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Inflation targeting, expectations formation, and macroeconomic stability

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Inflation targeting, expectations formation, and macroeconomic stability

Abstract

This paper explores the impact of inflation targeting on agents' expectations formation and macroeconomic stability. It provides the conditions for the expectational stability of the equilibrium in a New Keynesian model with inflation targeting. We find that inflation targeting not only determines the long-run inflation rate, but also stabilizes expectations formation processes. Fundamental and non-fundamental equilibria become expectationally stable even if the nominal interest rate is pegged at its steady state. The Taylor principle becomes unnecessary for the stability of the fundamental equilibrium, but necessary for preventing stable non-fundamental equilibria. This effect cannot be reproduced by announcing the output gap target and it is robust to the calibrated credibility of the central bank. These results modify the conventional wisdom of macroeconomic stability to fit the dynamics of recent economies near the zero lower bound. Unconventional monetary policies might have contributed to the recent stability by raising the credibility of inflation targeting.

JEL classification: C62; D83; D84; E32; E52

Keywords: Inflation target; Learning; Expectational stability; Taylor principle; Interest-rate peg; Sunspots

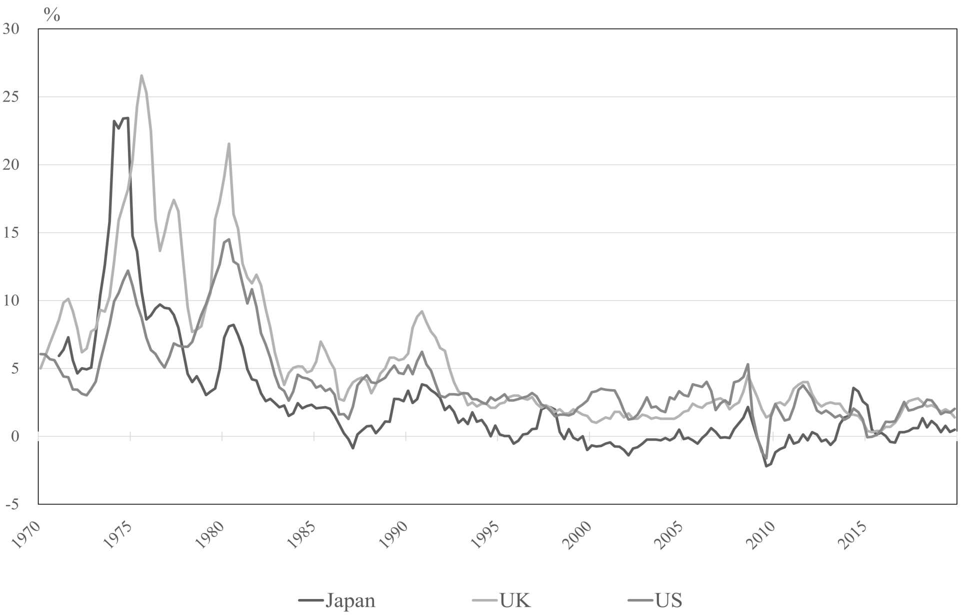

Figure 1: CPI inflation rate.

1 Introduction

Inflation targeting announces the central bank's objective and plan for the public by setting the target of the long-run inflation rate. Since the 1970s, it has been agreed that the long-run inflation rate is primarily determined by the central bank (see Bernanke et al., 1999). This perspective was reinforced in the early 1980s when Paul Volcker's aggressive stance reduced the inflation rate from 10% to around 4% (see Erceg and Levin, 2003). This induced central banks to announce inflation targets implicitly or explicitly, and inflation rates have been reduced and stabilized near the targets since the 1990s (see Figure 1).[1]

[1] In this paper, "inflation targeting" not only refers to the "explicit" targeting that makes an official announcement for the inflation target, but also the "implicit" targeting that pursues the target without any official announcement. The implicit targeting by the Federal Reserve has been recognized since the 1980s, while explicit targeting began in the 2010s (see Clarida et al., 1998; Goodfriend, 2004).

For the inflation-targeting policy to be effective, announcing the inflation target has to anchor agents' expectations. The mechanism of this process is trivial in the rational expectations framework in which agents have perfect knowledge of the economic structure, while it is still unclear under real expectations formation. Indeed, anchoring expectations remains a difficult task for inflation-targeting central banks. Inflation forecasts of the private sector often deviate from their targets. To prevent this, central banks introduce unconventional monetary policies (e.g., quantitative easing, forward guidance), but their effects are less than expected (see Del Negro et al., 2012).

This paper reexamines the impact of inflation targeting on agents' expectations formation and macroeconomic stability. To analyze the dynamics of expectations formation, the literature focuses on the *expectational stability* of an equilibrium under *adaptive learning*, where agents have no knowledge of the economic structure and hence form their expectations by estimating econometric models with available data (see Evans and Honkapohja, 2001). We provide the conditions for the expectational stability of an equilibrium in the presence of inflation targeting and examine whether such targeting affects the stability conditions imposed on monetary policy rules. To find desirable monetary policy rules, we explore the stability of both fundamental and non-fundamental equilibria. These studies are carried out in a New Keynesian (NK) model with alternative nominal interest rate rules. We further investigate whether such an impact, if any, can be reproduced by announcing the targets of other variables, particularly the output gap. Finally, we check the robustness of these results under the imperfect credibility of the central bank.

The impact of inflation targeting on macroeconomic stability has been extensively investigated in the learning literature. Orphanides and Williams (2004, 2005) show in simple macroeconomic models that inflation targeting reduces inflation and output gap variability by eliminating the uncertainty of the long-run inflation rate. Branch and Evans (2017) find that the imperfect information of targeting can generate instability in inflation rates (see also Florio and Gobbi, 2015). Hommes and Lustenhouwer (2019a,b) adopt a heuristic switching model and find that the credibility of the central bank expands the region of policy parameters that leads to a stable equilibrium.

Despite these studies, the impact of inflation targeting is yet to be fully clarified. In particular, the pure effect of announcing the inflation target remains unclear because the central bank is assumed to announce not only the

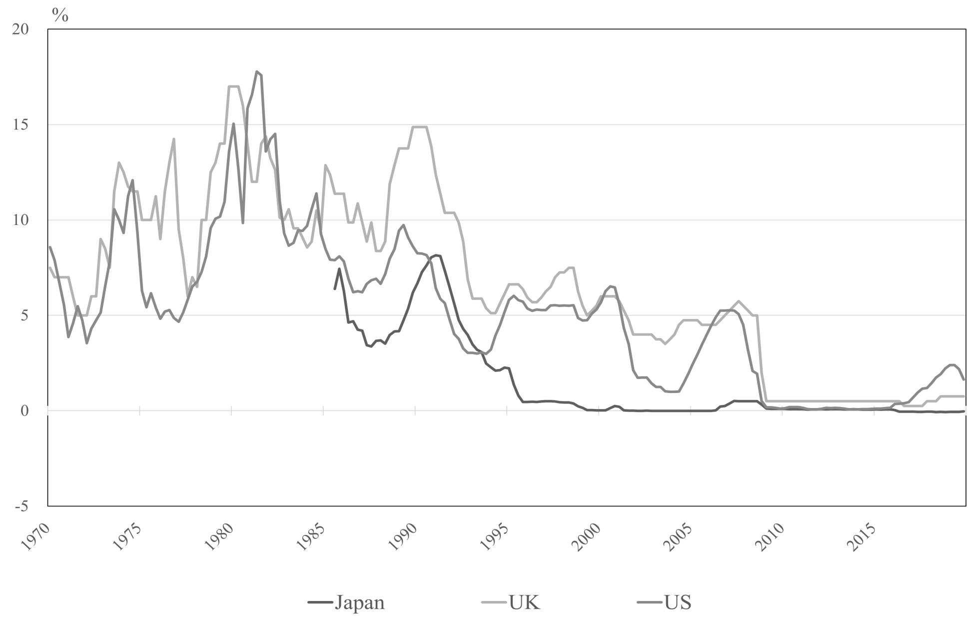

Figure 2: Nominal interest rates.

inflation target, but also the output gap target (see also Gaspar et al., 2006; Ho et al., 2021; Lustenhouwer, 2021); in this setting, any findings might stem from the output gap target rather than the inflation target. Further, the impact of targeting on the stability of non-fundamental business cycles is less clear. The robustness of the findings under different interest rate rules is less examined. In addition, whether the impact of targeting holds under the imperfect credibility of the central bank remains unclear.

