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Bottleneck Congestion and Endogenously Determined Departure Times under Bounded Rationality

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Bottleneck Congestion and Endogenously Determined Departure Times under Bounded Rationality

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Abstract

This study examines departure time decisions in Vickrey's bottleneck model by incorporating bounded rationality through a quantal response equilibrium (QRE) framework. Traditional bottleneck models assume perfect rationality and consequently may fail to capture real-world commuter behavior. To address this limitation, a discrete version of the bottleneck model was developed and experimentally tested to determine the predictive performance of QRE and symmetric mixed-strategy equilibrium (SMSE). In a controlled laboratory setting with two experimental conditions, the results revealed that while departure time choices initially deviated from SMSE predictions, they progressively became closer over successive rounds. Nevertheless, QRE consistently demonstrated a superior fit compared to SMSE under both conditions, particularly in explaining persistent delays in departure choices. Based on two out-of-sample validation measures, QRE outperformed SMSE in modeling commuters' departure time decision patterns. These results highlight the importance of integrating bounded rationality into bottleneck congestion models to improve the predictive power of commuter behavior modeling.

Keywords: Bottleneck congestion; Departure time choice; Bounded rationality; Quantal response equilibrium; Laboratory experiment

JEL Classifications: C72, C92

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1 Introduction

The bottleneck model proposed by Vickrey (1969) serves as a foundational framework for extensive research on rush hour traffic congestion dynamics. Since its inception, the model has been adapted by incorporating more realistic assumptions, thus enhancing its applicability in empirical research, urban planning, and transportation policy.[1] A significant innovation of Vickrey's model is its ability to endogenously determine patterns of commuters' departure time choices (Arnott et al., 1998; Arnott, 1998). In its simplest form, as articulated by Arnott et al. (1990, 1993), the bottleneck model describes commuters traveling from home to work on a single road with a single bottleneck. When the flow of commuters exceeds the bottleneck's limited service capacity, a queue forms, resulting in physical impossibility for all commuters to arrive at a common work-starting time. Consequently, some commuters experience schedule delays when arriving either early or late. In this framework, commuters independently and simultaneously select their departure times to minimize travel costs, which comprise both travel time and schedule delay costs. In equilibrium, all commuters incur the same travel cost and cannot get better off by unilaterally altering their departure times. Given that commuters differ in their schedule delay costs, this equilibrium condition necessitates that the level of bottleneck congestion, and consequently, the pattern of commuters' departure times, evolves throughout the peak period.

The equilibrium patterns of departure time choices and their theoretical implications hold true only when all the model assumptions are satisfied. The assumption that often draws criticism is perfect rationality. Commuters may exhibit bounded rationality, which limits their ability to make optimal decisions owing to cognitive constraints, insufficient information, and various external factors. However, many existing models do not account for the possibility of errors in commuters' departure time decisions, implying that individuals can always flawlessly optimize their choices. The first attempt to address this limitation in the commuter departure time choice problem was made by Mahmassani & Chang (1987), who introduced the concept of *bounded rational user equilibrium* (BRUE). This framework incorporates the satisficing approach proposed by Simon (1955), which allows commuters to select options that meet their minimum acceptability criteria rather than exhaustively pursuing an optimal choice.

This study incorporates bounded rationality into a discrete version of the simplest bottleneck model formulated by Arnott et al. (1990, 1993), using the notion of *quantal response equilibrium* (QRE), first proposed by McKelvey & Palfrey (1995). The discrete

[1] Li et al. (2020) is an excellent reference for a review of the bottleneck model literature over the last 50 years.

model is structured as a non-cooperative game involving a finite number of commuters, discrete departure times, and complete information. While Nash equilibrium, the canonical solution concept of non-cooperative games, requires that players form correct beliefs about the behavior of other players and best respond to these beliefs, these assumptions may not hold in practice. QRE relaxes the best-response requirement of Nash equilibrium while preserving the assumption of correct beliefs in a stochastic sense. QRE has successfully explained systematic deviations observed in a wide range of experimental games, including centipede games (McKelvey & Palfrey, 1998), traveler's dilemma games (Capra et al., 1999), private value auctions (Goeree et al., 2002), imperfect price competitions (Capra et al., 2002), all-pay auctions (Gneezy & Smorodinsky, 2006), voter participation games (Levine & Palfrey, 2007), and volunteer's dilemma games (Goeree et al., 2017; Kawagoe et al., 2018, 2023). In transportation research, QRE has been used to study route choice behavior (Zhao & Huang, 2014; Dechenaux et al., 2014), but not departure time choice behavior.[2]

This study examines the extent to which aggregate departure time choice behavior can be explained by QRE. To accomplish this goal, this study adopts an experimental economics approach, which provides a more reliable source of data due to its greater control over experimental conditions.[3] The bottleneck model is highly stylized, and all of its assumptions may not be satisfied by naturally occurring data, undermining the ability to either support or refute its theoretical predictions. In contrast, an experimental approach allows researchers to determine the method of data generation. Researchers maintain control over the strategies available to participants, the information they receive, and how they evaluate outcomes while holding constant other variables that may influence behavior outside the model.

There is a small but growing body of experimental research on bottleneck models (Schneider & Weimann, 2004; Gabuthy et al., 2006; Ziegelmeyer et al., 2008; Daniel et al., 2009; Sun et al., 2017; Yang et al., 2022; Otsubo et al., 2023; Liu et al., 2023).[4] Although these studies vary in their model setups and experimental designs, it is common to all of them that commuters' departure time choice behavior is characterized through equilibrium solutions, including user equilibrium (Sun et al., 2017; Liu et al., 2023), pure-strategy equilibrium (Schneider & Weimann, 2004; Gabuthy et al., 2006; Yang et al., 2022), and mixed-strategy equilibrium (Ziegelmeyer et al., 2008; Daniel et al., 2009; Yang et al., 2022; Otsubo et al., 2023). The findings of these studies have been mixed. Those employing user or pure-strategy equilibrium predictions find no supporting evidence for these equi-

[2]Di & Liu (2016) review the models of bounded rational route choice behavior.

[3]Dixit et al. (2017) reviews the past applications of experimental economics methods to transportation research.

[4]Only the experimental studies following the induced value theory (Smith, 1976) are listed here.

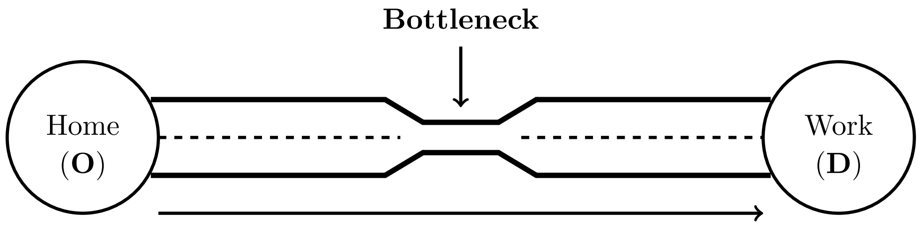

Figure 1: Road with a Single Bottleneck

libria. Conversely, studies using mixed-strategy equilibrium predictions confirm that the observed aggregate patterns of departure time choices closely correspond with equilibrium predictions, although some note slight deviations toward later departures.

