistence of U-specific labor wedge	Beta	0.5	0.2
ρ_{LH}	Persistence of HtM-specific labor wedge	Beta	0.5	0.2
ρ_g	Persistence of government wedge	Beta	0.5	0.2
σ_z	SD of efficiency wedge shock	IG	0.5	Inf
σ_x	SD of investment wedge shock	IG	0.5	Inf
σ_L	SD of neutral labor wedge shock	IG	0.5	Inf
σ_{LU}	SD of U-specific labor wedge shock	IG	0.5	Inf
σ_{LH}	SD of HtM-specific labor wedge shock	IG	0.5	Inf
σ_g	SD of government wedge shock	IG	0.5	Inf

Following standard Bayesian likelihood approaches, we use the Kalman filter to evaluate the likelihood function of the log-linearized equilibrium system and the Metropolis–Hastings algorithm to generate draws from the posterior distribution of the deep parameters.

4 Empirical Results

4.1 Posterior Estimates

The posterior estimates are presented in Table 3. Many posterior estimates are close between the US and Japan. Judging by the credible intervals, τ^X is higher in the US, implying that the distortion in investment activity is higher in the US. In addition, C^H/C is higher in Japan than in the US, implying that consumption inequality is less severe in Japan than in the US. The persistence parameter of the efficiency wedge is higher in

Table 3: Posterior Estimates

	US						Japan					
	Mean	90% credible interval					Mean	90% credible interval				
τ^X	1.1202	[	0.8730	,	1.3724	]	0.5673	[	0.3322	,	0.8041	]
$100(z-1)$	0.3566	[	0.2280	,	0.4808	]	0.4342	[	0.2243	,	0.6475	]
$\frac{C^H}{C}$	0.5706	[	0.5378	,	0.6072	]	0.7122	[	0.6790	,	0.7436	]
L	0.2000	[	0.1888	,	0.2094	]	0.2208	[	0.2041	,	0.2364	]
ρ_z	0.0788	[	0.0421	,	0.1158	]	0.1922	[	0.1454	,	0.2412	]
ρ_x	0.9493	[	0.9178	,	0.9810	]	0.9727	[	0.9551	,	0.9918	]
ρ_L	0.8359	[	0.6659	,	0.9890	]	0.9916	[	0.9847	,	0.9997	]
ρ_{LU}	0.9854	[	0.9741	,	0.9991	]	0.9905	[	0.9829	,	0.9990	]
ρ_{LH}	0.8706	[	0.8055	,	0.9369	]	0.4500	[	0.2646	,	0.6359	]
ρ_g	0.9885	[	0.9797	,	0.9979	]	0.9773	[	0.9636	,	0.9916	]
σ_z	0.9835	[	0.8841	,	1.0758	]	1.6196	[	1.4745	,	1.7714	]
σ_x	0.3863	[	0.3067	,	0.4696	]	0.5537	[	0.4586	,	0.6420	]
σ_L	0.1907	[	0.1046	,	0.2821	]	0.6791	[	0.5066	,	0.8369	]
σ_{LU}	0.7115	[	0.6186	,	0.8110	]	0.5893	[	0.3998	,	0.7649	]
σ_{LH}	3.4108	[	2.8603	,	3.9387	]	3.5238	[	2.9183	,	4.1320	]
σ_g	1.5785	[	1.4204	,	1.7267	]	2.1127	[	1.9144	,	2.3123	]

Table 4: Variance Decompositions

US

	z	τ^L	τ^{LH}	τ^{LU}	τ^x	g
$\Delta \log Y_t$	71.38	2.40	4.78	15.41	2.23	3.80
$\Delta \log C_t$	56.36	0.73	14.94	16.45	5.13	6.39
$\Delta \log X_t$	31.10	7.30	0.09	15.21	45.04	1.25
$\log L_t$	3.52	1.18	3.47	65.79	1.79	24.24
$\hat{c}_t^H - \hat{c}_t$	0.08	0.01	75.63	23.67	0.04	0.57

Japan

	z	τ^L	τ^{LH}	τ^{LU}	τ^x	g
$\Delta \log Y_t$	81.11	5.94	1.72	3.31	1.98	5.93
$\Delta \log C_t$	55.47	8.15	16.66	4.20	4.60	10.92
$\Delta \log X_t$	48.03	6.72	3.50	4.18	36.46	1.11
$\log L_t$	10.10	43.47	0.33	21.04	2.26	22.81
$\hat{c}_t^H - \hat{c}_t$	1.45	0.19	44.91	49.85	0.32	3.28

Note: Infinity-horizon forecast error variance decompositions are performed.

Japan, while the persistence of the HtM labor wedge is much lower in Japan. The standard deviations of efficiency, neutral labor, and government wedges are larger in Japan than in the US.

4.2 Driving sources of inequality and business cycles

Table 4 shows the variance decompositions of the growth rates of output ($100\Delta \log Y_t$), consumption ($100\Delta \log C_t$), and investment ($100\Delta \log X_t$), labor supply ($100 \log L_t$), and consumption inequality ($\hat{c}_t^H - \hat{c}_t$). We focus on consumption inequality generated in the model ($\hat{c}_t^H - \hat{c}_t$) rather than the observation data since the observation data is annual.

In the US, the efficiency wedge plays the dominant role in output growth, accounting for more than 70%. The U-specific labor wedge is the second, accounting for about 15%.

For consumption growth, the efficiency wedge is still the most significant factor, and 50% is accounted for. The U-specific and HtM-specific labor wedges also have substantial roles in consumption growth; each wedge accounts for about 15%. In the case of investment growth, the investment wedge is the most significant contributor, and the efficiency wedge is the second. The total labor supply is mainly accounted for by the U-specific labor wedge (about 65%) and the government wedge (about 20%). Except for total labor supply, the role of the efficiency wedge is crucial in these variables. On the other hand, consumption inequality is explained by household-specific labor wedges: about 75% by the HtM-specific labor wedge and about 20% by the U-specific labor wedge. In contrast, the efficiency wedge has almost no contribution to consumption inequality.

In Japan, the efficiency wedge is the main contributor to output, consumption, and investment growth, as in the US. Unlike the US, the role of the U-specific labor wedge is minor to them. The investment wedge significantly affects investment growth, as in the US. Unlike the US, the neutral labor wedge accounts for about 40% of the total labor supply. The U-specific labor wedge also significantly affects the total labor supply, accounting for about 20%. Consumption inequality is explained by household-specific labor wedges, as in the US. However, Japan's primary source of consumption inequality is the U-specific labor wedge, whereas it is the HtM-specific labor wedge in the US.

According to our results, household-specific labor wedges are essential for consumption inequality in the US and Japan. Bayer et al. (2024) and Bilbiie et al. (2022) find that markup shock is an important source of income inequality and wealth distribution dynamics. Since the markup shock distorts the factor prices, including the wage, our labor wedges might capture a similar effect. If so, our result implies that markup shock specific to each household is crucial, not neutral to two types of households.

We focus on the impulse response functions to interpret the mechanism behind the results of the variance decomposition. Figure 1 shows the estimated impulse responses to a one standard deviation shock to the wedges on output growth ($100\Delta \log Y_t$), consumptions of two households, and consumption inequality ($\hat{c}_t^H - \hat{c}_t$) in the US. For output growth, the response to the efficiency wedge shock is the largest among shocks. This result is consistent with our finding that the main driver for output growth is the efficiency wedge in the variance decomposition. The second largest response is to the U-specific labor wedge shock, which is also in line with the variance decomposition.

On the other hand, the consumption responses to the efficiency wedge shock are very close for the two types of households. Thus, only a slight consumption inequality is caused by the efficiency wedge shock. In the case of investment, neutral labor, and government wedges, the differences in consumption responses between the two types of households are also slight. The consumption responses differ significantly from the U-specific and HtM-specific labor wedge shocks; these significantly contribute to consumption inequality in the variance decomposition. Remarkably, the impact of the HtM-specific labor wedge is the most significant to consumption inequality. In the variance decomposition, the U-specific labor wedge accounts for about 15% of output growth and about 20% of consumption inequality. The impulse responses imply that the U-specific labor wedge shock decreases output growth and increases consumption inequality ($\hat{c}_t^H - \hat{c}_t$). So then, erasing the effects of the U-specific labor wedge would positively impact the business cycles while widening the consumption gap.

Figure 2 is the analogue of Figure 1 in Japan. As in the US, output growth responds most significantly to efficiency wedge shocks, and its magnitude is much larger than in the US case. Unlike the US, the magnitudes of the neutral labor wedge and the government wedge on output growth are almost the same as those of the U-specific labor wedge. On the other hand, in the US, the magnitude of the U-specific labor wedge on output growth is more significant than those of the neutral labor and the government wedges. This difference is consistent with the variance decomposition, which shows that the contributions of the neutral labor wedge and the government wedge to output growth are more significant than that of the U-specific labor wedge in Japan.