For these reasons, the dynamics of recent economies are not fully explained in the learning literature. Following the global financial crisis and the Great Recession, inflation-targeting countries have been stable, while central banks have fixed nominal interest rates near the zero lower bound (the so-called *quiet of the zero bound era* (Cochrane, 2022)) (see Figure 2). The Japanese economy, in particular, has experienced this situation over the last two decades (see Aruoba et al., 2018). Before the global financial crisis, non-fundamental business cycles were observed during the US and European housing booms in the 2000s, when nominal interest rates were irresponsive to inflation (Taylor, 2007a,b). However, the learning literature predicts that the economy is expectationally stable only if the central bank responds to inflation aggressively by following the *Taylor principle* (i.e., raising the nominal interest rate more than one-for-one in response to an increase in the inflation rate) (e.g., Howitt, 1992; Bullard and Mitra, 2002; Evans and Honkapohja, 2003; Evans and McGough, 2018). Non-fundamental business cycles are predicted to always be unstable in calibrated NK models (see Honkapohja and Mitra, 2004; Evans and McGough, 2005a).

The main finding of this paper bridges these gaps. We find that inflation targeting stabilizes agents' expectations formation. Targeting not only determines the long-run inflation rate, but also simplifies their expectations formation processes. If agents recognize the inflation target as the steady-state inflation rate, they can skip estimating the parameters or expectations of the steady-state inflation rate and fix them at or around the target. This simplification makes it easy for agents' expectations formation processes to converge and thus enhances macroeconomic stability.

We find that this mechanism significantly relaxes the stability conditions imposed on monetary policy rules. In the presence of inflation targeting, fundamental and sunspot equilibria are stable even if the central bank pegs the nominal interest rate at its steady state. Then, in contrast to the findings in the literature, the Taylor principle becomes unnecessary for the stability of the fundamental equilibrium, but necessary for preventing sunspot equilibria from being stable. These results are robust under different nominal interest rate rules. Calibrations show that this effect is unique to announcing the inflation target in the sense that it cannot be reproduced by announcing the output gap target. Although this effect reduces if the credibility of the central bank is imperfect, calibrations indicate that the effect is robust under the estimated credibility of modern central banks.

These results not only clarify the stabilizing effect of inflation targeting, but also demonstrate the plausible stability of the economy. The stability of the fundamental equilibrium under an interest-rate peg is consistent with the evidence that recent economies have been stable and nominal interest rates are fixed near the zero lower bound. The stability of sunspot equilibria in violation of the Taylor principle also fits past observations that central banks were irresponsive to inflation during non-fundamental fluctuations (e.g., Clarida et al., 2000; Lubik and Schorfheide, 2004; Taylor, 2007a,b). They imply that models with inflation targeting may better explain the dynamics of recent economies and past macroeconomic fluctuations.

Our results further reinforce the significance of the Taylor principle for macroeconomic stability. This principle is known as the nominal interest rate rule that prevents the indeterminacy of the stationary equilibrium (see Woodford, 2003). While the previous literature argues that the indeterminacy does not arise under learning because of the instability of sunspot equilibria, this paper shows that inflation targeting revives room for the indeterminacy in violation of the Taylor principle. This implies that the Taylor principle is essential for the determinacy of the equilibrium not only under rational expectations, but also under realistic expectations.

The results on credibility also imply the importance of central banks' efforts to improve their credibility. Over the last decade, unconventional monetary policies have been introduced to stimulate aggregate demand (see García-Schmidt and Woodford, 2019). These policies might have improved the credibility of the inflation-targeting stance of the central bank. If this is the case, unconventional policies might have contributed to the recent stable period by enhancing the effect of inflation targeting.

This paper is closely related to the literature on the expectational *in*stability of the economy under an interest-rate peg policy. The instability of pegging the nominal interest rate is originally shown by Howitt (1992) (see also Evans and Honkapohja, 2003). The peg corresponds to monetary policy in a *liquidity trap*, and the instability of the liquidity trap steady state is shown by Evans and Honkapohja (2005), Evans et al. (2008), and Benhabib et al. (2014). These findings are not necessarily consistent with recent economies that have been stable near the zero lower bound. To bridge this gap, Arifovic et al. (2018) show the stability of the liquidity trap equilibrium under social learning. Cochrane (2022) provides a similar result under fiscal underpinnings of monetary policy. While the present study does not directly analyze the economy at the zero lower bound, it does explain stability at such effective lower bounds.

This paper is also related to studies of the stability of sunspot equilibria under learning. Most of the literature shows that sunspot equilibria are expectationally *un*stable in calibrated NK models. In response to this puzzle, Carlstrom and Fuerst (2004) find stable sunspot equilibria under structural heterogeneity in learning. Airaudo (2013) considers limited asset market participation for stability. Arifovic et al. (2013) incorporate social learning rather than econometric learning. McGough and Nakagawa (2019) consider the impact of limited and heterogeneous information sets under learning. The contribution of the present study is to show stable sunspot equilibria in the presence of inflation targeting. This mechanism might explain the dynamics of past non-fundamental fluctuations in the eras of not only explicit, but also implicit inflation targeting, such as the US pre-Volcker period in the late 1970s (see Clarida et al., 2000; Lubik and Schorfheide, 2004) and the Japanese asset price bubble in the late 1980s (Clarida et al., 1998; Ahrend et al., 2008).

Other related studies have examined the credibility of the central bank. Recent experience with unconventional monetary policies has emphasized the importance of the credibility of modern central banks. Lamla and Vinogradov (2019) find that FOMC's announcements do not affect consumers' beliefs. Erceg and Levin (2003) and Michelis and Iacoviello (2016) demonstrate that the imperfect credibility of the central bank destabilizes agents' inflation expectations (see also Hommes and Lustenhouwer, 2019a,b). Bodenstein et al. (2012) argue that imperfect credibility caused the *forward guidance puzzle* focused on by Del Negro et al. (2012). Honkapohja and Mitra (2020) show that the credibility of the central bank improves macroeconomic stability on a different basis. The contribution of this paper is to suggest that the credibility of the central bank contributes to macroeconomic stability by raising the stabilizing effect of inflation targeting.

The remainder of this paper is structured as follows. The next section presents a standard NK model and the determinacy condition for the equilibrium. Section 3 reviews the conventional stability conditions for fundamental and sunspot equilibria in the absence of inflation targeting. Section 4 provides the stability conditions in the presence of targeting and clarifies its effects on expectations formation. Section 5 shows the characteristics of inflation targeting by comparing it with the output gap target. Section 6 shows the robustness of our results under the imperfect credibility of the central bank. The final section concludes.