In this experiment, participants were incentivized through monetary rewards to make a series of departure time choices that directly influenced their final earnings. The observed departure time patterns of the participants were compared with symmetric mixed-strategy equilibrium (SMSE) predictions. The results indicate that the aggregate behavior initially deviated from the SMSE predictions, demonstrating a trend toward later departures in the early rounds of the experiment, with the magnitude of this deviation diminishing in the subsequent rounds. The QRE model was estimated from the data using maximum likelihood estimation technique and compared with the SMSE based on two measures of goodness-of-fit: mean square deviation and Euclidean distance. The results demonstrate that QRE outperforms SMSE in explaining the observed behavior.

The remainder of this paper is organized as follows. Section 2 provides a detailed description of the discrete version of the bottleneck model and its associated equilibrium solution. Section 3 describes the experimental design, and Section 4 presents the results of the study. Finally, Section 5 provides the concluding remarks.

2 Theory

2.1 A Discrete Version of the Bottleneck Model

A group of n identical commuters travels by driving their cars along a single road connecting home (O) to work (D) (see Figure 1). Each commuter simultaneously chooses a departure time $t \in T = \{0, 1, 2, \ldots, t_{\mathrm{L}}\}$.

Two sources of disutility are associated with home-to-work trips. The first is the travel cost. When driving to work, no congestion occurs anywhere except at a single segment of the road called a *bottleneck* (e.g., toll gate, tunnel, or bridge). The bottleneck serves a single car at a time on a *first-come, first-served* (FCFS) basis. Following Arnott et al. (1990, 1993), zero travel time is assumed in the segment between home and bottleneck entrance and in the segment between bottleneck exit and work. Thus, the travel time from home to work is the sum of waiting and service times at the bottleneck.

In this study, the service time per car is assumed to be one unit of time. If multiple departures occur simultaneously, a queue forms behind the bottleneck, and the order of entering the bottleneck is determined randomly. Let $Q(t)$ be the length of the queue at time t such that

$$Q(t) = \begin{cases} 0 & \text{if } t < 0 \\ \max\{Q(t-1)-1, 0\} & \text{if } t \geq 0. \end{cases}$$

Suppose that commuter i and m other commuters depart at t_i. If the tie is broken in that k ($\leq m$) other commuters are ahead of commuter i in the queue, commuter i's travel time is

$$T(t) = Q(t) + k + 1.$$

The second source of disutility is the cost of schedule delay, i.e., arriving early or late at the destination. All commuters want to arrive at work at the common arrival time t^*, but the limited service capacity of the bottleneck makes it impossible to occur. Some commuters must arrive at work early and bear the cost of waiting until work begins, whereas some others must arrive late and pay a penalty for doing so.

When commuter i departs at t and her travel time is $T(t)$, she arrives at work at $t + T(t)$. Then, commuter i's schedule delay is given by

$$\begin{cases} t^* - \big(t + T(t)\big) & \text{if } t^* > t + T(t) \\ 0 & \text{if } t^* = t + T(t) \\ \big(t + T(t)\big) - t^* & \text{if } t^* < t + T(t) \end{cases}$$

Travel cost (TC) is the sum of the travel time cost (TTC) and schedule delay cost (SDC). The travel cost of commuter i departing at t is given by

$$TC(t) = \alpha T(t) + \beta \max\Big\{0, t^* - \big(t + T(t)\big)\Big\} + \gamma \max\Big\{0, \big(t + T(t)\big) - t^*\Big\},$$

where α, β, and γ are the shadow prices of travel time, early arrival, and late arrival, respectively, such that $\gamma > \alpha > \beta > 0$ (Small, 1982). Commuters independently and simultaneously decide their departure times to minimize travel costs.

Table 1 presents a numerical example of a game with a set of parameters $(n, t_{\mathrm{L}}, t^*, \alpha, \beta, \gamma) = (10, 18, 12, 120, 25, 125)$. Commuter 1 departing alone at $t = 2$ experienced no waiting at the bottleneck. The travel cost is

$$120 \cdot 1 + 25 \cdot \max\Big\{0, 12 - \big(2+1\big)\Big\} + 125 \cdot \max\Big\{0, \big(2+1\big) - 12\Big\} = 345.$$

Table 1: Numerical Example with $(n, t_{\mathrm{L}}, t^*, \alpha, \beta, \gamma) = (10, 18, 12, 120, 25, 125)$

Commuter	Departure Time	Waiting Time	Arrival Time	Travel Cost
1	2	0	3	345
2	5	0	6	270
3	5	1	7	365
4	8	0	9	195
5	8	1	10	290
6	8	2	11	385
7	8	3	12	480
8	10	2	13	485
9	10	3	14	730
10	13	1	15	615

Commuters 8 and 9 departed at $t = 10$. The tie was broken in favor of commuter 8. Commuters 6 and 7 waited ahead of commuter 8 in queue. Commuter 9 had to wait until commuters 6, 7, and 8 left the bottleneck. The travel costs of commuters 8 and 9 are

$$120 \cdot 3 + 25 \cdot \max\Big\{0, 12 - \big(10 + 3\big)\Big\} + 125 \cdot \max\Big\{0, \big(10 + 3\big) - 12\Big\} = 485$$

and

$$120 \cdot 4 + 25 \cdot \max\Big\{0, 12 - \big(10 + 4\big)\Big\} + 125 \cdot \max\Big\{0, \big(10 + 4\big) - 12\Big\} = 730,$$

respectively.

2.2 Symmetric Mixed-Strategy Equilibrium (SMSE)

The experiment employed a common set of parameters $(n, t_{\mathrm{L}}, t^*, \beta, \gamma) = (10, 18, 12, 25, 125)$. This configuration encompassed 19 departure times, ranging from $t = 0$ to $t = 18$, with a common desired arrival time of $t = 12$. Two experimental conditions, High Alpha and Low Alpha, were established based on distinct values of α: $\alpha = 120$ per unit of time in the High Alpha condition and $\alpha = 30$ per unit of time in the Low Alpha condition.