In Japan, the consumption response to the HtM-specific labor wedge is less persistent than in the US. Then, the gap in consumption between the two types of households disappears quickly in Japan. In the US, on the other hand, the HtM-specific labor wedge has a long-lasting impact on consumption inequality. This difference in the shapes of these impulse responses can be interpreted as creating the fact that the U-specific labor wedge explains more consumption inequality in Japan, whereas the HtM-specific labor wedge is the major contributor to consumption inequality in the US.

What is the central message of these results? In the US, the labor market distor-

Figure 1: Impulse Responses to a One Standard Deviation Shock to Wedges (1): US

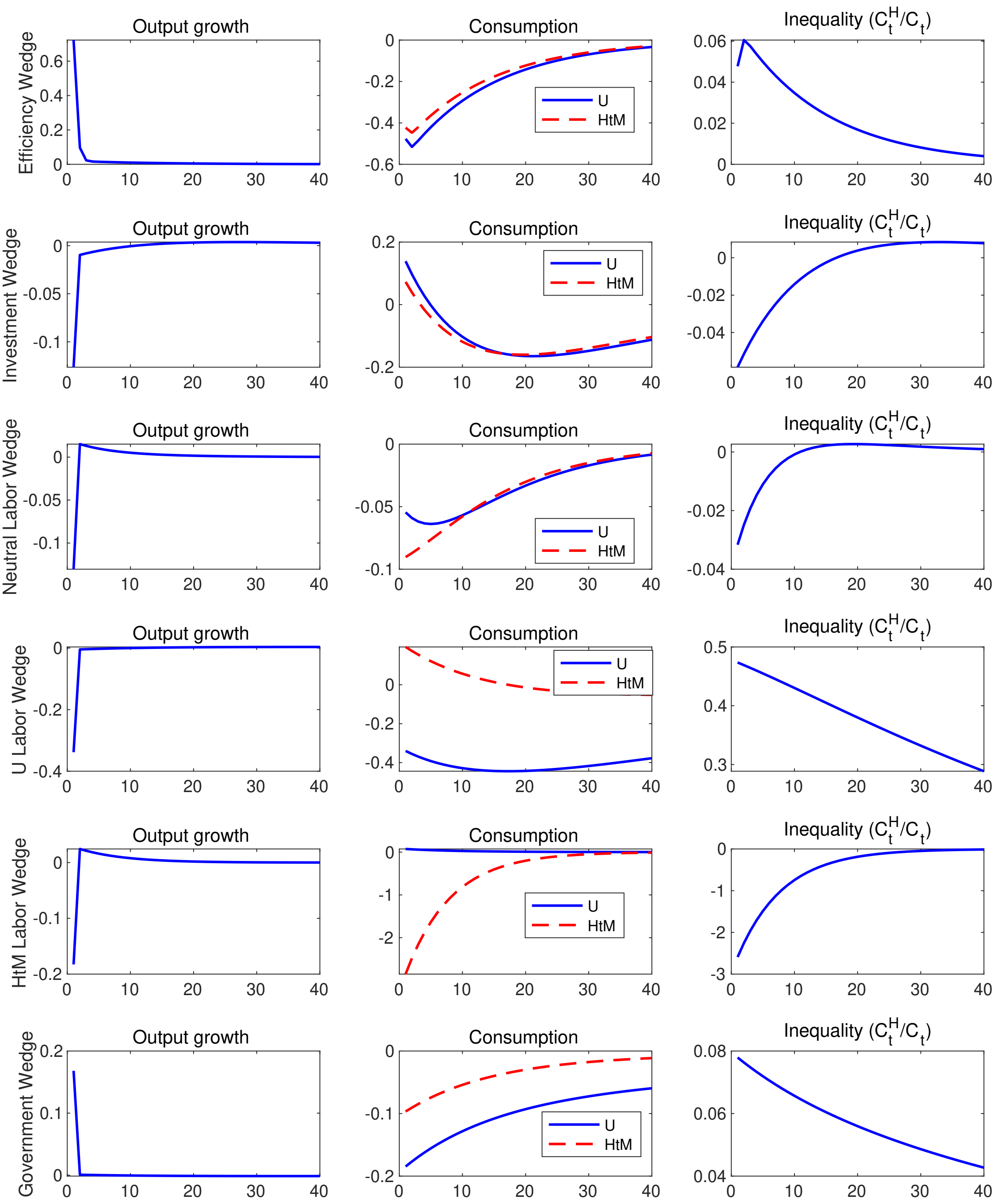
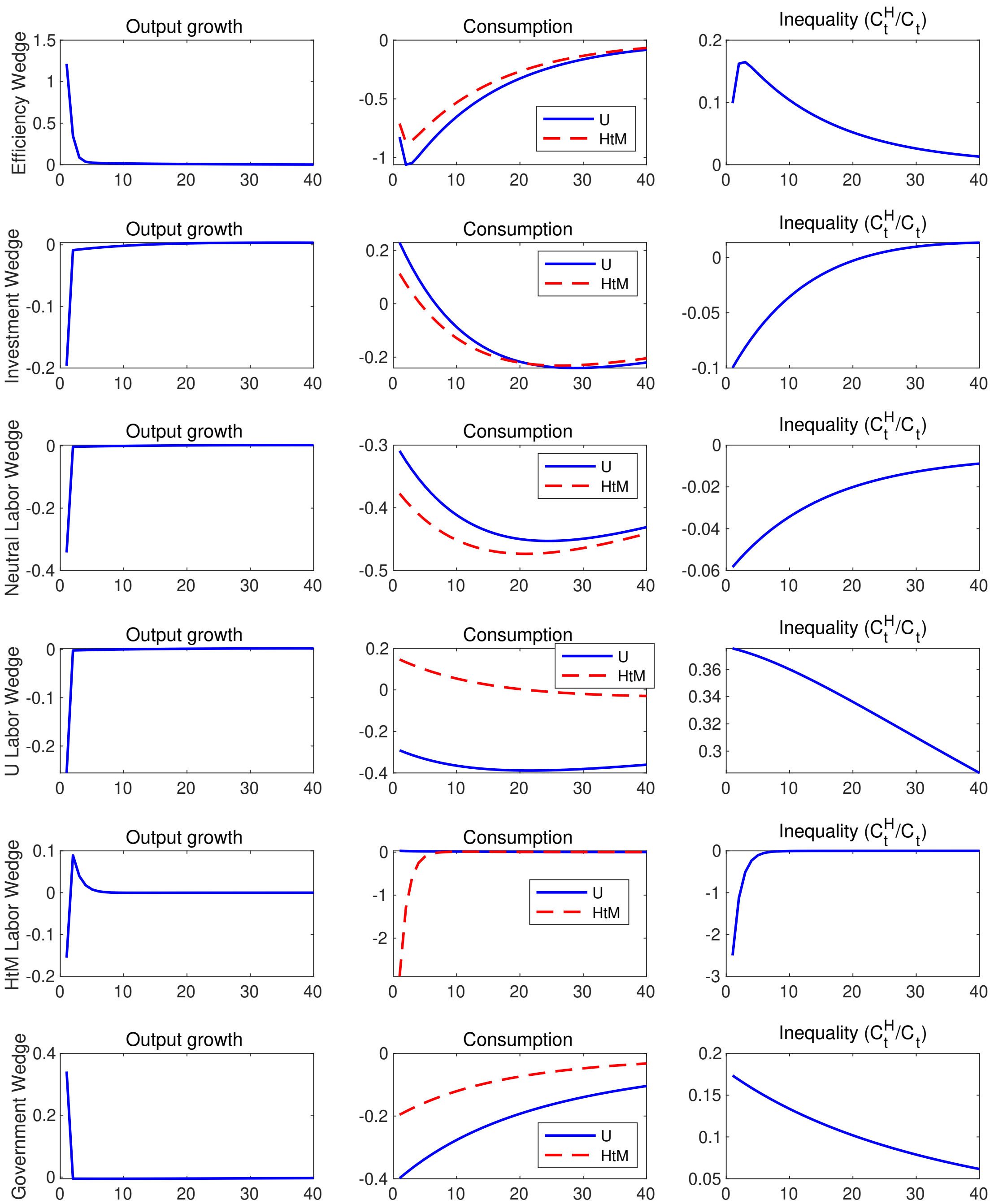


Figure 2: Impulse Responses to a One Standard Deviation Shock to Wedges (2): Japan



tions specific to the U households significantly impact business cycles and consumption inequality, while the primary source of business cycles is the distortion in aggregate productivity (i.e., the efficiency wedge), and that of consumption inequality is the labor market distortion specific to the HtM household, the poor. Then, the labor market distortions specific to the U households are essential to understand the relation between business cycles and consumption inequality in the US. On the other hand, we find that no common factors significantly impact both business cycles and consumption inequality in Japan. In business cycles, the distortion in aggregate productivity is dominant, and the labor market distortions are less critical. The key to understanding consumption inequality is the U households: non-poor households. In both countries, the effects of labor market distortions common to the two household types (i.e., the neutral labor wedge) are insignificant except for aggregate labor supply. In particular, the effects on output growth and consumption inequality are minor.