2 Model

2.1 NK model

We use a standard NK model (see Woodford, 2003):

$$x_t = -\alpha\left(i_t - E_t^*\pi_{t+1}\right) + E_t^* x_{t+1}, \tag{1}$$
$$\pi_t = \kappa x_t + \beta E_t^* \pi_{t+1}. \tag{2}$$
$$i_t = \phi_\pi \pi_t + \phi_x x_t + v_t. \tag{3}$$

The endogenous variables x_t, π_t, and i_t represent the output gap, inflation rate, and nominal interest rate, respectively. The exogenous variable v_t is a monetary policy shock, and we assume that v_t follows an independent and identically distributed (iid) process for simplicity. Eq. (1) is a log-linearized intertemporal Euler equation derived from households' optimal choice of consumption. Eq. (2) is the Phillips curve derived from the optimizing behavior of monopolistically competitive firms with Calvo price setting. E_t^* is the operator of agents' expectations at time t, which may or may not be rational. $\alpha > 0$, $\kappa > 0$, and $0 < \beta < 1$ are assumed. Eq. (3) is a central bank's nominal interest rate rule, which responds to contemporaneous variables by controlling $\phi_\pi \geq 0$ and $\phi_x \geq 0$. The non-stochastic steady state of this model is $(x, \pi, i) = (0, 0, 0)$.

To check the robustness of our analysis, we consider alternative nominal interest rate rules responding to the expected variables: a forward-looking rule (see McCallum and Nelson, 1999) and a forward-looking variant rule (see Clarida et al., 2000):

$$i_t = \phi_\pi E_t^* \pi_{t+1} + \phi_x E_t^* x_{t+1} + v_t, \tag{4}$$
$$i_t = \phi_\pi E_t^* \pi_{t+1} + \phi_x x_t + v_t. \tag{5}$$

To focus on empirically plausible equilibria, we assume that the model has positive feedback from expectations, which is one of the typical characteristics of calibrated NK models (see McGough and Nakagawa, 2019). For this purpose, let us represent the system (1)–(2) with each of the policy rules (3)–(5) as the process $y_t = (x_t, \pi_t)'$ following

$$y_t = B E_t^* y_{t+1} + C v_t, \tag{6}$$

where B is the 2×2 coefficient matrix of $E_t^* y_{t+1}$ and C is the 2×1 coefficient vector of v_t. The expectational feedback is positive if and only if all the eigenvalues of matrix B have positive real parts, that is,

$$\det(B) > 0, \quad \operatorname{tr}(B) > 0. \tag{7}$$

The details of this restriction imposed on the parameters (ϕ_π, ϕ_x) are provided in Appendix A. Hereafter, we focus on the parameter region for (ϕ_π, ϕ_x) that satisfy the positive feedback restriction (7).

2.2 Determinacy

Under rational expectations ($E_t^* = E_t$), if and only if both the eigenvalues of matrix B exist inside the unit circle, the stationary equilibrium is uniquely determined at the fundamental rational expectations equilibrium (REE):

$$y_t = Cv_t. \tag{8}$$

Otherwise, the stationary equilibrium is indeterminate such that a continuum of stationary sunspot REEs also exist. Further, if and only if the eigenvalue outside the unit circle is real, sunspot REEs can have the following form (see Evans and McGough, 2005b):

$$\begin{aligned} y_t &= Cv_t + \bar{d}\xi_t, \\ \xi_t &= \theta^{-1}\xi_{t-1} + \varepsilon_t, \end{aligned} \tag{9}$$

where ξ_t is a serially correlated extrinsic shock that follows a martingale difference sequence ε_t, so-called "sunspot," with the real eigenvalue θ satisfying $-1 < \theta^{-1} < 1$. $E(v_t\varepsilon_t) = E(v_t\xi_t) = 0$ is assumed. $\bar{d} \equiv s\hat{d}$, where s is an arbitrary real constant and $\hat{d}$ is the real eigenvector corresponding to the eigenvalue θ.[2]

The determinacy (or indeterminacy) condition of the stationary REE is summarized as follows.

Proposition 1 *Under any of the policy rules (3)–(5), the stationary equilibrium is determinate at the fundamental REE (8) if and only if*

$$\kappa(\phi_\pi - 1) + \phi_x(1 - \beta) > 0; \tag{10}$$

otherwise, the equilibrium is indeterminate such that stationary sunspot REEs of the form (9) also exist.

[2] In the indeterminate case, other forms of sunspot REEs also exist. In the current paper, those forms are not analyzed, as they are unstable under learning (see Evans and McGough, 2005b, 2018).

The proof is presented in Appendix B. In the indeterminate case, the eigenvalues are all real; hence, sunspot REEs can have the form (9). Eq. (10) is known as the (generalized) *Taylor principle*, which prevents the indeterminacy of the stationary equilibrium. Under positive feedback (7), this principle is the unique determinacy condition under any type of policy rule.

3 Benchmark

This section reviews the conventional stability conditions for fundamental and sunspot REEs in the absence of inflation targeting.

3.1 Expectations formation

Under adaptive learning, agents have imperfect knowledge of the economic structure to form rational expectations. Following the methodology in the learning literature (e.g., Evans and Honkapohja, 2001), we assume that agents specify the perceived law of motion (PLM), which follows the form of the fundamental REE (8) or sunspot REEs (9):

$$y_t = a + cv_t + d\xi_t + e_t, \tag{11}$$

where $a \equiv (a_x, a_\pi)'$ is a constant term vector perceived as the non-stochastic steady state, while c and d are the 2×1 vectors of the coefficients for v_t and for sunspot ξ_t, respectively. e_t is the 2×1 vector of the error terms at time t, which are perceived as white noise. When the fundamental REE is learned, the parameter d is fixed at zero instead of being estimated. The parameters $\phi' \equiv (a, c, d)$ are estimated as the optimal linear projection of y_{t-1} on $z'_{t-1} \equiv \left(1, v_{t-1}, \xi_{t-1}\right)$, which satisfies the least-squares orthogonality condition: $Ez_{t-1}\left(y_{t-1} - \phi'_t z_{t-1}\right) = 0$.

Using the PLM (11) and estimated parameters, agents form the forecast:

$$E^*_t y_{t+1} = a + \theta^{-1} d\xi_t.$$

By incorporating this forecast into Eq. (6), the actual law of motion (ALM) of the economy is obtained as

$$y_t = Ba + Cv_t + B\theta^{-1} d\xi_t. \tag{12}$$

3.2 Expectational stability

The local dynamics of ϕ are governed by the associated ordinary differential equation (ODE):

$$\frac{\mathrm{d}\phi}{\mathrm{d}\tau} = T(\phi) - \phi, \tag{13}$$

where τ denotes the notional time and $T(\phi) \equiv \begin{pmatrix} Ba & C & B\theta^{-1}d \end{pmatrix}$ is the map from the PLM to the ALM. The fixed point $\bar{\phi}$ of the ODE is either the fundamental REE $(\mathbf{0}, C, 0)$ (when d is fixed at zero) or sunspot REEs $\left(\mathbf{0}, C, \bar{d}\right)$ (when d is estimated). Note that $B\theta^{-1}\bar{d} = \bar{d}$. If the ODE is locally asymptotically stable around either of the fixed points, the parameters ϕ converge to the fixed point under real-time learning and the economy is determined at the process (12). In this case, a corresponding REE is said to be locally stable under learning.

Under the assumption that v_t follows an iid process, the stability of the ODE (13) is governed by the stability of the smaller ODE for the constant term vector a (see Evans et al., 2008):

$$\frac{\mathrm{d}a}{\mathrm{d}\tau} = Ba - a.$$

The ODE is locally stable if and only if the Jacobian, $D(Ba - a) = B - I_2$, has all eigenvalues with negative real parts. This provides the conditions for the expectational stability of REEs as follows.