This investigation exclusively examines symmetric equilibrium, based on the assumption that all commuters adopt an identical mixed strategy in equilibrium. Table 2 presents the symmetric mixed-strategy equilibrium (SMSE) for each experimental condition. These equilibria were computed numerically using the nonstationary Markov chain method described by Otsubo & Rapoport (2008). Comparisons between the High Alpha and Low Alpha conditions indicate that commuters are more likely to avoid departing at the same

Table 2: Departure time distributions under SMSE

Departure Time	High Alpha		Low Alpha	
	Probability	Cumulative Probability	Probability	Cumulative Probability
0	0	0	0	0
1	0.022	0.022	0	0
2	0.077	0.099	0	0
3	0.098	0.197	0.077	0.077
4	0.111	0.308	0.761	0.839
5	0.12	0.428	0	0.839
6	0.126	0.554	0	0.839
7	0.126	0.68	0.039	0.877
8	0.11	0.79	0.037	0.914
9	0.083	0.874	0.026	0.94
10	0.065	0.939	0.028	0.968
11	0.047	0.986	0.019	0.987
12	0.014	1	0.013	1
13	0	1	0	1
14	0	1	0	1
15	0	1	0	1
16	0	1	0	1
17	0	1	0	1
18	0	1	0	1

time as others when α is high than when it is low. In the High Alpha condition, departures are spread across a wider range of departure times, with each commuter departing at $t = 1, 2, 3, \ldots, 11, 12$ with positive probabilities. Conversely, in the Low Alpha condition, commuters predominantly depart at $t = 4$ with a probability of 0.761, whereas departure times at $t = 3, 7, 8, 9, 10, 11$, and 12 have small probabilities.

2.3 Quantal Response Equilibrium (QRE)

In a Nash equilibrium, players choose optimal strategies based on their correct beliefs about other players' behaviors. The quantal response equilibrium (QRE) model proposed by McKelvey & Palfrey (1995) relaxes this strict optimality by allowing players to select better strategies with higher probability and worse strategies with lower probability, while still requiring that beliefs about others' strategies match with the equilibrium choice

Table 3: Expected departure and travel times under QRE

Condition		QRE (λ) 0.002	0.005	0.02	0.5	SMSE
High Alpha	Departure Time	7.794	7.082	6.402	6.137	6.122
	Travel Time	1.476	1.638	1.941	2.195	2.209
Low Alpha	Departure Time	7.797	7.177	6.332	4.848	4.721
	Travel Time	1.487	1.712	2.623	4.125	4.229

probabilities.

Denote a mixed strategy by $p = \big(p(0), p(1), \ldots, p(t_{\mathrm{L}})\big)$. Given the belief that all other commuters play p, a commuter computes the expected travel cost for each departure time. Denote by $\mathrm{ETC}\big(t|p\big)$ the expected travel cost of a commuter departing at t, given p. Each commuter chooses departure times according to a probability distribution that takes the following logit form:

$$p(t) = \frac{e^{-\lambda \cdot \mathrm{ETC}(t|p)}}{\displaystyle\sum_{k=0}^{18} e^{-\lambda \cdot \mathrm{ETC}(k|p)}},$$

where λ is a precision parameter ranging from 0 to ∞.[5] The QRE for a given value of λ is the mixed strategy $p_\lambda = \big(p_\lambda(0), p_\lambda(1), \ldots, p_\lambda(t_{\mathrm{L}})\big)$ such that for $t \in T$,

$$p_\lambda(t) = \frac{e^{-\lambda \cdot \mathrm{ETC}(t|p_\lambda)}}{\displaystyle\sum_{k=0}^{18} e^{-\lambda \cdot \mathrm{ETC}(k|p_\lambda)}}.$$

When $\lambda = 0$, the QRE dictates that all departure times are chosen with equal probability, that is, $p_\lambda(t) = \frac{1}{19}$ for all $t \in T$. As λ approaches infinity, the QRE converges to a Nash equilibrium. Figure 2 shows the cumulative departure time distributions of QRE across $\lambda = 0.002$, 0.005, 0.02, and 0.5. The solid line represents the SMSE distribution. Under both conditions, the QRE assigns positive probabilities to departure times outside the support of the SMSE. With $\lambda = 0.002$, the QRE distribution is nearly uniform across departure times. As λ increases, the QRE distribution progressively moves toward the SMSE distribution. When $\lambda = 0.5$, the QRE distribution almost perfectly overlaps with the SMSE distribution under both conditions.

The expected values for departure and travel times under QRE are summarized in Table 3. Lower λ values correspond to more dispersed departure time choices under both experimental conditions. As λ increases, the expected departure and travel times approach

[5] This study assumes homogeneity in bounded rationality, i.e., a common λ value across all players. For a review of heterogeneous QRE models, see Chapter 4 of Goeree et al. (2002).

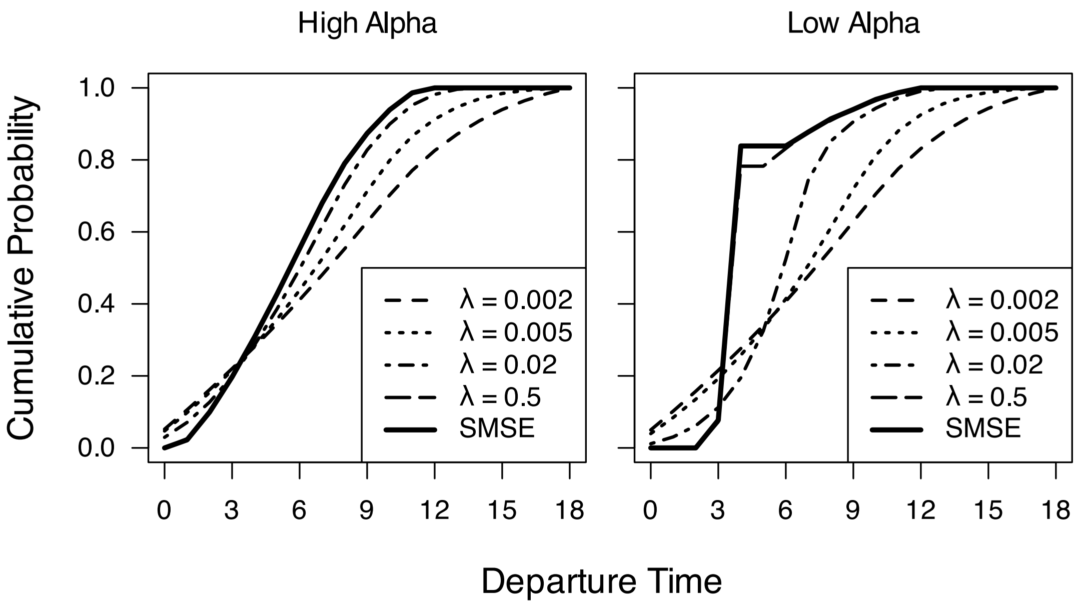

Figure 2: Cumulative probability distributions under QRE by Condition

the SMSE predictions. If participants exhibit bounded rationality, the mean values of the observed departure and travel times are expected to be larger and smaller, respectively, than those of the SMSE under both conditions.

3 Experiment

The experiment was conducted in October 2023 at the Experimental Economics Laboratory of the Research Institute for Socionetwork Strategies (RISS), Kansai University, Osaka, Japan. A total of 120 student participants from various fields of study were recruited using ORSEE software (Greiner, 2015). The experiment used a between-subjects design with two experimental conditions: High Alpha and Low Alpha. There were six sessions, with three sessions per condition, and 20 participants were invited for each session. None of the participants were allowed to participate in more than one session.