5 Does a Reduction in Inequality Increase or Reduce the Volatility of Business Cycles?

This section analyzes how reducing consumption inequality would affect the business cycle based on the estimates in Section 4. Here, we consider the following two types of counterfactual simulations. The first is the labor market reforms, that means eliminating the effects of the U-specific and HtM-specific labor wedges. These wedges are the main drivers of consumption inequality in the variance decomposition. Since each labor wedge implies distortions in the labor market, we can analyze what happens to the business cycle if the government removes those distortions and reduces consumption inequality. The second is by reducing consumption inequality through redistribution policies by the government. In this approach, we examine the effects of government policies while the market distortions represented by the wedges are left. Bilbiie et al. (2022) also employ a similar method to our second strategy.

Table 5 shows the standard deviations of output growth. It implies that the effects of reducing inequality on the business cycle depend on the country and how to do it. The labor market reform increases output growth volatility in the US while it reduces in Japan.

Table 5: Output Growth Volatilities by Reducing Consumption Inequality

	Data	Labor Market Reforms		Redistribution Policy	
		(1)	(2)	(3)	(4)
US	0.583	0.783	0.782	0.547	0.554
Japan	1.023	1.006	1.008	1.153	1.203

Note: Standard deviations of $100\Delta \log Y_t$ are reported.

(1) τ_t^{LU} and τ_t^{LH} are constant at their estimated steady-state levels.

(2) τ_t^{LU} is constant at its estimated steady-state levels, and τ_t^{LH} is constant such that $C^H/C = 1$ at the steady state.

(3) C_t^H/C_t is constant at the estimated steady-state level.

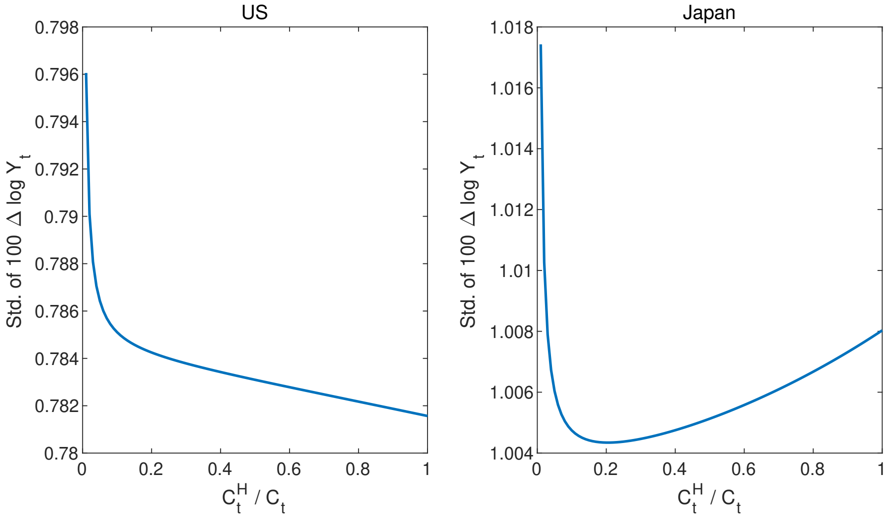
(4) C_t^H/C_t is constant at its maximum level. (US: 0.663, Japan: 0.847)

On the other hand, redistribution policy decreases output growth volatility in the US while it increases in Japan.

Labor market reforms: Cases (1) and (2) in Table 5 are the results of the labor market reforms. In this experiment, we perform a counterfactual experiment in which we fix the values of either the U-specific or the HtM-specific labor wedges or both at their steady-state values while the other wedges are the same as the estimated ones. Case (1) reduces cyclical inequality by setting the HtM-specific and U-specific labor wedges constant at their estimated steady-state levels. Case (2) reduces both cyclical and level inequality by setting the U-specific labor wedges constant at its estimated steady-state levels and the HtM-specific labor wedges constant such that $C^H/C = 1$ at the steady state.

As in Case (1), eliminating the cyclical effects of the U-specific and the HtM-specific labor wedges increases output growth volatility in the US. In contrast, it reduces the volatility of output growth in Japan. This difference comes from the correlations between wedges and output growth. In the US, the U-specific and the HtM labor wedges are positively correlated with output growth: $corr(\tau_t^{LH}, \Delta \log Y_t) = 0.25$ and $corr(\tau_t^{LU}, \Delta \log Y_t) = 0.12$. In contrast, in Japan, they are negatively correlated with output growth: $corr(\tau_t^{LH}, \Delta \log Y_t) = -0.12$ and $corr(\tau_t^{LU}, \Delta \log Y_t) = -0.26$. In other

Figure 3: Output growth volatility and steady-state inequality (1): Labor market reforms



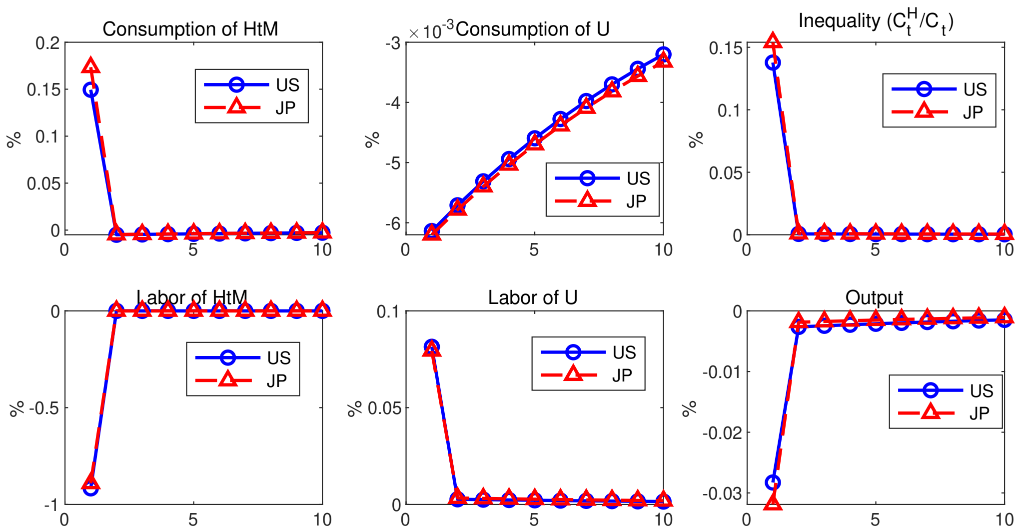
words, the labor market distortions specific to these households are severe in a boom in the US, while they are severe in a recession in Japan. These wedges have adverse effects on output growth. Thus, removing the effects of these wedges increases the volatility of the output growth in the US, whereas it reduces in Japan.[4]

The magnitudes of removing the effects of these wedges are quantitatively small in Japan. This would be because the contributions of τ_t^{LH} and τ_t^{LU} are small in the variance decomposition. On the other hand, in the US, the U-specific labor wedge is the second contributor to output growth and removing wedges significantly impacts output growth volatility.

Case (2) shows that eliminating level inequality (i.e., $C^H/C = 1$ at the steady state), similar to reducing cyclical inequality, increases output growth volatility in the US, but it reduces in Japan. However, the effects of reducing the level of inequality might not be monotonic. Figure 3 shows the relations between the output growth volatilities and the steady-state consumption inequality C^H/C via labor market reforms. In the US, the output growth volatility is monotonically decreasing in consumption inequality. In contrast, this relation is U-shaped in Japan. If consumption inequality is quite severe, reducing level inequality decreases the output growth volatility. On the other hand, if consumption

[4]It is noticed that our labor market reforms always improve welfare in both countries since the labor market distortions are improved.

Figure 4: Impulse responses to redistribution shock



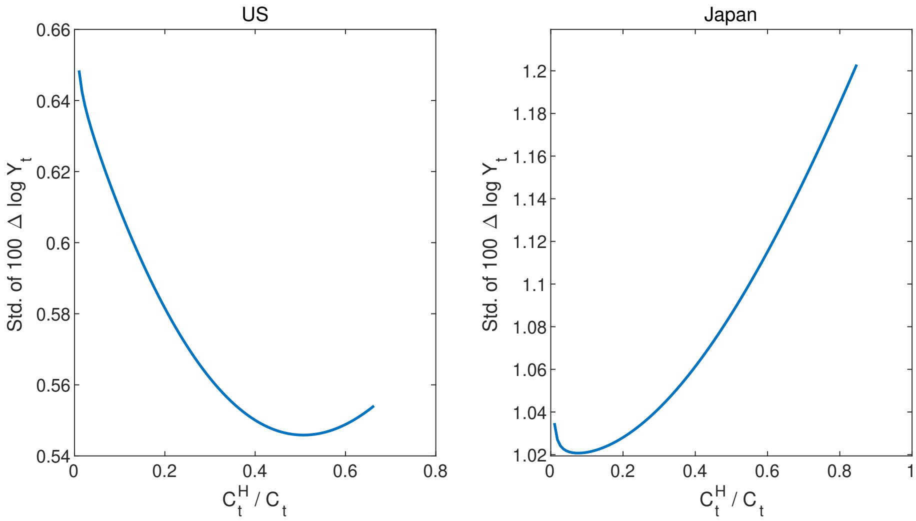
inequality is mild, reducing level inequality increases output growth volatility.