Proposition 2 *Under any of the policy rules (3)–(5), the fundamental REE (8) and sunspot REEs (9) are locally stable under learning if and only if the Taylor principle (10) is satisfied.*

The proof is presented in Appendix C.

Stationary sunspot REEs must satisfy the indeterminacy condition (Proposition 1) as well, but this contradicts the above stability condition. Hence, we obtain the following result.

Proposition 3 *Under any of the policy rules (3)–(5), sunspot REEs (9) are stationary and locally unstable under learning.*

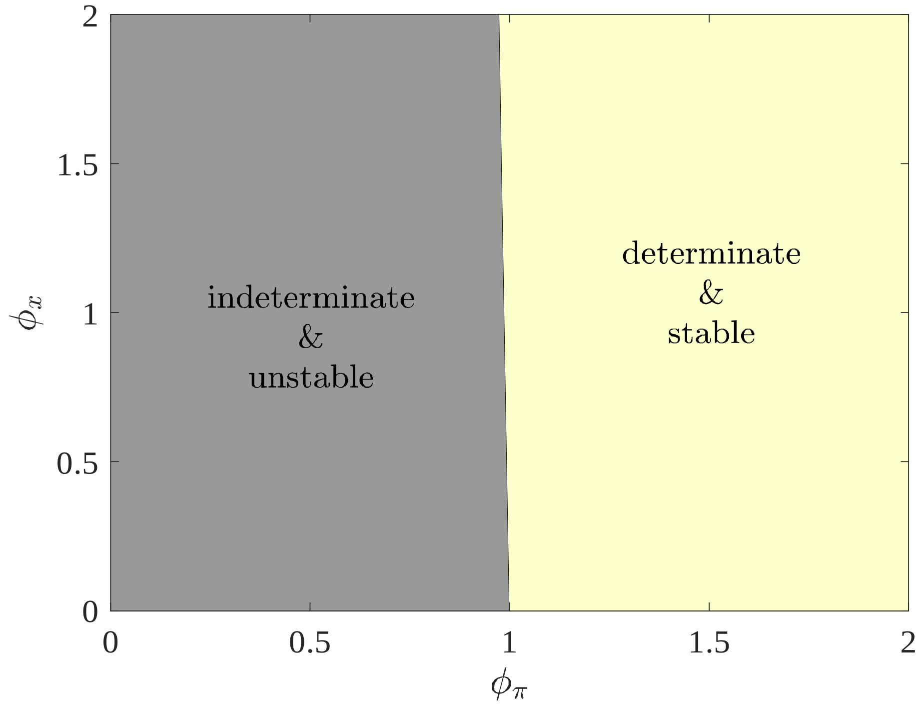

Figure 3: Region for determinacy and stability under the contemporaneous rule (3) and $(\alpha, \kappa, \beta) = (1/1.45, 0.77, 0.99)$ in Lubik and Schorfheide (2004, Table 3).

These results are conventional in the literature. Figure 3 calibrates the region for determinacy and stability. The Taylor principle is essential not only for the determinacy, but also for the stability of the fundamental REE (see Bullard and Mitra, 2002). It follows that the fundamental REE is unstable under an interest-rate peg at the steady state ($\phi_\pi = \phi_x = 0$) (see Howitt, 1992; Evans and Honkapohja, 2003). Further, stationary sunspot REEs are always unstable in NK models with positive feedback (see Honkapohja and Mitra, 2004; Evans and McGough, 2005a).

4 Inflation targeting

This section clarifies the impact of inflation targeting by providing the stability conditions under targeting. The central bank announces the inflation target $\pi = 0$, which corresponds to the steady-state inflation rate, and controls the nominal interest rate to achieve this target. The announcement is fully credible; hence, all agents recognize the inflation target as the steady-state inflation rate.

4.1 Stability conditions

In the spirit of Orphanides and Williams (2004), we assume that if agents recognize the steady-state inflation rate, they fix the second element a_π in the constant term vector a in the PLM (11) at the steady state and estimate only the first element a_x. If agents learn the fundamental REE, they form the PLM $y_t = (a_x, 0)' + cv_t$ and a forecast $E_t^* y_{t+1} = (a_x, 0)'$. The ALM is determined as $y_t = B(a_x, 0)' + Cv_t$.

As v_t follows an iid process, the stability of REEs is determined by the convergence of the estimate of a_x. The parameter a_x is estimated by the sample mean of x_t, which is represented by a recursive algorithm:

$$a_{xt} = a_{x,t-1} + t^{-1}\left(b_{11}a_{x,t-1} - a_{x,t-1}\right).$$

a_{xt} is the estimate of a_x at time t and b_{11} is the element in the first row and first column of matrix B. The convergence of the algorithm is governed by the following ODE:

$$\frac{\mathrm{d}a_x}{\mathrm{d}\tau} = b_{11}a_x - a_x.$$

The same ODE is obtained even when sunspot REEs are learned.

The ODE is locally stable if and only if its Jacobian is negative:

$$D\left(b_{11}a_x - a_x\right) = b_{11} - 1 < 0,$$

where $b_{11} = \frac{1}{\alpha\phi_x + \alpha\kappa\phi_\pi + 1}$ under the contemporaneous rule (3), $b_{11} = 1 - \alpha\phi_x$ under the forward-looking rule (4), and $b_{11} = \frac{1}{\alpha\phi_x + 1}$ under the forward-looking variant rule (5). This provides the stability conditions as follows.

Proposition 4 *When the inflation target $\pi = 0$ is announced, the fundamental REE (8) and sunspot REEs (9) are locally stable under learning if and only if*

1. *under the contemporaneous rule (3),*

$$\phi_\pi + \phi_x > 0;$$

2. *under the forward-looking and forward-looking variant rules (4), (5),*

$$\phi_x > 0.$$

Comparing these results with Proposition 2, we find that the stability conditions are significantly relaxed by the presence of inflation targeting. Previously, REEs were stable only under the Taylor principle, while here they are stable for any $\phi_\pi, \phi_x \geq 0$ except the interest-rate peg ($\phi_\pi = \phi_x = 0$). That is, the Taylor principle becomes unnecessary for the stability of REEs.

In practice, the interest-rate peg can also lead to stability if plausible assumptions are added. An example is as follows.

Corollary 1 *If there exists an agent who is informed of not only the inflation target, but also the steady-state output gap $x = 0$, the fundamental REE (8) and sunspot REEs (9) are locally stable for any $\phi_\pi, \phi_x \geq 0$ under any of the policy rules (3)–(5).*

The proof is shown in Appendix D. Since the perfect absence of such informed agents is implausible, it is practical to conclude that REEs are stable even under the interest-rate peg.

These results differ from the results in the literature (e.g., Hommes and Lustenhouwer, 2019a,b) in that the present results are obtained only by announcing the inflation target. Previous studies incorporate the inflation target such that agents fix all the parameters or expectations of the inflation rate and output gap at or around their steady-state values (the so-called *inflation and output gap targets*) (see also Gaspar et al., 2006; Orphanides and Williams, 2007; Ho et al., 2021; Lustenhouwer, 2021). This is as if the central bank announces not only the inflation target, but also the output gap target. While this assumption does simplify the analysis, their findings might stem from announcing the output gap target rather than the inflation target. Our analysis, by contrast, identifies the pure effect of announcing the inflation target.

We find that announcing the inflation target alone has a significant effect on expectations formation and macroeconomic stability. REEs are stable even if the nominal interest rate is pegged at the steady state. This result is obtained despite the assumption that the steady state of the output gap remains unobservable to agents. Hence, stable economies under the interest-rate peg are plausible in countries in which central banks announce inflation targets. The findings in the above related literature might be driven by the assumption of announcing the inflation target.