Upon arrival at the laboratory, the participants were randomly seated in individual computer terminals. They were given written instructions and asked to read them silently at their own pace.[6] Any form of communication between them was strictly prohibited, and the questions were answered individually by the experimenter.

The experiment was programmed using oTree (Chen et al., 2016). At the beginning of each session, two equal-sized groups of 10 participants each were randomly formed. The

[6] The instructions (written in Japanese) are available upon request.

group composition was the same throughout each session. Participants played the same game 40 times (40 rounds). Although there was no interaction between the two groups, the entire session progressed from one round to the next and ended simultaneously.

Each of the 40 rounds is structured in the same manner. The computer showed a decision screen that displayed all 19 departure times restricted to 5 min intervals between 8:00 ($t = 0$) and 9:30 ($t = 18$), with a common desired arrival time of 9:00 ($t = 12$). Each participant was asked to choose their departure time without time pressure. Once all 20 participants submitted their departure time decisions, a results screen provided each participant with feedback information limited to their own decision (i.e., departure time, arrival time, travel time, etc.) and associated results (i.e., travel cost, payoff, etc.) for the round, not the decisions and results of other participants. Hence, it was impossible for any participant to picture the distribution of departure time choices of other participants. The shadow price of traveling time, α, was set to 120 points per 5 min (24 points per min) in the High Alpha condition and 30 points per 5 min (6 points per min) in the Low Alpha condition.

At the end of the session, a summary screen displayed the total number of points that the participants had accumulated and the corresponding earnings in Japanese yen. They were instructed to remain seated until they were asked to come forward and receive cash payments. Points were converted into Japanese yen at the rate of 1 point = 0.41 yen in the High Alpha condition and 1 point = 0.18 yen in the Low Alpha condition. The average individual earnings were 2642 yen in the High Alpha condition and 2368 yen in the Low Alpha condition, including a 500 yen show-up bonus.[7] Each session lasted approximately 80 min, including reading instructions and receiving payments.

4 Results

This section first assesses how well SMSE describes the observed behavior, and then compares SMSE to QRE using two measures of goodness-of-fit.

4.1 Assessment of SMSE

Figure 3 exhibits the predicted and observed cumulative relative frequency distributions of departure time choices by condition and blocks of 10 rounds. In each panel, there are six gray lines and one thick black line representing the group-level distributions and predicted distribution under SMSE, respectively. Common to both conditions was the observation that the participants tended to depart later than predicted. In block 1 of the Low Alpha

[7] As of the first day of the experiment, the US dollar to Japanese yen exchange rate was \$1=149 yen, and the minimum wage was 1064 yen in Osaka Prefecture.

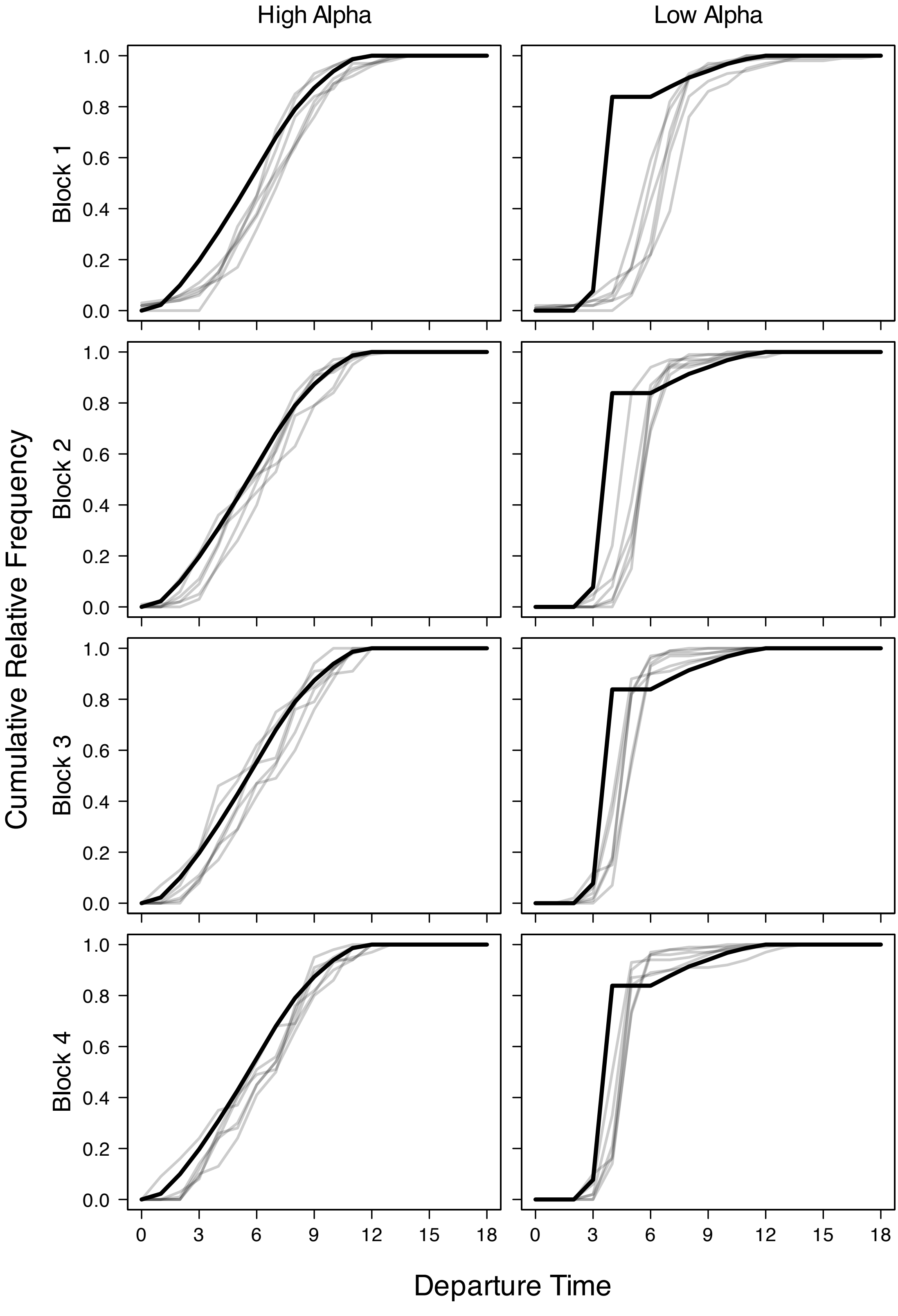

Figure 3: Observed and predicted cumulative departure time distributions by condition and blocks of 10 rounds

condition (top-right panel), the observed distributions are far to the right of the predicted distribution. This means that the participants in this condition left the origin much later than was predicted. As they gained more experience with the game, the observed distributions approached the predicted distribution; however, delayed departures persisted throughout the sessions. A similar pattern of deviation from the predicted distribution was observed for the High Alpha condition.