Redistribution policy: Cases (3) and (4) in Table 5 are the results of redistribution policy. In the second experiment, we change the lump-sum taxes to U and HtM, Equations (13) and (14), to achieve constant C_t^H/C_t. To implement counterfactual simulations, we replace the HtM budget constraint in the equilibrium system with the equation where C_t^H/C_t is constant. Case (3) is where C_t^H/C_t is constant at their estimated steady-state levels. Case (4) is where C_t^H/C_t is constant at their maximum steady-state levels. In the US, $C_t^H/C_t = 0.663$, and in Japan, $C_t^H/C_t = 0.847$.[5]

Cases (3) informs that in the US, eliminating cyclical consumption inequality reduces output growth volatility, whereas it increases output growth volatility in Japan. The US case is consistent with the result of Bilbiie et al. (2022), who also find that eliminating fluctuations of consumption inequality reduces output growth volatility. Our results show that it is different in Japan.

[5]Our numerical simulations imply that, in our model, $C_t^H/C_t = 1$ cannot be achieved by the redistribution policy. This result would be because the lump-sum transfer to HtM becomes enormous. The income effect from this transfer makes HtM reduce its labor supply to zero, and the assumption of interim solutions does not hold.

Figure 5: Output growth volatility and steady-state inequality (2): Redistribution policy



To understand this difference in two countries, the keys are the impulse of redistribution policy and the correlation between consumption inequality and the business cycle. Fig 4 shows the impulse response to redistribution for 1% of the steady-state HtM consumption from U to HtM. Redistribution from U to HtM increases consumption and leisure of HtM, and decreases HtM labor supply by the positive income effects. On the other hand, it decreases consumption and labor supply of U and increases U labor supply by the negative income effects. Since the effect on U households is dynamically smoothed, the decrease in labor supply of HtM is larger than the increase in labor supply of U, and aggregate labor supply decreases. Therefore, redistribution policy reduces output. According to our estimation, $corr(C_t^H/C_t, \Delta \log Y_t) = -0.20$ in the US, indicating that the consumption gap worsens during boom times. Therefore, the government redistributes from U households to HtM households during booms. Since redistribution policy negatively impacts the economy, as shown in Figure 4, it puts the brakes on the boom and stabilizes business cycles. On the other hand, $corr(C_t^H/C_t, \Delta \log Y_t) = -0.05$ in Japan, then the correlation is almost zero. As in Figure 4, redistribution policy causes economic fluctuations during both booms and recessions. Thus, the output growth volatility increases in Japan.

Case (4) shows the effects of reducing both level and cyclical inequality (increasing in C_t^H/C_t) via redistribution policy. It reduces output growth volatility in the US, but it increases in Japan. These effects are not monotonic. Figure 5 shows the relations between

Table 6: Welfare Level by Reducing Consumption Inequality

			Labor Market Reforms		Redistribution Policy	
		Baseline	(1)	(2)	(3)	(4)
US	SW	-191.51	-191.06	-185.71	-191.57	-192.27
	W^U	-190.76	-190.34	-185.45	-190.69	-193.12
	W^H	-194.51	-193.95	-186.78	-195.09	-188.86
Japan	SW	-188.78	-188.68	-186.24	-188.85	-190.54
	W^U	-188.74	-188.64	-186.50	-188.90	-192.67
	W^H	-188.97	-188.84	-185.20	-188.64	-182.01

Note: Social welfare SW is given by $(1-\theta)W^U + \theta W^H$, where W^U and W^H denote the welfare of U and HtM households, respectively.

Baseline: The welfare is calculated by original estimation results.

(1) τ_t^{LU} and τ_t^{LH} are constant at their estimated steady-state levels.

(2) τ_t^{LU} is constant at its estimated steady-state levels, and τ_t^{LH} is constant such that $C^H/C = 1$ at the steady state.

(3) C_t^H/C_t is constant at the estimated steady-state level.

(4) C_t^H/C_t is constant at its maximum level. (US: 0.663, Japan: 0.847)

the output growth volatilities and the steady-state consumption inequality C^H/C via labor market reforms. In both the US and Japan, the relation is U-shaped.

Welfare implications of reducing inequality: So far we have investigated the effects of reducing inequality via labor market reform and redistribution policy on output growth volatility. In the rest of this section, we explore the welfare implications of these policies. Table 6 shows the welfare level of baseline and counterfactual simulations. The social welfare level SW is defined as the weighted average of the welfare levels of two households: $SW = (1-\theta)W^U + \theta W^H$, where W^U and W^H denote the welfare levels of U and HtM households, respectively. The details of the calculation of welfare are in Appendix D.

Cases (1) and (2) show that labor market reforms improve the social welfare levels and welfare levels of U and HtM households from the baseline. It is unsurprising since labor market reforms mean eliminating the fluctuations of distortionary labor wedges in our model. The welfare level of Case (2) is higher than that of Case (1) since the level of HtM-specific labor wedge also improves in this case, in addition to eliminating fluctuations of labor wedges.

Cases (3) and (4) show that redistribution policy worsens the social welfare levels from the baseline in both the US and Japan. In addition, redistribution policy has different effects on U and HtM households. Case (3) shows that the welfare level of U households improves, whereas that of HtM worsens in the US. Contrary to this, the welfare level of HtM households improves, whereas that of U worsens in Japan. This difference between the two countries can be interpreted in the following way. Case (3) involves transfers among U and HtM households so that consumption inequality is constant at the estimated steady-state level. According to our estimation in Section 4, in the US, consumption inequality is more moderate than its steady-state level for about 66% of the total period. Under Case, (3) additional transfer from HtM to U occurred during that period. Then, the welfare level of U households improves, and that of HtM worsens. In Japan, consumption inequality is more severe than its steady-state level for about 66% of the total period. Additional transfers from U to HtM occurred during that period. Then, the welfare level of HtM households improves, and that of U worsens in Japan. Case (4) shows that an additional transfer from U to HtM occurs, in addition to Case (3), to improve the steady-state level of consumption inequality. This effect is more potent than that of Case (3), and then the welfare level of HtM households improves, and that of U worsens in both countries.

In summary, labor market reforms and redistributive policies have different effects on business cycle volatility in the US and Japan. On the other hand, from the perspective of welfare, labor market reform is welfare-improving in both countries, while redistribution policy is not. These results imply that labor market reforms work well in Japan because they help reduce inequality, stabilize business cycles, and improve welfare. On the other hand, there is no such policy in the US. Labor market reforms reduce inequality and improve welfare but destabilize business cycles. Redistribution policy reduces inequality and stabilizes business cycles but lowers welfare.

In this section, our results depend on the AR(1) specification of wedges. In Appendix E, we consider the case where the wedges follow the VAR(1) process. Even if the wedges are VAR(1) and the shocks are correlated, the main implication of this section doesn't change.

6 Concluding Remarks

In this paper, we have investigated the sources of inequality and business cycles in the US and Japan, using a tractable heterogeneous-agent business cycle model with U and HtM households. Following the BCA approach, we introduced six wedges, interpreted as distortions in the economy: efficiency, neutral labor, U-specific labor, HtM-specific labor, investment, and government wedges. While our model is simple, including these wedges enables the model to cover various detailed models. We have highlighted consumption inequality as inequality.

We have found that, in the US, the U-specific labor wedge significantly impacts both business cycles and consumption inequality, while the primary source of business cycles is the efficiency wedge, and that of consumption inequality is the HtM-specific labor wedge. On the other hand, we have found that no common factors significantly impact both business cycles and consumption inequality in Japan. In Japan, the key for business cycles is the distortion in aggregate productivity, and that for consumption inequality is the labor market distortions specific to U and HtM households. The most significant contributor to consumption inequality is the HtM (poor) in the US, whereas it is the U (non-poor) in Japan.

We have also investigated the relation between inequality and business cycles. Especially we have conducted two types of counterfactual simulations to analyze the effects of the reduction in consumption inequality on output growth volatility. The first is the labor market reforms by removing the effects of the U-specific and HtM-specific labor wedges, which are the primary driving sources of consumption inequality. We have found that it increases the output growth volatility in the US, whereas it reduces the output growth volatility in Japan. The second is the redistribution policy. We have found that it reduces the output growth volatility in the US, whereas it increases in Japan. Then, the effects of reducing inequality on the business cycle depend on the country and how to do it. We also explore the welfare implications of these policies. According to our results, labor market reform is welfare-improving in both countries, while redistribution policy is not.

One of our key findings is that the effects of reducing inequality depend on the country and how to reduce inequality. Bilbiie et al. (2022) find that reducing inequality stabilizes business cycles by estimating their model by the data of the US and considering redistribution policy to reduce inequality. On the other hand, we provide the possibility that the result can be overturned by using Japanese data in addition to US data and introducing labor market reforms as alternative ways to reduce inequality.

Our model is tractable and simple, and there are remaining future tasks. For example, labor wedges are black boxes in our model. To identify the detailed sources of changing labor wedge would be one of them. Nonetheless, we have found a complex relation between inequality and the business cycle, and our work has a certain contribution to this field.