4.2 Stationary sunspot REEs

Combined with the indeterminacy condition (Proposition 1) and stability conditions (Proposition 4), the instability of stationary sunspot REEs (Proposition 3) is modified in the presence of inflation targeting.

Proposition 5 *When the inflation target $\pi = 0$ is announced, sunspot REEs (9) are stationary and locally stable under learning if and only if the stability conditions in Proposition 4 are satisfied in violation of the Taylor principle (10).*

Further, suppose the existence of informed agents considered in Corollary 1.

Corollary 2 *If there exists an agent who is also informed of the steady-state output gap as in Corollary 1, sunspot REEs (9) are stationary and locally stable under learning if and only if the Taylor principle (10) is violated.*

We find that inflation targeting relaxes the stability condition of stationary sunspot REEs as well. Figure 4 calibrates the region for determinacy and stability under inflation targeting. While stationary sunspot REEs are unstable without such targeting, they become stable under inflation targeting as long as the Taylor principle is violated. This means that inflation targeting makes room for the indeterminacy of the stable REE in violation of the Taylor principle.

4.3 Simulation

The stability under inflation targeting is illustrated using real-time simulations. Unless otherwise stated, we use $(\alpha, \kappa, \beta) = (1/1.45, 0.77, 0.99)$ in Lubik and Schorfheide (2004, Table 3) and assume that the central bank adopts the

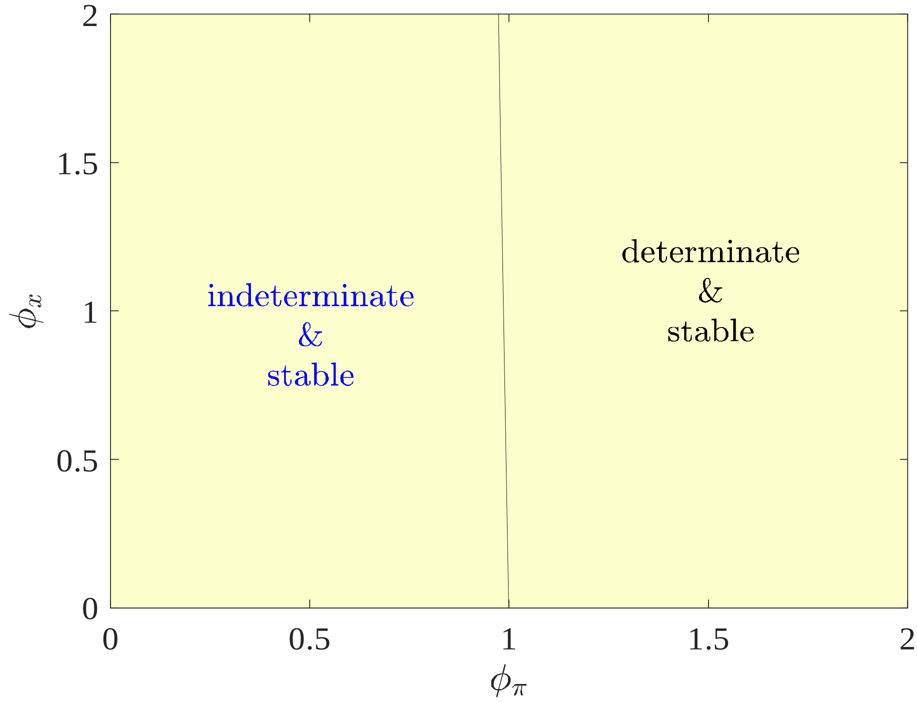

Figure 4: Region for determinacy and stability under inflation targeting with the contemporaneous rule (3) and $(\alpha, \kappa, \beta) = (1/1.45, 0.77, 0.99)$ in Lubik and Schorfheide (2004, Table 3).

contemporaneous rule (3). We set the policy parameters $(\phi_\pi, \phi_x) = (0, 0.1)$, which violate the Taylor principle and represent nearly an interest-rate peg at the steady state.

Figure 5 illustrates the instability of REEs in the absence of inflation targeting (Proposition 2). We simulate the updating of the parameter estimates (a, c, d) for sunspot REEs in recursive least-squares estimations and the corresponding paths for the temporary sunspot equilibrium (x_t, π_t). The simulations for the fundamental REE are omitted, as they are essentially the same as those for sunspot REEs, except d being fixed at zero. The initial values for the parameters and equilibria are set at the fixed points, each of which is indicated by the horizontal dashed line in each panel. The initial value for the arbitrary real constant s in the sunspot coefficient d is set to $s = 1$. A decreasing gain algorithm is used for the simulation.[3] The shock v_t follows $N(0, 1)$.

[3]Following Evans and McGough (2020), we use a decreasing gain sequence with $\gamma_t = t^{-0.8}$

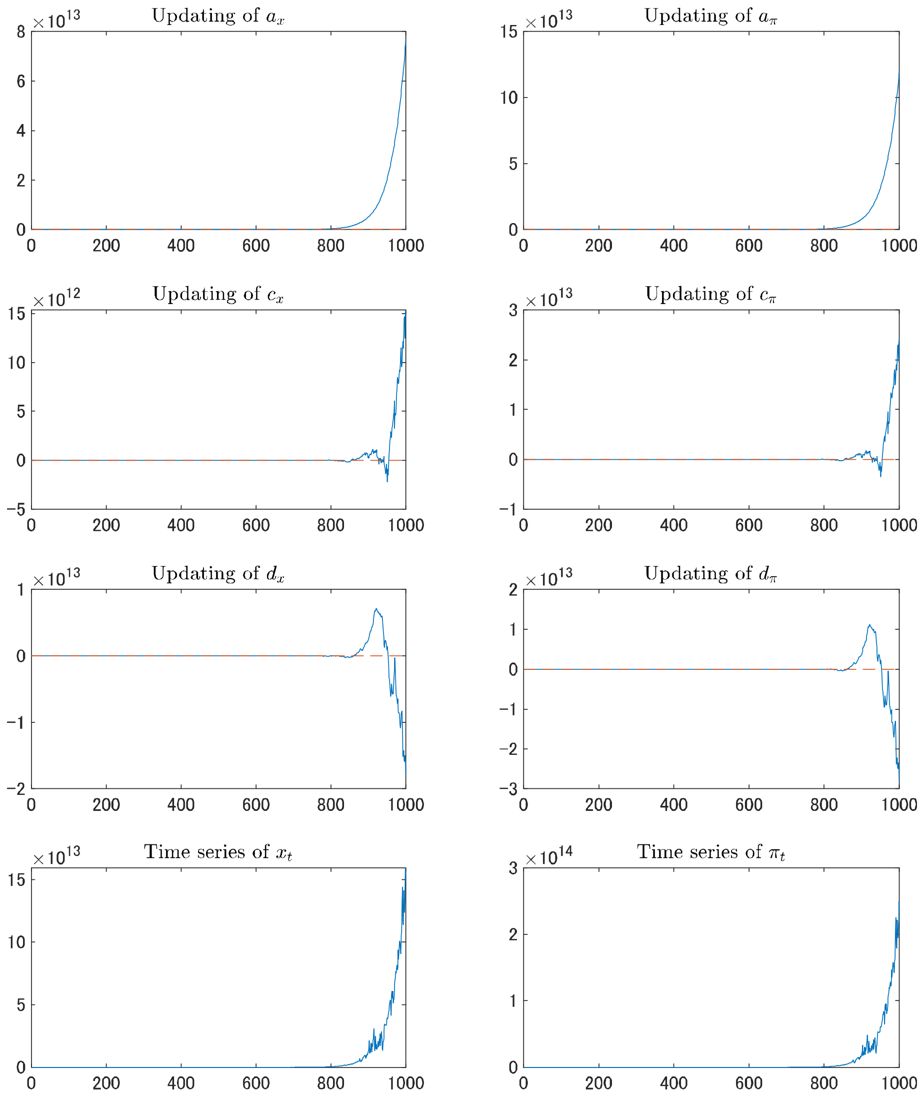

Figure 5: Simulation for parameters $a = (a_x, a_\pi)$, $c = (c_x, c_\pi)$, $d = (d_x, d_\pi)$ (upper panels) and endogenous variables (x, π) (bottom panels) in the absence of inflation targeting.