Table 4 reports the means of departure and travel times by condition, group, and blocks of 10 rounds. Across both conditions, the mean departure times gradually approached the SMSE predictions over successive blocks, yet persistently remain higher than the predicted values throughout all four blocks. The mean travel times also generally fell below the SMSE predictions, with the exception of the first block in the High Alpha condition. In contrast, mean travel times in the Low Alpha condition consistently remained below the SMSE predictions, although they tended to move closer to the predicted values over time.

As shown in Table 3, if participants exhibit bounded rationality, the mean departure times are expected to exceed the SMSE predictions. To test this hypothesis, a one-sided Wilcoxon signed-rank test was conducted, treating each group as an independent observation and using data from the final block, where the observed distributions of departure time choice behavior were closest to the predicted distributions (Figure 3). The results revealed statistically significant differences at the 5% level for both the High Alpha ($p = 0.03125$) and Low Alpha ($p = 0.01562$) conditions, suggesting that, even after gaining experience, later departures did not disappear.

Bounded rationality also suggests that the mean travel times are lower than the SMSE predictions. The same test was applied to test the hypothesis that the observed mean travel times in the final block are significantly lower than those in the SMSE predictions. The results revealed a significant difference in the High Alpha condition ($p = 0.01563$), while no significant difference was found in the Low Alpha condition ($p = 0.2813$). This indicates consistency between the observed mean travel times and SMSE predictions in the Low Alpha condition, but not in the High Alpha condition.

In summary, participants in both conditions displayed behavior indicative of bounded rationality, persisting in the later rounds of the experiment, although the degree of consistency with bounded rationality appears to vary between conditions. The following subsection assesses the comparative fit of the SMSE and QRE models using two goodness-of-fit measures.

4.2 Comparison between SMSE and QRE

Unlike parameter-free Nash equilibrium, QRE includes a precision parameter λ, which must be estimated from the experimental data. Following Camerer & Ho (1999), the first

Table 4: Means of departure and travel times by condition, group, and blocks of 10 rounds

			Block				
Condition		Group	1	2	3	4	SMSE
High Alpha	Departure Time	All	7.047	6.605	6.532	6.748	6.122
		1	7.5	6.17	7.03	6.84	
		2	6.99	6.91	6.39	6.64	
		3	6.55	6.75	7.02	7.2	
		4	6.59	6.79	7.05	6.9	
		5	7.43	6.3	5.79	6.86	
		6	7.22	6.71	5.91	6.05	
	Travel Time	All	2.462	2.163	1.972	1.96	2.209
		1	2.45	1.86	1.69	1.98	
		2	2.10	2.53	2.40	1.97	
		3	3.08	1.81	1.81	2.02	
		4	2.69	1.70	1.66	1.93	
		5	2.47	2.59	1.95	2.10	
		6	1.98	2.49	2.32	1.76	
Low Alpha	Departure Time	All	6.987	5.877	5.158	5.132	4.721
		1	7.01	6.09	5.43	5.2	
		2	6.57	5.81	5.1	5.28	
		3	6.78	6.04	5.33	5.11	
		4	6.37	5.04	4.96	4.75	
		5	7.55	6.12	5.08	5.52	
		6	7.64	6.16	5.05	4.93	
	Travel Time	All	3.523	4.167	4.227	4.147	4.229
		1	3.78	4.51	4.47	4.3	
		2	3.86	4.29	4.1	4.12	
		3	3.38	3.43	3.68	3.84	
		4	3.77	4.26	4.04	4.16	
		5	3.74	4.38	4.32	3.99	
		6	2.61	4.13	4.75	4.47	

70% of the data were used for parameter estimation, whereas the remaining 30% were used for out-of-sample validation. In this study, the precision parameter λ was estimated separately for each group, based on data from the first 28 rounds, by maximizing the

Table 5: Estimated precision parameter of QRE by condition and group

High Alpha			Low Alpha		
Group	λ	LL	Group	λ	LL
1	0.042	-680.121	1	0.029	-485.287
2	0.042	-665.167	2	0.035	-520.367
3	0.103	-662.394	3	0.028	-537.093
4	0.112	-667.055	4	0.061	-491.819
5	0.028	-678.840	5	0.029	-534.971
6	0.034	-676.849	6	0.023	-598.009

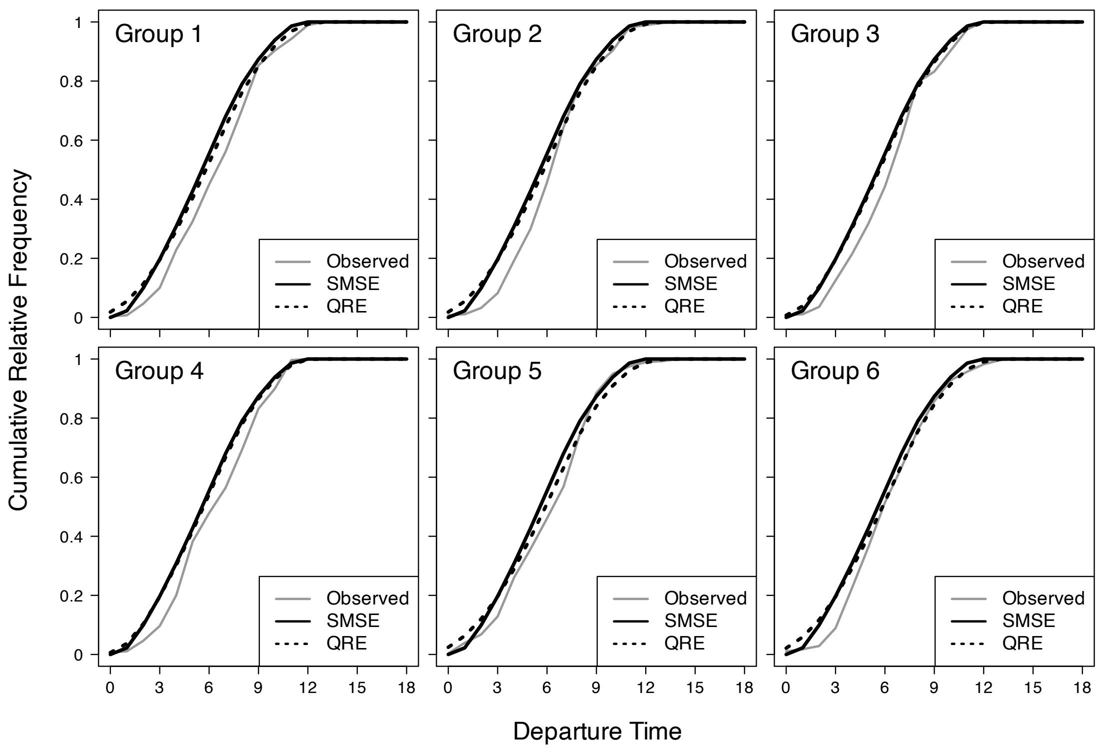

Figure 4: Observed and estimated QRE cumulative departure time distributions by group for the High Alpha condition

following log-likelihood function:

$$\mathrm{LL}(\lambda) = \sum_{t=0}^{18} n(t) \ln p_\lambda(t),$$

where $n(t)$ is the departure frequency at t.