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A Equilibrium System

The equilibrium system of the model is given by

$$
\begin{aligned}
&\Lambda_t^U = \frac{1}{C_t^U}, \\
&(1+\tau_t^X)\Lambda_t^U = \beta E_t \left\{ s\Lambda_{t+1}^U \left[(1+\tau_{t+1}^X)(1-\delta) + sR_{t+1} \right] \right. \\
&\qquad\qquad \left. + \Lambda_{t+1}^H (1-s)^2 \frac{1-\theta}{\theta} R_{t+1} \right\}, \\
&\frac{1}{1-L_t^U} = \Lambda_t^U (1-\tau_t^L - \tau_t^{LU}) W_t, \\
&\Lambda_t^H = \frac{1}{C_t^H}, \\
&\frac{1}{1-L_t^H} = \Lambda_t^H (1-\tau_t^L - \tau_t^{LH}) W_t, \\
&C_t^H = W_t L_t^H + (1-s)\frac{1-\theta}{\theta} R_t K_{t-1}^U, \\
&K_t^U = (1-\delta) K_{t-1}^U + X_t^U, \\
&Y_t = [K_t]^\alpha (Z_t L_t)^{1-\alpha}, \\
&R_t = \alpha \frac{Y_t}{K_t}, \\
&W_t = (1-\alpha)\frac{Y_t}{L_t}, \\
&Y_t = C_t + X_t + G_t, \\
&K_t = (1-\theta) K_{t-1}^U, \\
&X_t = (1-\theta) X_t^U, \\
&C_t = (1-\theta) C_t^U + \theta C_t^H, \\
&L_t = (1-\theta) L_t^U + \theta L_t^H.
\end{aligned}
$$

Let the detrended variables as

$$
c_t = \frac{C_t}{Z_t}, \qquad c_t^U = \frac{C_t^U}{Z_t}, \qquad c_t^H = \frac{C_t^H}{Z_t}, \qquad \lambda_t^H = \Lambda_t^H Z_t, \qquad \lambda_t^H = \Lambda_t^H Z_t,
$$
$$
x_t = \frac{X_t}{Z_t}, \qquad x_t^U = \frac{X_t^U}{Z_t}, \qquad k_t = \frac{K_t}{Z_t}, \qquad k_t^U = \frac{K_t^U}{Z_t}, \qquad y_t = \frac{Y_t}{Z_t}, \qquad w_t = \frac{W_t}{Z_t},
$$

and $z_t = Z_t / Z_{t-1}$.

Then, the detrended equilibrium system is given by

$$\begin{aligned}
\lambda_t^U &= \frac{1}{c_t^U},\\
(1+\tau_t^X)\lambda_t^U &= \beta E_t \{ s\frac{\lambda_{t+1}^U}{z_{t+1}} \left[(1+\tau_{t+1}^X)(1-\delta) + sR_{t+1}\right] \\
&\quad + \frac{\lambda_{t+1}^H}{z_{t+1}}(1-s)^2 \frac{1-\theta}{\theta} R_{t+1} \},\\
\xi \frac{1}{1-L_t^U} &= \lambda_t^U (1-\tau_t^L - \tau_t^{LU}) w_t,\\
\xi \frac{1}{1-L_t^H} &= \lambda_t^H (1-\tau_t^L - \tau_t^{LH}) w_t,\\
c_t^H &= w_t L_t^H + (1-s)\frac{1-\theta}{\theta} R_t \frac{k_{t-1}^U}{z_t},\\
\lambda_t^H &= \frac{1}{c_t^H},\\
k_t^U &= (1-\delta)\frac{k_{t-1}^U}{z_t} + x_t^U,\\
y_t &= (k_t)^\alpha L_t^{1-\alpha},\\
R_t &= \alpha \frac{y_t}{k_t},\\
w_t &= (1-\alpha)\frac{y_t}{L_t},\\
y_t &= c_t + x_t + g_t,\\
k_t &= (1-\theta)\frac{k_{t-1}^U}{z_t},\\
x_t &= (1-\theta) x_t^U,\\
c_t &= (1-\theta)c_t^U + \theta c_t^H,\\
L_t &= (1-\theta)L_t^U + \theta L_t^H.
\end{aligned}$$

At a balanced growth path, the equilibrium system becomes

$$\begin{aligned}
\lambda^U &= \frac{1}{c^U},\\
(1+\tau^X)\lambda^U &= \beta \{ s\frac{\lambda^U}{z} \left[(1+\tau^X)(1-\delta) + sR\right] + \frac{\lambda^H}{z}(1-s)^2 \frac{1-\theta}{\theta} R \},\\
\xi \frac{1}{1-L^U} &= \lambda^U (1-\tau^L - \tau^{LU}) w,\\
\xi \frac{1}{1-L^H} &= \lambda^H (1-\tau^L - \tau^{LH}) w,
\end{aligned}$$

$$
\begin{aligned}
&c^H = wL^H + (1-s)\frac{1-\theta}{\theta}R\frac{k^U}{z},\\
&\lambda^H = \frac{1}{c^H},\\
&\left[1-\frac{1-\delta}{z}\right]k^U = x^U,\\
&y = k^\alpha L^{1-\alpha},\\
&R = \alpha\frac{y}{k},\\
&w = (1-\alpha)\frac{y}{L},\\
&y = c + x + g,\\
&k = (1-\theta)\frac{k^U}{z},\\
&x = (1-\theta)x^U,\\
&c = (1-\theta)c^U + \theta c^H,\\
&L = (1-\theta)L^U + \theta L^H.
\end{aligned}
$$

If c^H/c and L are known, this balanced growth path can be computed as follows. First, if c^H/c is known, then

$$
\begin{aligned}
\frac{\lambda^H}{\lambda^U} &= \frac{c^U}{c^H}\\
&= \frac{\left(\frac{c^H}{c}\right)^{-1} - \theta}{1-\theta}.
\end{aligned}
$$

R can be calculated by the Euler equation as

$$
R = \frac{(1+\tau^X) - \frac{\beta s}{z}(1+\tau^X)(1-\delta)}{\frac{\beta s^2}{z} + \frac{\beta(1-s)^2}{z}\left(\frac{\lambda^H}{\lambda^U}\right)\left(\frac{1-\theta}{\theta}\right)}.
$$

Given the value of R, the following values are calculated as

$$
\begin{aligned}
\frac{k}{y} &= \frac{\alpha}{R},\\
\frac{x}{y} &= \left[1-\frac{1-\delta}{z}\right]\left(\frac{k}{y}\right)z,\\
\frac{c}{y} &= 1 - \frac{x}{y} - \frac{g}{y},\\
\frac{y}{L} &= \left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}},\\
w &= (1-\alpha)\left(\frac{y}{L}\right).
\end{aligned}
$$

If L is known, then

$$y = \left(\frac{y}{L}\right) L,$$
$$c = \left(\frac{c}{y}\right) y,$$
$$k = \left(\frac{k}{y}\right) y,$$
$$k^U = \frac{kz}{1-\theta}.$$

Finally, since c^H/c is given, then

$$c^H = \left(\frac{c^H}{c}\right) c,$$
$$L^H = \frac{c^H - (1-s)\frac{1-\theta}{\theta} R \frac{k^U}{z}}{w},$$
$$L^U = \frac{L - \theta L^H}{1-\theta},$$
$$c^U = \frac{c - \theta c^H}{1-\theta}.$$

The ratios of the labor disutility parameter ξ and labor wedge are given by

$$\frac{\xi}{(1-\tau^L-\tau^{LU})} = \lambda^U (1-L^U) w,$$
$$\frac{\xi}{(1-\tau^L-\tau^{LH})} = \lambda^H (1-L^H) w.$$

Let $\hat{v}_t$ denote the log-deviation from its steady-state: $\hat{v}_t = \log(v_t) - \log(v)$. In the case of investment and labor wedges, $\tilde{\tau}_t^J$ is defined as the difference from its steady-state: $\tilde{\tau}_t^J = \tau_t^J - \tau^J$ for $J = X$, L, LU, and LH.