As clarified theoretically, the violation of the Taylor principle leads to the instability of REEs in the absence of inflation targeting. The constant terms $a = (a_x, a_\pi)'$ diverge from the fixed points to explode with the equilibrium paths. This explosion then destabilizes the updating of the coefficients c and d, leading to further economic fluctuations.

Figure 6 shows the corresponding simulations in the presence of inflation targeting, under which the parameter a_π is fixed at the steady state ($\pi = 0$). We see that inflation targeting stabilizes agents' expectations formation. The constant term parameter a_x converges to the fixed point even though the nominal interest rate is almost pegged at the steady state. Interestingly, announcing the inflation target helps estimate the steady-state output gap. This leads to the fast convergence of the other parameter estimates, making the economy stable around the steady state.

Similar results are obtained under the forward-looking rule (4) with the calibrated parameters $(\alpha, \kappa, \beta) = (0.164, 0.3, 0.99)$ (McCallum and Nelson, 1999) and under the forward-looking variant rule (5) with the parameters $(4, 0.075, 0.99)$ (Clarida et al., 2000).

4.4 Discussion

These results not only explain the stabilizing effect of inflation targeting on agents' expectations formation, but also provide plausible stability conditions. In the absence of inflation targeting, the fundamental REE is unstable under learning if the Taylor principle is violated. We find that under inflation targeting, this conventional result is modified to be consistent with the empirical evidence that several countries have been stable with nominal interest rates fixed near the zero lower bound over the last decade. This implies that NK models with inflation targeting can describe the dynamics of recent economies following the global financial crisis.

The plausibility of stationary sunspot REEs is also improved by considering inflation targeting. Without inflation targeting, stationary sunspot REEs are always unstable under the indeterminacy and stability conditions. This conventional wisdom contradicts the evidence that central banks have violated the Taylor principle during past non-fundamental fluctuations (e.g., Clarida et al., 2000; Lubik and Schorfheide, 2004; Taylor, 2007a,b). Our results resolve this

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rather than $\gamma_t = t^{-1}$ to increase the speed of convergence.

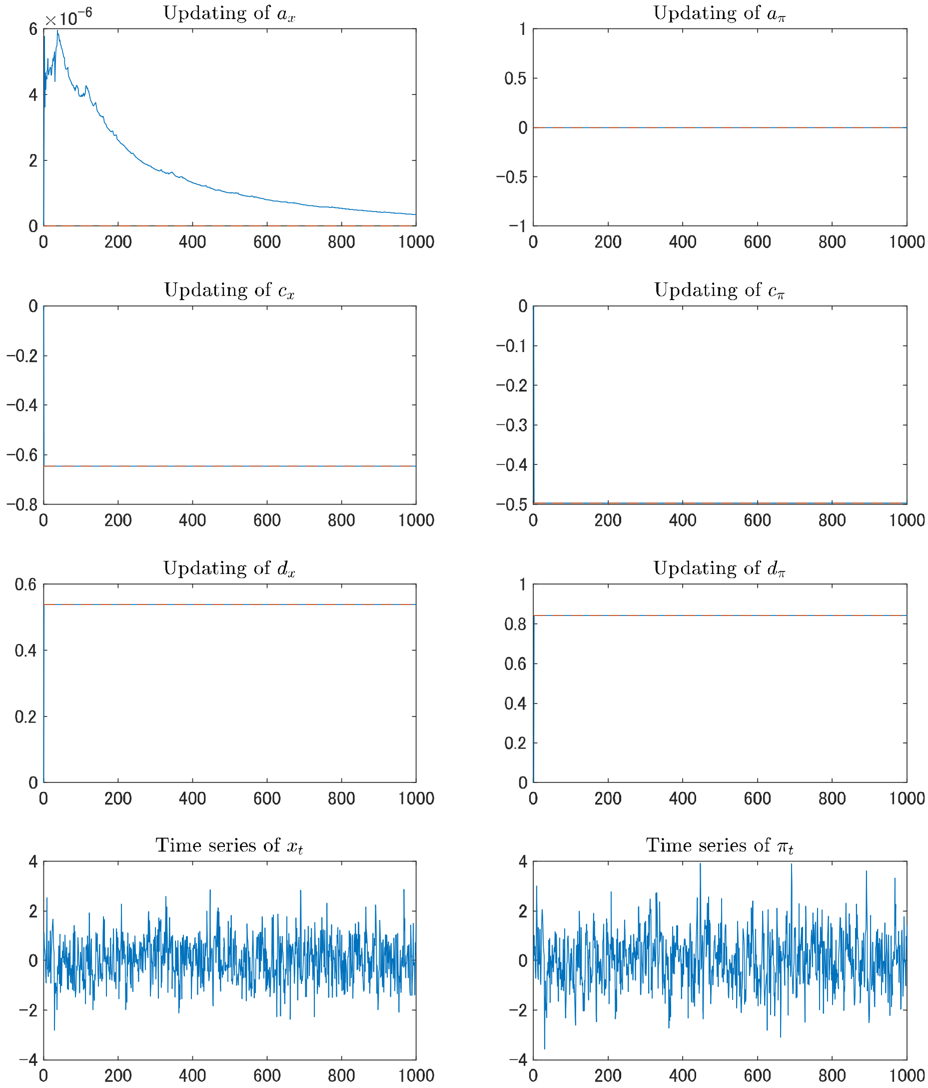

Figure 6: Simulation for parameters $a = (a_x, a_\pi)$, $c = (c_x, c_\pi)$, $d = (d_x, d_\pi)$ (upper panels) and endogenous variables (x, π) (bottom panels) in the presence of inflation targeting.

puzzle and suggest that sunspot REEs with inflation targeting may fit these empirical characteristics.

As a policy implication, these results reinforce the role of the Taylor principle for macroeconomic stability. The Taylor principle was originally emphasized to ensure the determinacy of the stationary equilibrium at the fundamental equilibrium (see Woodford, 2003). In this paper, we find that inflation targeting makes room for the indeterminacy of the stationary and stable equilibrium and that the Taylor principle is the sufficient and necessary condition for determinacy. Hence, the Taylor principle is important for the determinacy of the equilibrium not only under rational expectations, but also under realistic expectations, and the principle is more important in inflation-targeting countries.

5 Output gap target

According to our analysis, one might think that a similar effect could be attained by announcing the output gap target instead of the inflation target. This section examines whether the stabilizing effect can be reproduced by the output gap target or whether it is a unique characteristic of the inflation target.

Suppose that, instead of the inflation target, the steady-state output gap ($x = 0$) is announced. Then, agents fix the first element a_x in the constant term vector a in the PLMs at the steady state and estimate only a_π. Similar to in the previous analysis, the stability conditions for REEs are obtained as follows.