Table 5 reports the estimated λ values and corresponding log-likelihood values by condition and group. The cumulative departure time choice distributions under the estimated

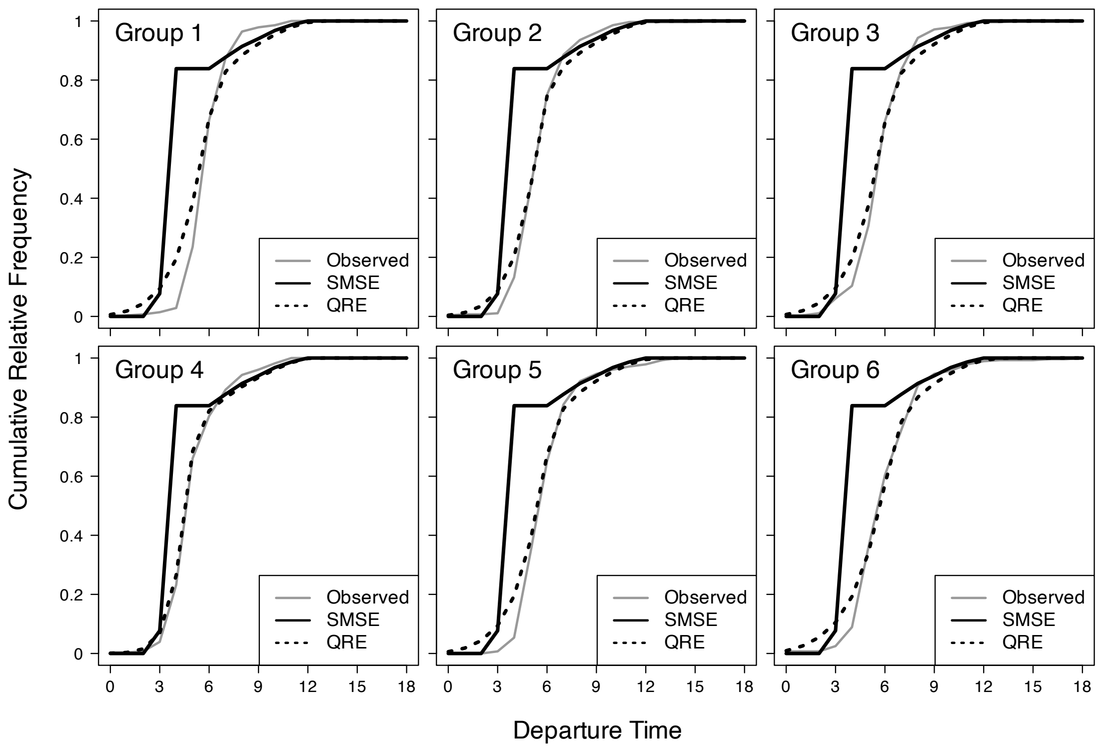

Figure 5: Observed and estimated QRE cumulative departure time distributions by group for the Low Alpha condition

QRE are displayed with the observed and SMSE distributions in Figures 4 (High Alpha) and 5 (Low Alpha), respectively. In the High Alpha condition, it is unclear which model, SMSE or QRE, better fits the data from the first 28 rounds, and in each group, the distributions of these theoretical models were almost identical. Meanwhile, it was visually clear that QRE outperformed SMSE in the Low Alpha condition: the observed distribution was accounted well by the estimated QRE in each group.

Data from the last 12 rounds were used to perform an out-of-sample validation for each group using two measures of goodness-of-fit. Let $p(t)$ be the probability of departing at t as specified by a theoretical model, either the SMSE or QRE model. The first measure is the mean squared deviation (MSD), defined as

$$\mathrm{MSD} = \frac{1}{120} \sum_{i=1}^{10} \sum_{t=0}^{18} \sum_{r=29}^{40} \Big(d_i(t,r) - p(t) \Big)^2,$$

where

$$d_i(t,r) = \begin{cases} 1 & \text{if participant } i \text{ departs at } t \text{ in round } r \\ 0 & \text{otherwise.} \end{cases}$$

Table 6: Comparison between models by condition, measure, and group

High Alpha				Low Alpha			
Measure	Group	SMSE	QRE	Measure	Group	SMSE	QRE
MSD	1	0.90082	0.89784	MSD	1	1.3726	0.76869
	2	0.90011	0.90127		2	1.2978	0.81103
	3	0.89958	0.89683		3	1.4956	0.79328
	4	0.90835	0.90743		4	0.95787	0.68375
	5	0.16023	0.15743		5	1.2571	0.84344
	6	0.20069	0.18951		6	1.1042	0.88512
ED	1	0.20341	0.19594	ED	1	0.88983	0.43342
	2	0.14803	0.15192		2	0.87094	0.52128
	3	0.20547	0.19867		3	0.9383	0.42207
	4	0.1849	0.18237		4	0.58151	0.25303
	5	0.16023	0.15743		5	0.83652	0.53486
	6	0.20069	0.18951		6	0.72853	0.55825

The second goodness-of-fit measure is the Euclidian distance (ED), calculated as the sum of the squared differences between the observed and predicted distributions of departure time choices:

$$\mathrm{ED} = \sum_{t=0}^{18} \Big(f(t) - p(t) \Big)^2,$$

where $f(t)$ is the relative frequency of departures at time t during the last 12 rounds. Both measures had a maximum of two, with lower values indicating a better fit of the theoretical model to the data.

Table 6 compares the SMSE and QRE models by condition, measure, and group. The model with shaded values demonstrates a superior fit. In the High Alpha condition, the values of MSD and ED were very close to each other. Except group 2, these values were slightly smaller for QRE than SMSE. In contrast, the Low Alpha condition shows a very different picture; in all six groups, QRE had much smaller MSD and ED values than SMSE. QRE provided a better fit across all six groups.

5 Conclusion

Since the pioneering work by Vickrey (1969), which provided a framework for the dynamic analysis of rush-hour bottleneck congestion, his bottleneck model has been extended in a variety of directions. Despite its important policy implications, the model's ability to predict actual behavior has long remained unanswered. This study experimentally examined the extent to which the aggregate behavior of decision makers is accounted for by two symmetric equilibrium models, SMSE and QRE, the former assuming full rationality and the latter bounded rationality. The bottleneck model was tailored to its discrete version by relaxing the assumptions of a continuum of commuters and a continuous strategy space. The discrete model was then subjected to a controlled laboratory experiment in which a group of 10 participants interacted repeatedly 40 times. The experiment consisted of two conditions that differed from one another with respect to the shadow price of travel time. The results showed a strong initial tendency of participants to delay their departures, which was in line with QRE predictions. However, this tendency diminished with increased experience as participants' behavior gradually approached SMSE predictions.