Then, the log-linearized equilibrium system is given by

$$\hat{\lambda}_t^U = -\hat{c}_t^U,$$
$$\tilde{\tau}_t^X + \hat{\lambda}_t^U, = \frac{\beta s(1-\delta)}{z}\left(E_t\hat{\lambda}_{t+1}^U - E_t\hat{z}_{t+1} + E_t\tilde{\tau}_{t+1}^X\right)$$
$$+ \frac{\beta s^2 R}{z(1+\tau^X)}\left(E_t\hat{\lambda}_{t+1}^U + E_t\hat{R}_{t+1} - E_t\hat{z}_{t+1}\right)$$
$$+ \left(1 - \frac{\beta s(1-\delta)}{z} - \frac{\beta s^2 R}{z(1+\tau^X)}\right)\left(E_t\hat{\lambda}_{t+1}^H - E_t\hat{z}_{t+1} + E_t\hat{R}_{t+1}\right),$$
$$\frac{L^U}{1-L^U}\hat{L}_t^U = \widehat{\lambda}_t^U - \tilde{\tau}_t^L - \tilde{\tau}_t^{LU} + \hat{w}_t,$$
$$\frac{L^H}{1-L^H}\hat{L}_t^H = \widehat{\lambda}_t^H - \tilde{\tau}_t^L - \tilde{\tau}_t^{LH} + \hat{w}_t,$$

$$
\begin{aligned}
\hat{c}_t^H &= \frac{wL^H}{c^H}(\hat{w}_t + \hat{L}_t^H) + \left(1 - \frac{wL^H}{c^H}\right)\left[\hat{R}_t + \hat{k}_{t-1}^U - \hat{z}_t\right], \\
\hat{\lambda}_t^H &= -\hat{c}_t^H, \\
\hat{k}_t^U &= \frac{1-\delta}{z}(\hat{k}_{t-1}^U - \hat{z}_t) + \left[1 - \frac{1-\delta}{z}\right]\hat{x}_t^U, \\
\hat{y}_t &= \alpha(\hat{k}_{t-1}^U - \hat{z}_t) + (1-\alpha)\hat{L}_t, \\
\hat{R}_t &= \hat{y}_t - \hat{k}_{t-1}^U + \hat{z}_t, \\
\hat{w}_t &= \hat{y}_t - \hat{L}_t, \\
\hat{y}_t &= \frac{c}{y}\hat{c}_t + \frac{(1-\theta)x^U}{y}\hat{x}_t^U + \frac{g}{y}\hat{g}_t, \\
\hat{c}_t &= \left(1 - \frac{\theta c^H}{c}\right)\hat{c}_t^U + \frac{\theta c^H}{c}\hat{c}_t^H, \\
\hat{L}_t &= \left(1 - \frac{\theta L^H}{L}\right)\hat{L}_t^U + \frac{\theta L^H}{L}\hat{L}_t^H.
\end{aligned}
$$

The evolution of wedges is given by

$$
\begin{aligned}
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \varepsilon_t^z, \\
\tilde{\tau}_t^X &= \rho_X \tilde{\tau}_{t-1}^X + \varepsilon_t^X, \\
\tilde{\tau}_t^L &= \rho_L \tilde{\tau}_{t-1}^L + \varepsilon_t^L, \\
\tilde{\tau}_t^{LU} &= \rho_{LU} \tilde{\tau}_{t-1}^{LU} + \varepsilon_t^{LU}, \\
\tilde{\tau}_t^{LH} &= \rho_{LH} v_{t-1}^{LH} + \varepsilon_t^{LH}, \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \varepsilon_t^g.
\end{aligned}
$$

B Data Appendix

The data sources are summarized in Table A1.

B.1 US Data

GDP, consumption, investment, and government spending are all obtained in nominal values and deflated using the GDP deflator. These data are seasonally adjusted quarterly series. All per capita variables are obtained by dividing by labor force population. We use Personal Consumption Expenditures as consumption. Investment is a series of Gross

Table A1: Data Sources

Variables	Sources (FRED codes)
Quarterly series	US: NIPA tables
	Japan: ESRI, Quarterly Estimates of the GDP
Labor force population	US: CNP16OV
	Japan: MIC, Labour Force Survey
Average hours worked per worker	US: PRS85006023
	Japan: Ministry of Health, Labour and Welfare, Monthly Labor Survey
The employed persons	US: CE16OV
	Japan: MIC, Labour Force Survey
Consumption inequality	US: Consumer Expenditure Survey (See below)
	Japan: MIC, Family Income and Expenditure Survey

Variables for consumption inequality for the US	Sources (FRED codes)
Number of people in consumer units: all	CXU980010LB0101M
Number of people in consumer units: lowest 20%	CXU980010LB0102M
Total average annual expenditures: all	CXUTOTALEXPLB0101M
Total average annual expenditures: lowest 20%	CXUTOTALEXPLB0102M
Expenditures: vehicle insurance: all	CXU500110LB0101M
Expenditures: vehicle insurance: lowest 20%	CXU500110LB0102M
Expenditures: Health Insurance: all	CXUHLTHINSRLB0101M
Expenditures: Health Insurance: lowest 20%	CXUHLTHINSRLB0102M
Expenditures: personal insurance and pensions: all	CXUINSPENSNLB0101M
Expenditures: personal insurance and pensions: lowest 20%	CXUINSPENSNLB0102M

Notes: For the US, all data are taken from the FRED Database available through the Federal Reserve Bank of St. Louis. FRED codes are reported in the last column.

Private Domestic Investment. The government spending is defined as the sum of Government Consumption Expenditures and Gross Investment, and Net Exports of Goods and

Services. The reason for including net exports of goods and services as an item of government expenditure is to assume a closed economy model following Chari et al. (2007). The GDP deflator is from Implicit Price Deflators for Gross Domestic Product.

Labor supply L_t, following Hayashi and Prescott (2002) and Kobayashi and Inaba (2006), is constructed by

$$L_t = \frac{\text{Weekly average hours worked per employed person} \times \text{Employed person}}{24 \times 7 \times \text{Labor force}}.$$

For the data of the consumption inequality C_t^H/C_t, we calculate the ratio of consumption of the lowest 20% to that of all consumer units in the "Quintiles of income before taxes." We use average monthly consumption expenditures per household by quintile in annual income before taxes. We use adult-equivalent consumption expenditures as the observables for C_t^H and C_t. The adult-equivalent consumption expenditure of households is defined by dividing the average consumption expenditure by the square root of the number of household members according to the square root scale, which is also used by OECD. To make the definitions consistent with consumption in the model, we deduct "expenditures for vehicle insurance," "health insurance," and "personal insurance and pensions" from average annual expenditures.

B.2 Japanese Data

The data for Japan are constructed basically in the same way as for the US data. We use Private Consumption as consumption. Investment is defined as Gross Capital Formation by the private sector, which consists of Private Residential Investment, Private Non-residential Investment, and Change in Private Inventories. The government spending is defined as the sum of Government Consumption, Public Investment, Changes in Public Inventries, and Net Exports. The Economic and Social Research Institute (ESRI) releases a series on the 2008 SNA basis for years from 1994 to 2022. In addition, the ESRI provides a series on the 2008 SNA basis for years from 1980 to 1993 as provisional estimates. To connect the data for the entire period, we splice the nominal variables and GDP deflators using the overlapping years.

The aggregate labor L_t, following Hayashi and Prescott (2002) and Kobayashi and

Inaba (2006), is constructed by

$$L_t = \frac{\text{Monthly average hours worked per employed person} \times \text{Employed person}}{24 \times 7 \times 4 \times \text{Labor force}}.$$

For the consumption inequality C_t^H / C_t, we use average monthly consumption expenditures per household by quintile in annual income before taxes for working households with two or more members. We also use adult-equivalent consumption expenditures as well as US data. The Ministry of Internal Affairs and Communications (MIC) releases consistent series for periods 1863–1975, 1976–2007, and 2008–2021, respectively. To connect the data for the entire period, we splice data using the overlapping years.

C Derivation of the Observation Equation of Consumption Inequality

The available data on consumption inequality is annual. Suppose the period t is the 4th quarter of a year. The annual consumption of HtM ($HtMC$) is given by $C_{t-3}^H + C_{t-2}^H + C_{t-1}^H + C_t^H$. The aggregate consumption (AC) is given by $C_{t-3} + C_{t-2} + C_{t-1} + C_t$. The consumption inequality is $HtMC/AC$.

First, dividing the both numerator and denominator by Z_t, it is obtained

$$\begin{aligned}
\frac{HtMC}{AC} &= \frac{C_{t-3}^H + C_{t-2}^H + C_{t-1}^H + C_t^H}{C_{t-3} + C_{t-2} + C_{t-1} + C_t} \\
&= \frac{\frac{C_{t-3}^H}{Z_{t-3}} \times \frac{Z_{t-3}}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-2}^H}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-1}^H}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_t^H}{Z_t}}{\frac{C_{t-3}}{Z_{t-3}} \times \frac{Z_{t-3}}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-2}}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-1}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_t}{Z_t}} \\
&= \frac{\frac{c_{t-3}^H}{z_{t-2}z_{t-1}z_t} + \frac{c_{t-2}^H}{z_{t-1}z_t} + \frac{c_{t-1}^H}{z_t} + c_t^H}{\frac{c_{t-3}}{z_{t-2}z_{t-1}z_t} + \frac{c_{t-2}}{z_{t-1}z_t} + \frac{c_{t-1}}{z_t} + c_t} \\
&= \frac{c_{t-3}^H + c_{t-2}^H z_{t-2} + c_{t-1}^H z_{t-1} z_{t-2} + c_t^H z_t z_{t-1} z_{t-2}}{c_{t-3} + c_{t-2} z_{t-2} + c_{t-1} z_{t-1} z_{t-2} + c_t z_t z_{t-1} z_{t-2}}.
\end{aligned}$$