Proposition 6 *When the output gap target $x = 0$, instead of the inflation target $\pi = 0$, is announced, the fundamental REE (8) and sunspot REEs (9) are locally stable under learning if and only if*

1. *under the contemporaneous and forward-looking variant rules (3), (5),*

$$\kappa \left(\phi_\pi - 1\right) + \phi_x \left(1 - \beta\right) > -\frac{1-\beta}{\alpha}; \tag{14}$$

2. *under the forward-looking rule (4),*

$$\kappa \left(\phi_\pi - 1\right) > -\frac{1-\beta}{\alpha}.$$

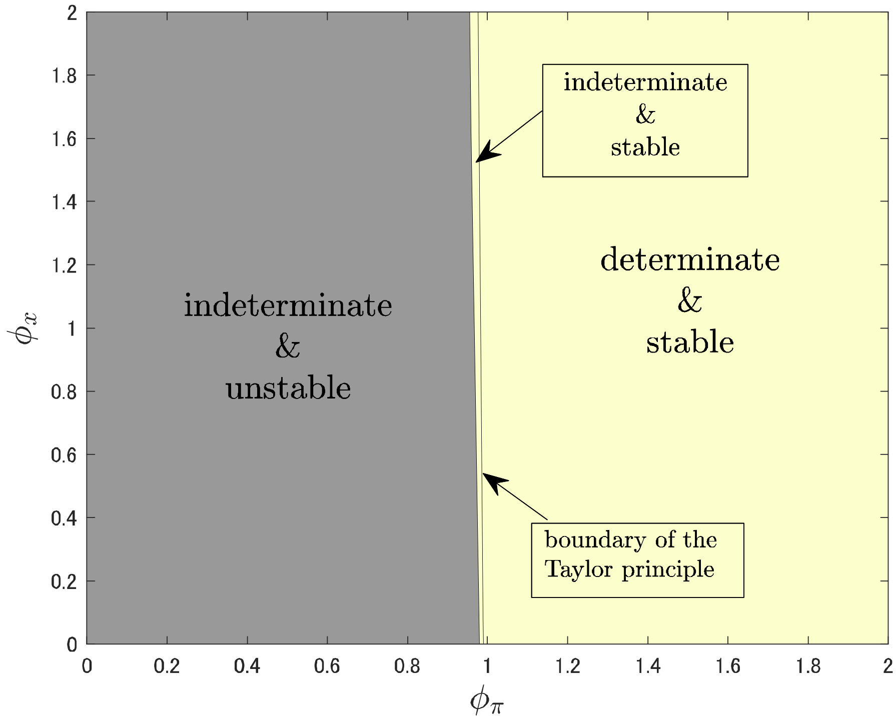

Figure 7: Region for determinacy and stability under output gap targeting with the contemporaneous rule (3) and $(\alpha, \kappa, \beta) = (1/1.45, 0.77, 0.99)$ in Lubik and Schorfheide (2004, Table 3).

The proof is presented in Appendix E.

We find that the stabilizing effect can be reproduced by the output gap target. Under positive feedback (7), the conditions in Proposition 6 are all less restrictive than the Taylor principle (10) (see also Appendix A). That is, announcing the steady-state output gap also relaxes the stability conditions under any type of policy rule. Similar to the case of announcing the inflation target, the fundamental REE and sunspot REEs can be stable in violation of the Taylor principle.

However, the effect of announcing the output gap target seems economically negligible. Figure 7 calibrates the region for stability under the output gap target. This target creates the region for stability in which the Taylor principle is violated, but this region is too narrow to include the calibrated parameters of the NK models. For example, suppose $(\phi_\pi, \phi_x) = (0.77, 0.17)$ estimated in Lubik and Schorfheide (2004, Table 3). Given $\phi_x = 0.17$, the stability condition

(14) is slightly relaxed from $\phi_\pi > 0.998$ (no target) to $\phi_\pi > 0.979$ (with the target). This effect is negligible for the estimated $\phi_\pi = 0.77$ to be included in this region.

Figure 8 illustrates this result by using a simulation with the current policy parameters $(\phi_\pi, \phi_x) = (0, 0.1)$. Similar to the case of no inflation targeting, the parameter a_π diverges from the fixed point to explode unboundedly. This leads to the explosion of the economy and fluctuations of the parameter estimates. A similar result is obtained even if the nominal interest rate rule becomes aggressive to inflation. Figure 9 shows the updating under the policy parameters $(\phi_\pi, \phi_x) = (0.9, 0.1)$. As long as the Taylor principle is violated, the parameter a_π does not stop diverging.

These results imply that the stabilizing effect is unique to announcing the inflation target. Announcing the output gap target has the same effect, but it seems economically negligible.

6 Imperfect credibility

Finally, this section considers the imperfect credibility of the central bank. The previous sections depended on the assumption that the central bank is perfectly credible to achieve the announced inflation target. In fact, our experience with unconventional monetary policies since the Great Recession has cast doubt on their credibility (see Bodenstein et al., 2012). Let us further reexamine the robustness of our results under imperfect credibility.

Suppose that some agents (proportion $\lambda \in [0, 1)$) believe the inflation target, while other agents $(1 - \lambda)$ do not and estimate the constant term a_π. The parameter λ is defined as the degree of the credibility of the central bank.

Proposition 7 *When the inflation target $\pi = 0$ is announced but trusted only by agents of proportion $\lambda \in [0, 1)$, the fundamental REE (8) and sunspot REEs (9) are locally stable under learning if and only if*

1. *under the contemporaneous rule (3),*

$$\kappa (\phi_\pi - 1) + \phi_x (1 - \beta) > -\frac{\lambda}{\alpha} (1 - \beta (1 - \lambda) + \alpha (\kappa + \beta \phi_x)); \qquad (15)$$