To evaluate the predictive performance of the QRE model relative to that of the SMSE model, 70% of the data were used to estimate the precision parameter of the QRE model, and the remaining 30% of the data were reserved for out-of-sample validation using two goodness-of-fit measures, MSD and ED. The validation confirmed that QRE consistently outperformed SMSE under both experimental conditions. QRE captures bounded-rational commuter behavior, highlighting the limitations of equilibrium models based on fully rational decision-making assumptions.

Although this study focuses on the predictive power of two equilibrium models, SMSE and QRE, for aggregate behavior, it may be instructive to discuss individual behavior. The most striking finding is that given no feedback information about the behavior of others at the end of each round, participants in the current experiment generated systematic and replicable patterns of aggregate behavior that differed slightly from SMSE. They exhibited a variety of departure time choice behaviors that defy a simple classification. For example, Figure 6 plots the cumulative relative frequency distributions of the departure time choices of the 10 participants in group 1 of the High Alpha condition, with the SMSE distribution in the solid line. The figure was generated from the data pooled over 40 rounds. Some participants (e.g., participants 1 and 9) departed earlier and some others (e.g., participants 6 and 7) departed later than the SMSE. Participant 4 repeatedly chose the same departure time. The behavior of participants 2 and 5 differed only slightly from the SMSE, while participant 3 behaved almost perfectly in accordance with the SMSE. The finding of highly ordered patterns of aggregate behavior coupled with highly chaotic patterns of individual behavior has also been reported in previous experimental studies, such as market entry games (Sundali et al., 1995; Erev & Rapoport, 1998), route choice games (Selten et al., 2007; Morgan et al., 2009; Rapoport et al., 2009), and queuing games (Rapoport et al., 2004; Seale et al., 2005; Rapoport et al., 2010).

In conclusion, incorporating bounded rationality into Vickrey's bottleneck models, such as QRE, improves the ability to predict commuter behavior at the aggregate level. This approach provides valuable insights for policymakers seeking to design interventions

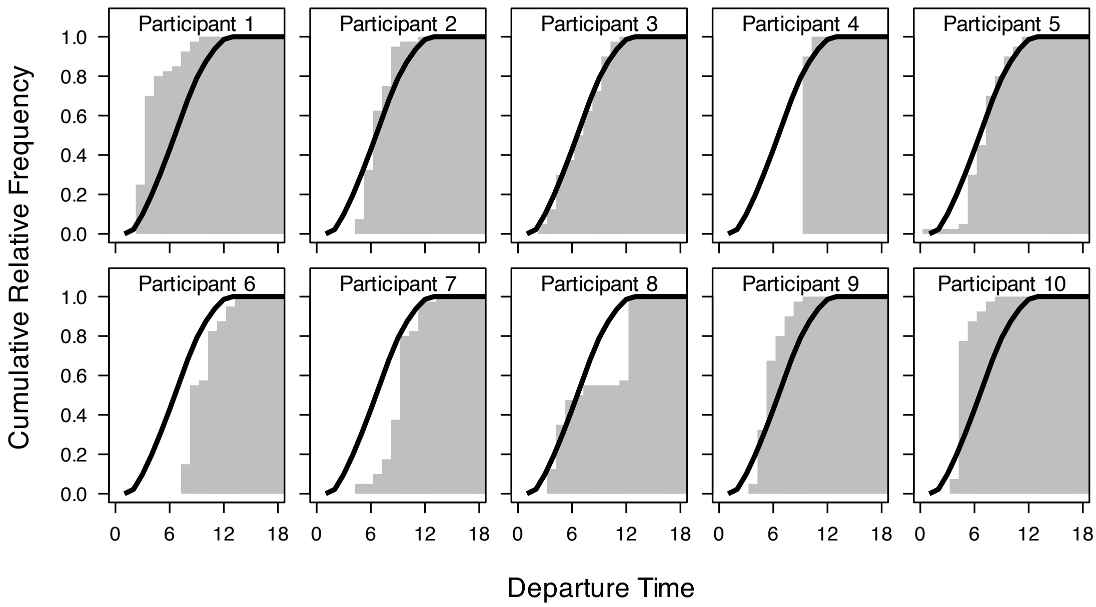

Figure 6: Cumulative observed relative frequency distributions of departure time choices of the 10 participants in group 1 of the High Alpha condition

that account for actual behavioral patterns, which may include systematic deviations from perfect rationality. Future research should further investigate bounded rationality allowing for individual heterogeneity influencing commuter departure time decisions and explore the application of QRE to more complex traffic networks to validate and extend the model's predictive capabilities in various urban contexts.

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Ethical Compliance

All procedures performed in this study involving human participants were approved by the Internal Review Board of the Research Institute for Socionetwork Strategies (RISS) at Kansai University (No. 2023016).

Conflicts of Interest

The author declares no conflicts of interest associated with this study.

References

Arnott, R. (1998). William Vickrey: Contributions to Public Policy. *International Tax and Public Finance*, *5*(1), 93–113.

Arnott, R., de Palma, A., & Lindsey, R. (1990). Economics of a bottleneck. *Journal of Urban Economics*, *27*(1), 111–130.

Arnott, R., de Palma, A., & Lindsey, R. (1993). A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand. *The American Economic Review*, *83*(1), 161–179.

Arnott, R., de Palma, A., & Lindsey, R. (1998). Recent Development in the Bottleneck Model. In K. J. Button, & E. Verhoef (Eds.) *Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility*, (pp. 79–110). Cheltenham, UK: Edward Elgar.

Camerer, C., & Ho, T.-H. (1999). Experience-weighted Attraction Learning in Normal Form Games. *Econometrica*, *67*(4), 827–874.

Capra, C. M., Goeree, J. K., Gomez, R., & Holt, C. A. (1999). Anomalous Behavior in a Traveler's Dilemma? *The American Economic Review*, *89*(3), 678–690.

Capra, C. M., Goeree, J. K., Gomez, R., & Holt, C. A. (2002). Learning and Noisy Equilibrium Behavior in an Experimental Study of Imperfect Price Competition. *International Economic Review*, *43*(3), 613–636.

Chen, D. L., Schonger, M., & Wickens, C. (2016). oTree—An open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, *9*, 88–97.

Daniel, T. E., Gisches, E. J., & Rapoport, A. (2009). Departure Times in Y-Shaped Traffic Networks with Multiple Bottlenecks. *The American Economic Review*, *99*(5), 2149–2176.

Dechenaux, E., Mago, S. D., & Razzolini, L. (2014). Traffic congestion: An experimental study of the Downs-Thomson paradox. *Experimental Economics*, *17*(3), 461–487.