Log-linearizing this equation yields

$$\widehat{\left(\frac{HtMC}{AC}\right)} = \widehat{HtMC} - \widehat{AC}.$$

Since the log-linearization of the numerator and denominator imply

$$\widehat{HtMC} = \frac{1}{1+z+z^2+z^3}\left[\hat{c}_{t-3}^H + z(\hat{c}_{t-2}^H + \hat{z}_{t-2}) + z^2(\hat{c}_{t-1}^H + \hat{z}_{t-1} + \hat{z}_{t-2}) + z^3(\hat{c}_t^H + \hat{z}_t + \hat{z}_{t-1} + \hat{z}_{t-2})\right]$$

$$\widehat{AC} = \frac{1}{1+z+z^2+z^3}\left[\hat{c}_{t-3} + z(\hat{c}_{t-2} + \hat{z}_{t-2}) + z^2(\hat{c}_{t-1} + \hat{z}_{t-1} + \hat{z}_{t-2}) + z^3(\hat{c}_t + \hat{z}_t + \hat{z}_{t-1} + \hat{z}_{t-2})\right],$$

then, the annual consumption inequality is given by

$$\left(\frac{\widehat{HtMC}}{AC}\right) = \frac{1}{1+z+z^2+z^3}\left[(\hat{c}^H_{t-3} - \hat{c}_{t-3}) + z(\hat{c}^H_{t-2} - \hat{c}_{t-2}) + z^2(\hat{c}^H_{t-1} - \hat{c}_{t-1}) + z^3(\hat{c}^H_t - \hat{c}_t)\right].$$

D Calculation of Welfare

In this appendix, we describe the procedure to calculate welfare in Table 6. The instantaneous utility functions, U^U_t and U^H_t, are

$$U^U_t = \log(C^U_t) + \xi \log(1 - L^U_t),$$
$$U^H_t = \log(C^H_t) + \xi \log(1 - L^H_t).$$

The detrended instantanous utility function, u^U_t and $u^H_t$is defined by

$$u^U_t = U^U_t - \log(Z_t),$$
$$u^H_t = U^H_t - \log(Z_t).$$

Then, we obtain

$$u^U_t = \log(c^U_t) + \xi \log(1 - L^U_t) \tag{A.1}$$
$$u^H_t = \log(c^H_t) + \xi \log(1 - L^H_t), \tag{A.2}$$

where $c^U_t = C^U_t/Z_t$ and $c^H_t = C^H_t/Z_t$.

Letting ν^U_t and ν^H_t denote the detrended variables of V^U_t and V^H_t such that

$$\nu^U_t = V^U_t - \frac{1}{1+\beta}\log(Z_t),$$
$$\nu^H_t = V^H_t - \frac{1}{1+\beta}\log(Z_t),$$

the detrended versions of (1) and its analogue for V^H_t are

$$\nu^U_t = u^U_t + \frac{\beta}{1+\beta}E_t \log(z_{t+1}) + E_t\left[s\nu^U_{t+1} + (1-s)\nu^H_{t+1}\right], \tag{A.3}$$
$$\nu^H_t = u^H_t + \frac{\beta}{1+\beta}E_t \log(z_{t+1}) + E_t\left[\frac{1-\theta}{\theta}(1-s)\nu^U_{t+1} + \left(1 - \frac{1-\theta}{\theta}(1-s)\right)\nu^H_{t+1}\right]. \tag{A.4}$$

Let $\boldsymbol{P}$ denote the transition probability matrix among U and HtM such that

$$\boldsymbol{P} = \begin{bmatrix} s & 1-s \\ \frac{1-\theta}{\theta}(1-s) & 1-\frac{1-\theta}{\theta}(1-s) \end{bmatrix}.$$

Now, each component of $\boldsymbol{P}^j$ can be interepreted as follows.

- $\boldsymbol{P}^j(1,1)$: Probability where an individual, who is U at the current period, will be U j periods later
- $\boldsymbol{P}^j(1,2)$: Probability where an individual, who is U at the current period, will be HtM j periods later
- $\boldsymbol{P}^j(2,1)$: Probability where an individual, who is HtM at the current period, will be U j periods later
- $\boldsymbol{P}^j(2,2)$: Probability where an individual, who is HtM at the current period, will be HtM j periods later

Let $n-1$ denote the data length. We compute the welfare levels of U and HtM households, $W_t^U$and W_t^H, as follows.

$$\begin{aligned} W_t^U = u_t^U + \sum_{j=1}^{n-1} \beta^j \left[\frac{1}{1+\beta}\log(z_{t+j}) + \boldsymbol{P}^j(1,1)u_{t+j}^U + \boldsymbol{P}^j(1,2)u_{t+j}^H\right] \\ + \beta^n \left[\frac{1}{1+\beta}\log(z_{t+n}) + \boldsymbol{P}^n(1,1)\nu_{t+n}^U + \boldsymbol{P}^n(1,2)\nu_{t+n}^H\right], \\ W_t^H = u_t^H + \sum_{j=1}^{n-1} \beta^j \left[\frac{1}{1+\beta}\log(z_{t+j}) + \boldsymbol{P}^j(2,1)u_{t+j}^U + \boldsymbol{P}^j(2,2)u_{t+j}^H\right] \\ + \beta^n \left[\frac{1}{1+\beta}\log(z_{t+n}) + \boldsymbol{P}^n(2,1)\nu_{t+n}^U + \boldsymbol{P}^n(2,2)\nu_{t+n}^H\right], \end{aligned}$$

based on (A.3), (A.4), and the realized equilibrium path of consumptions and labor supplies. We calculate instantaneous utilities u_{t+j}^U and u_{t+j}^H by (A.1) and (A.2) without any approximations. On the other hand, we use the values of ν_{t+n}^U and ν_{t+n}^H as ones obtained by the log-linear approximation for simplicity of analyses.

Finally, the social welfare level SW_t is calculated by

$$SW_t = (1-\theta)W_t^U + \theta W_t^H.$$

E The Case Where Wedges Follow the VAR(1) Process

In this appendix, we consider the case where the wedges follow the VAR(1) process. Let $\boldsymbol{s}_t$ and $\boldsymbol{\varepsilon}_t$ denote the vector of wedges and shocks such that $\boldsymbol{s}_t = [z_t, \tau_t^x, \tau_t^L, \tau_t^{LH}, \tau_t^{LU}, g_t]'$ and $\boldsymbol{\varepsilon}_t = [\varepsilon_t^z, \varepsilon_t^x, \varepsilon_t^L, \varepsilon_t^{LH}, \varepsilon_t^{LU}, \varepsilon_t^g]'$. Then, the wedge $\boldsymbol{s}_t$ evolves according the following VAR(1) process:

$$\boldsymbol{s}_t = \boldsymbol{V}\boldsymbol{s}_{t-1} + \boldsymbol{\varepsilon}_t.$$

We specify the VAR(1) coefficient matrix $\boldsymbol{V}$ as

$$\boldsymbol{V} = \begin{bmatrix} \rho_z & \rho_{zx} & \rho_{zL} & \rho_{zLH} & \rho_{zLU} & 0 \\ \rho_{xz} & \rho_x & \rho_{xL} & \rho_{xLH} & \rho_{xLU} & 0 \\ \rho_{Lz} & \rho_{Lx} & \rho_L & 0 & 0 & 0 \\ \rho_{LHz} & \rho_{LHx} & 0 & \rho_{LH} & 0 & 0 \\ \rho_{LUz} & \rho_{LUx} & 0 & 0 & \rho_{LU} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_g \end{bmatrix}.$$

Following Chari et al. (2007), we assume that the government wedge is independent of other wedges. We also assume that each of the three labor wedges is independent of the other two. In this specification, we allow the correlations among shocks. We assume that the government wedge shock is independent of other wedge shocks as in Chari et al. (2007). In addition, we also assume that each of the three labor wedge shocks is independent of the other two.

In the estimation, we set the prior distribution of the off-diagonal elements of $\boldsymbol{V}$, ρ_{ij}, is Normal distribution with mean zero, and the standard deviation is 0.3. The prior distribution of the correlations between wedge shocks i and j, σ_{ij}, is the generalized Beta distribution with support [-1,1], mean zero, and the standard deviation is 0.3. The prior distributions of other parameters are the same as in Table 2.

Estimation results: The posterior estimates are in Tables A2 and A3. σ_{ij} is the correlation between wedge i and j. Judging by the credible intervals, the efficiency wedge shock positively correlates with U-specific and HtM-specific labor wedge shock in the US. In Japan, the efficiency wedge positively correlates with the neutral labor wedge.