Di, X., & Liu, H. X. (2016). Boundedly rational route choice behavior: A review of models and methodologies. *Transportation Research Part B: Methodological*, *85*, 142–179.

Dixit, V. V., Ortmann, A., Rutström, E. E., & Ukkusuri, S. V. (2017). Experimental Economics and choice in transportation: Incentives and context. *Transportation Research Part C: Emerging Technologies*, *77*, 161–184.

Erev, I., & Rapoport, A. (1998). Coordination, "Magic," and Reinforcement Learning in a Market Entry Game. *Games and Economic Behavior*, *23*(2), 146–175.

Gabuthy, Y., Neveu, M., & Denant-Boemont, L. (2006). The Coordination Problem in a Structural Model of Peak-Period Congestion: An Experimental Study. *Review of Network Economics*, *5*(2).

Gneezy, U., & Smorodinsky, R. (2006). All-pay auctions—an experimental study. *Journal of Economic Behavior & Organization*, *61*(2), 255–275.

Goeree, J. K., Holt, C. A., & Palfrey, T. R. (2002). Quantal Response Equilibrium and Overbidding in Private-Value Auctions. *Journal of Economic Theory*, *104*(1), 247–272.

Goeree, J. K., Holt, C. A., & Smith, A. M. (2017). An experimental examination of the volunteer's dilemma. *Games and Economic Behavior*, *102*, 303–315.

Greiner, B. (2015). Subject pool recruitment procedures: Organizing experiments with ORSEE. *Journal of the Economic Science Association*, *1*(1), 114–125.

Kawagoe, T., Matsubae, T., & Takizawa, H. (2018). Quantal response equilibria in a generalized Volunteer's Dilemma and step-level public goods games with binary decision. *Evolutionary and Institutional Economics Review*, *15*(1), 11–23.

Kawagoe, T., Takizawa, H., & Yamamori, T. (2023). Asymmetric volunteer's dilemma game: Theory and experiment. *Games and Economic Behavior*, *142*, 955–977.

Levine, D. K., & Palfrey, T. R. (2007). The Paradox of Voter Participation? A Laboratory Study. *The American Political Science Review*, *101*(1), 143–158.

Li, Z.-C., Huang, H.-J., & Yang, H. (2020). Fifty years of the bottleneck model: A bibliometric review and future research directions. *Transportation Research Part B: Methodological*, *139*, 311–342.

Liu, Q., Lu, D., Jiang, R., Han, X., Liu, R., & Gao, Z. (2023). Departure time choice behavior in commute problem with stochastic bottleneck capacity: Experiments and modeling. *Transportmetrica A: Transport Science*, *19*(2), 1978590.

Mahmassani, H. S., & Chang, G.-L. (1987). On Boundedly Rational User Equilibrium in Transportation Systems. *Transportation Science*, *21*(2), 89–99.

McKelvey, R. D., & Palfrey, T. R. (1995). Quantal Response Equilibria for Normal Form Games. *Games and Economic Behavior*, *10*(1), 6–38.

McKelvey, R. D., & Palfrey, T. R. (1998). Quantal Response Equilibria for Extensive Form Games. *Experimental Economics*, *1*(1), 9–41.

Morgan, J., Orzen, H., & Sefton, M. (2009). Network architecture and traffic flows: Experiments on the Pigou–Knight–Downs and Braess Paradoxes. *Games and Economic Behavior*, *66*(1), 348–372.

Otsubo, H., Gisches, E. J., & Rapoport, A. (2023). The Downs-Thomson Paradox with Endogenously Determined Departure Times.

Otsubo, H., & Rapoport, A. (2008). Vickrey's model of traffic congestion discretized. *Transportation Research Part B: Methodological*, *42*(10), 873–889.

Rapoport, A., Kugler, T., Dugar, S., & Gisches, E. J. (2009). Choice of routes in congested traffic networks: Experimental tests of the Braess Paradox. *Games and Economic Behavior*, *65*(2), 538–571.

Rapoport, A., Stein, W. E., Mak, V., Zwick, R., & Seale, D. A. (2010). Endogenous arrivals in batch queues with constant or variable capacity. *Transportation Research Part B: Methodological*, *44*(10), 1166–1185.

Rapoport, A., Stein, W. E., Parco, J. E., & Seale, D. A. (2004). Equilibrium play in single-server queues with endogenously determined arrival times. *Journal of Economic Behavior & Organization*, *55*(1), 67–91.

Schneider, K., & Weimann, J. (2004). Against all Odds: Nash Equilibria in a Road Pricing Experiment. In M. Schreckenberg, & R. Selten (Eds.) *Human Behaviour and Traffic Networks*, (pp. 133–153). Berlin, Heidelberg: Springer.

Seale, D. A., Parco, J. E., Stein, W. E., & Rapoport, A. (2005). Joining a Queue or Staying Out: Effects of Information Structure and Service Time on Arrival and Staying Out Decisions. *Experimental Economics*, *8*(2), 117–144.

Selten, R., Chmura, T., Pitz, T., Kube, S., & Schreckenberg, M. (2007). Commuters route choice behaviour. *Games and Economic Behavior*, *58*(2), 394–406.

Simon, H. A. (1955). A Behavioral Model of Rational Choice. *The Quarterly Journal of Economics*, *69*(1), 99–118.

Small, K. A. (1982). The Scheduling of Consumer Activities: Work Trips. *The American Economic Review*, *72*(3), 467–479.

Smith, V. L. (1976). Experimental Economics: Induced Value Theory. *The American Economic Review*, *66*(2), 274–279.

Sun, X., Han, X., Bao, J.-Z., Jiang, R., Jia, B., Yan, X., Zhang, B., Wang, W.-X., & Gao, Z.-Y. (2017). Decision dynamics of departure times: Experiments and modeling. *Physica A: Statistical Mechanics and its Applications*, *483*, 74–82.

Sundali, J. A., Rapoport, A., & Seale, D. A. (1995). Coordination in Market Entry Games with Symmetric Players. *Organizational Behavior and Human Decision Processes*, *64*(2), 203–218.

Vickrey, W. S. (1969). Congestion Theory and Transport Investment. *The American Economic Review*, *59*(2), 251–260.

Yang, Y., Jiang, R., Han, X., Jia, B., & Gao, Z. (2022). Experimental study and modeling of departure time choice behavior in the bottleneck model with staggered work hours. *Travel Behaviour and Society*, *27*, 79–94.

Zhao, C.-L., & Huang, H.-J. (2014). Modeling Bounded Rationality in Congestion Games with the Quantal Response Equilibrium. *Procedia - Social and Behavioral Sciences*, *138*, 641–648.

Ziegelmeyer, A., Koessler, F., My, K. B., & Denant-Boèmont, L. (2008). Road Traffic Congestion and Public Information: An Experimental Investigation. *Journal of Transport Economics and Policy*, *42*(1), 43–82.