Table A2: Posterior Estimates: Model with VAR(1) Wedge

	US		Japan	
	Mean	90% credible interval	Mean	90% credible interval
τ^X	1.0389	[0.7818 , 1.2861]	0.5543	[0.2809 , 0.8225]
$100(z-1)$	0.2800	[0.0994 , 0.4489]	0.2407	[0.0411 , 0.4105]
$\frac{C^H}{C}$	0.5804	[0.5661 , 0.5952]	0.7200	[0.7034 , 0.7400]
L	0.1961	[0.1916 , 0.2003]	0.2093	[0.2016 , 0.2169]
ρ_z	0.1427	[0.0395 , 0.2445]	0.0665	[0.0106 , 0.1140]
ρ_x	0.9087	[0.8406 , 0.9826]	0.6248	[0.4030 , 0.8248]
ρ_L	0.8107	[0.7123 , 0.9193]	0.9804	[0.9661 , 0.9955]
ρ_{LU}	0.8884	[0.8126 , 0.9737]	0.8041	[0.6143 , 0.9783]
ρ_{LH}	0.8507	[0.7850 , 0.9235]	0.7485	[0.6161 , 0.8833]
ρ_g	0.9764	[0.9622 , 0.9919]	0.9733	[0.9602 , 0.9880]
ρ_{zx}	-0.5914	[-0.8777 , -0.3441]	0.0822	[-0.3071 , 0.4243]
ρ_{zL}	0.0490	[-0.1201 , 0.2088]	-0.0681	[-0.0990 , -0.0381]
ρ_{zLU}	-0.2559	[-0.3847 , -0.1196]	-0.0747	[-0.3050 , 0.1734]
ρ_{zLH}	-0.0156	[-0.0337 , 0.0028]	0.0445	[0.0073 , 0.0841]
ρ_{xz}	0.0977	[0.0288 , 0.1640]	0.1005	[0.0387 , 0.1627]
ρ_{xL}	-0.0061	[-0.0402 , 0.0260]	0.0223	[-0.0079 , 0.0588]
ρ_{xLU}	0.0038	[-0.0524 , 0.0620]	0.2605	[0.1061 , 0.4073]
ρ_{xLH}	-0.0061	[-0.0148 , 0.0026]	0.0184	[-0.0024 , 0.0401]
ρ_{Lz}	0.0627	[-0.0878 , 0.2137]	-0.0759	[-0.2455 , 0.0780]
ρ_{Lx}	-0.7016	[-0.9516 , -0.4581]	-0.0765	[-0.1684 , 0.0072]
ρ_{LUz}	-0.1900	[-0.3513 , -0.0193]	-0.1493	[-0.3218 , -0.0080]
ρ_{LUx}	-0.0833	[-0.2947 , 0.1281]	0.2442	[-0.0100 , 0.4720]
ρ_{LHz}	0.1475	[-0.2473 , 0.5421]	0.3146	[0.0051 , 0.6729]
ρ_{LHx}	-0.3004	[-0.6817 , 0.0638]	-0.1731	[-0.4369 , 0.0965]

Table A3: Posterior Estimates: Model with VAR(1) Wedge

	US			Japan		
	Mean	90% credible interval		Mean	90% credible interval	
σ_z	1.0271	[0.9190	, 1.1436]	1.5209	[1.3648	, 1.6604]
σ_x	0.3118	[0.1942	, 0.4316]	0.2334	[0.1447	, 0.3162]
σ_L	0.1650	[0.1066	, 0.2205]	0.8228	[0.6217	, 1.0033]
σ_{LU}	0.5163	[0.4131	, 0.6192]	0.5781	[0.3383	, 0.8873]
σ_{LH}	3.1857	[2.7068	, 3.6515]	2.4231	[1.7768	, 3.0337]
σ_g	1.5924	[1.4374	, 1.7500]	2.1094	[1.9104	, 2.2958]
σ_{zx}	0.2200	[-0.0212	, 0.4669]	0.1131	[-0.3257	, 0.5522]
σ_{zL}	0.1431	[-0.3372	, 0.6034]	0.2642	[0.0285	, 0.5052]
σ_{zLU}	0.3694	[0.1783	, 0.5766]	0.0701	[-0.2902	, 0.4388]
σ_{zLH}	0.4943	[0.3628	, 0.6303]	-0.1710	[-0.4059	, 0.0554]
σ_{xL}	-0.2775	[-0.7804	, 0.2092]	0.0074	[-0.4341	, 0.4468]
σ_{xLU}	-0.2530	[-0.5362	, -0.0004]	0.0567	[-0.3981	, 0.5092]
σ_{xLH}	0.0616	[-0.1755	, 0.2922]	0.1039	[-0.3402	, 0.5520]

In this specification of the wedges, the variance decomposition is not appropriate for investigating the driving sources of business cycles and inequality since the shocks are correlated. In addition, since a wedge shock affects other wedges through the VAR(1) process, it is unclear the relation between the importance of a wedge shock and the importance of the wedge itself. Then, we conduct a counterfactual simulation by setting U-specific and HtM-specific labor wedges at their estimated steady-state levels. Table A4 reports the volatilities of consumption inequality $100(\hat{c}_t^H - \hat{c}_t)$ of the baseline and the counterfactual simulations. This table shows that two household-specific labor wedges are the key drivers of consumption inequality as in Section 4. Therefore, our labor market reform is still useful to reduce cyclical consumption inequality.

Table A4: Consumption Inequality Volatilities with Constant Labor Wedges

	Baseline	Counterfactual
US	4.275	0.638
Japan	3.063	1.212

Note: Standard deviations of $100(\hat{c}_t^H - \hat{c}_t)$ are reported. In the baseline, $100(\hat{c}_t^H - \hat{c}_t)$ are estimated one in Section 4. In the counterfactual simulations, τ_t^{LU} and τ_t^{LH} are constant at their estimated steady-state levels.

Table A5: Output Growth Volatilities by Reducing Consumption Inequality: with VAR(1) Wedges

	Data	Labor Market Reforms		Redistribution Policy	
		(1)	(2)	(3)	(4)
US	0.583	0.673	0.670	0.530	0.534
Japan	1.023	0.939	0.938	1.183	1.222

Note: Standard deviations of $100\Delta \log Y_t$ are reported.

(1) τ_t^{LU} and τ_t^{LH} are constant at their estimated steady-state levels.

(2) τ_t^{LU} is constant at its estimated steady-state levels, and τ_t^{LH} is constant such that $C^H/C = 1$ at the steady state.

(3) C_t^H/C_t is constant at the estimated steady-state level.

(4) C_t^H/C_t is constant at its maximum level. (US: 0.674, Japan: 0.850)

Effects of reducing inequality on business cylces: Table A5 is the analogue of Table 5. As in Table 5, it implies that the effects of reducing inequality on the business cycle depend on the country and how to do it. The labor market reform increases output growth volatility in the US while it reduces it in Japan. On the other hand, redistribution policy decreases output growth volatility in the US while it increases it in Japan.

Welfare implications: Table A6 is the analogue of Table 6. The results are almost the same as in Table 6; Labor market reform is welfare improvement, while redistribution

Table A6: Welfare Level by Reducing Consumption Inequality

			Labor Market Reforms		Redistribution Policy	
		Baseline	(1)	(2)	(3)	(4)
US	SW	-190.24	-190.16	-184.88	-190.35	-191.25
	W^U	-189.50	-189.45	-184.67	-189.66	-192.28
	W^H	-193.19	-193.02	-185.75	-193.11	-187.12
Japan	SW	-186.48	-186.61	-184.28	-186.62	-188.39
	W^U	-186.44	-186.60	-184.55	-186.77	-190.55
	W^H	-186.67	-186.66	-183.18	-186.01	-179.78

Note: Social welfare SW is given by $(1-\theta)W^U + \theta W^H$, where W^U and W^H denote the welfare of U and HtM households, respectively.

Baseline: The welfare is calculated by original estimation results.

(1) τ_t^{LU} and τ_t^{LH} are constant at their estimated steady-state levels.

(2) τ_t^{LU} is constant at its estimated steady-state levels, and τ_t^{LH} is constant such that $C^H/C = 1$ at the steady state.

(3) C_t^H/C_t is constant at the estimated steady-state level.

(4) C_t^H/C_t is constant at its maximum level. (US: 0.674, Japan: 0.850)

policy is welfare-reducing for both countries.

One difference is Case (1) in Japan: the social welfare and the welfare level of U households are reduced. The reason is as follows. According to our estimates, the U-specific labor wedges in Japan during the sample period are lower than the steady-state level by about -1.7% points on average. The value of -1.7 is a significantly more negative value than the other estimation results (-0.4% points for the US in the VAR(1) wedges, and $+1.6\%$ points for the US and -0.2% points for Japan in the AR(1) wedges). Case (1) increases labor wedges by this number. Increases in labor wedges hurt welfare, and their effects are more significant than the positive effects on welfare by eliminating fluctuations of distortionary labor wedges. Social welfare is much affected by the welfare level of U (they are 80% in total households), then the social welfare in Japan worsens in Case (1). On the other hand, in Case (2), in addition to Case (1), the level of HtM-specific labor wedge is improved. The positive effect of improvement in HtM-specific labor wedge dominates the negative effect of Case (1), and the welfare levels improve.

Another different point is Case (3) in the US: the redistribution policy improves U's welfare and worsens HtM's welfare as in Japan. The reason is as follows. In the baseline estimation, consumption inequality is more moderate than its steady-state level for about 66% of the total period in the US. On the other hand, under the VAR (1) specification, US consumption inequality is more moderate for about 50% of the total period. In addition, the average of C_t^H/C_t is less than its steady-state level. Then, additional transfer from U to HtM occurs under the VAR(1) specification, and it improves HtM's welfare and worsens U's welfare.