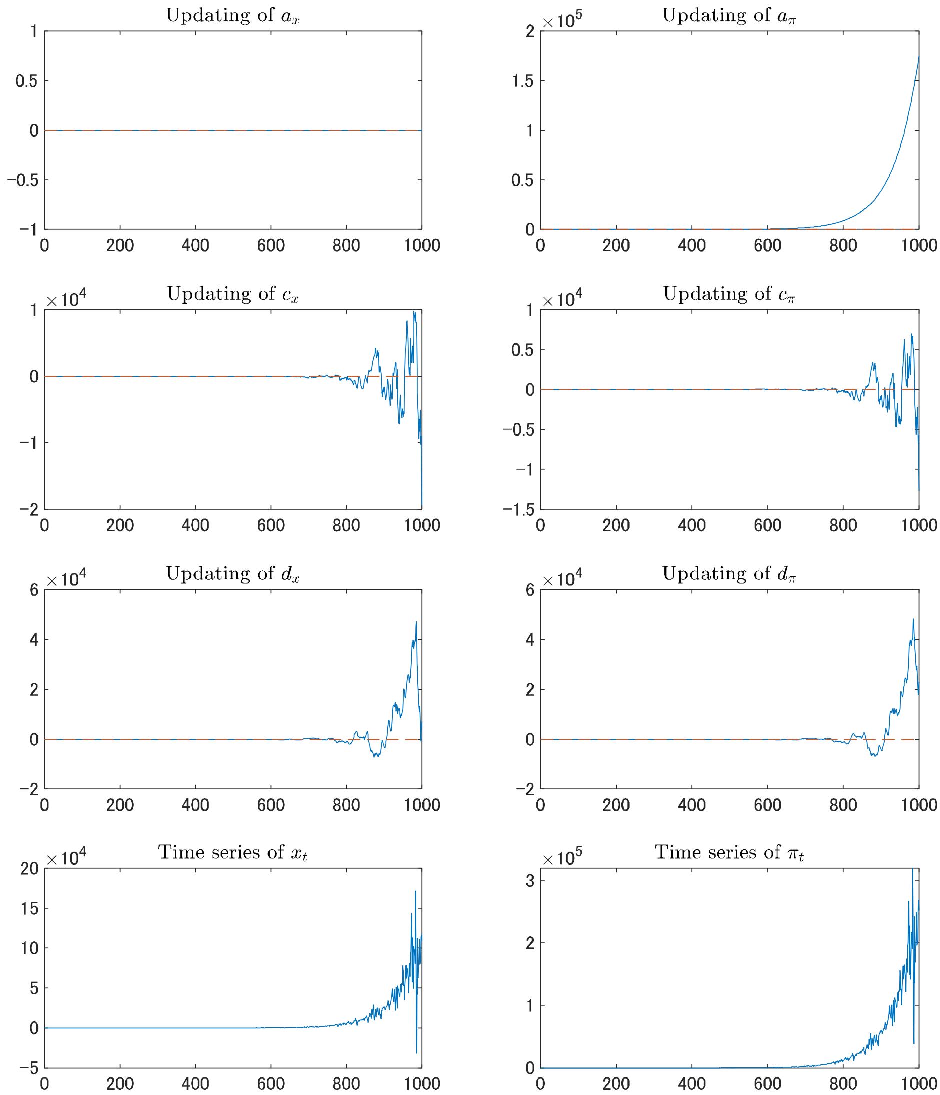

Figure 8: Simulation for parameters $a = (a_x, a_\pi)'$, $c = (c_x, c_\pi)'$, $d = (d_x, d_\pi)'$ (upper panels) and endogenous variables (x, π) (bottom panels) under output gap targeting.

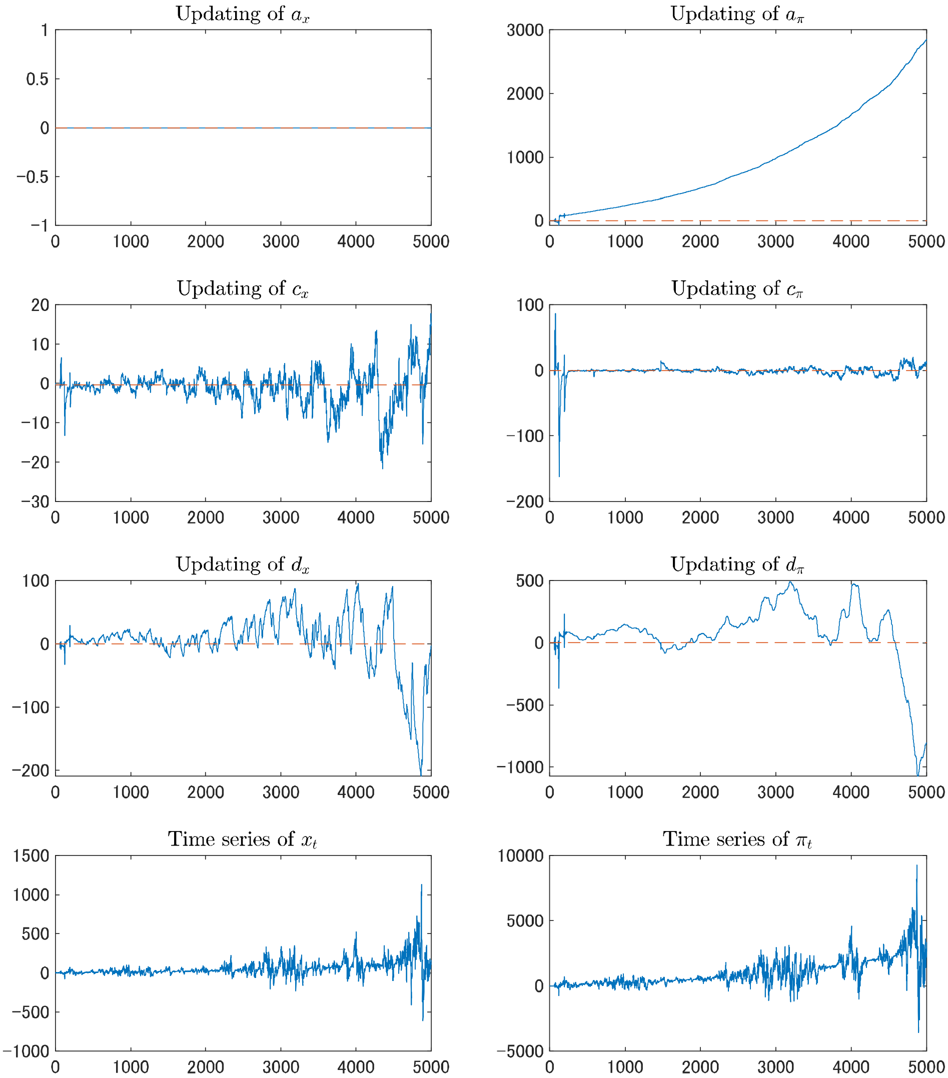

Figure 9: Simulation for parameters $a = (a_x, a_\pi)'$, $c = (c_x, c_\pi)'$, $d = (d_x, d_\pi)'$ (upper panels) and endogenous variables (x, π) (bottom panels) under output gap targeting with $(\phi_\pi, \phi_x) = (0.9, 0.1)$.

2. under the forward-looking rule (4),

$$\kappa\left(\phi_{\pi}-1\right)+\phi_{x}\left(1-\beta\right)>-\frac{\lambda}{\alpha\left(1-\lambda\right)}\left(1-\beta\left(1-\lambda\right)\left(1-\alpha\phi_{x}\right)\right); \quad (16)$$

3. under the forward-looking variant rule (5),

$$\kappa\left(\phi_{\pi}-1\right)+\phi_{x}\left(1-\beta\right)>-\frac{\lambda}{\alpha\left(1-\lambda\right)}\left(1-\beta\left(1-\lambda\right)+\alpha\phi_{x}\right). \quad (17)$$

The proof is presented in Appendix F.

Consistent with the literature, the stabilizing effect of inflation targeting is reduced by imperfect credibility. The right-hand sides of the stability conditions (15)–(17) are all non-positive and decreasing in λ under positive feedback (7) (see Appendix A). Hence, the credibility of the central bank is important for the effectiveness of inflation targeting.

In practice, even if the credibility of modern central banks is imperfect, the stabilizing effect of targeting can hold like under perfect credibility.

Corollary 3 *For any of the policy rules (3)–(5) with the inflation target* $\pi=0$*, the fundamental REE (8) and sunspot REEs (9) are locally stable for any* $\phi_{\pi},\phi_{x}\geq 0$ *if and only if*

$$\lambda>\frac{1}{2\beta}\left(\sqrt{\left(1-\beta+\alpha\kappa\right)^{2}+4\alpha\kappa\beta}-\left(1-\beta+\alpha\kappa\right)\right).$$

The proof is trivial according to Proposition 7 with $\phi_{\pi}=\phi_{x}=0$.

This result is illustrated in Figure 10, which calibrates the boundary of the stability region for each degree of credibility λ. The right-hand side of the boundary is the region for stability. The value on each line represents the degree of credibility that determines the boundary. The boundary shifts to the lower left with an increase in the degree of credibility and reaches the origin when $\lambda=0.508$. That is, if the central bank is credible for at least more than a half of agents, the stability condition is the same as that under perfect credibility.

This degree of credibility is highly plausible for modern central banks. van der Cruijsen, Jansen, and de Haan (2015) find that more than 50% of Dutch households know that the main objective of the ECB is to achieve the

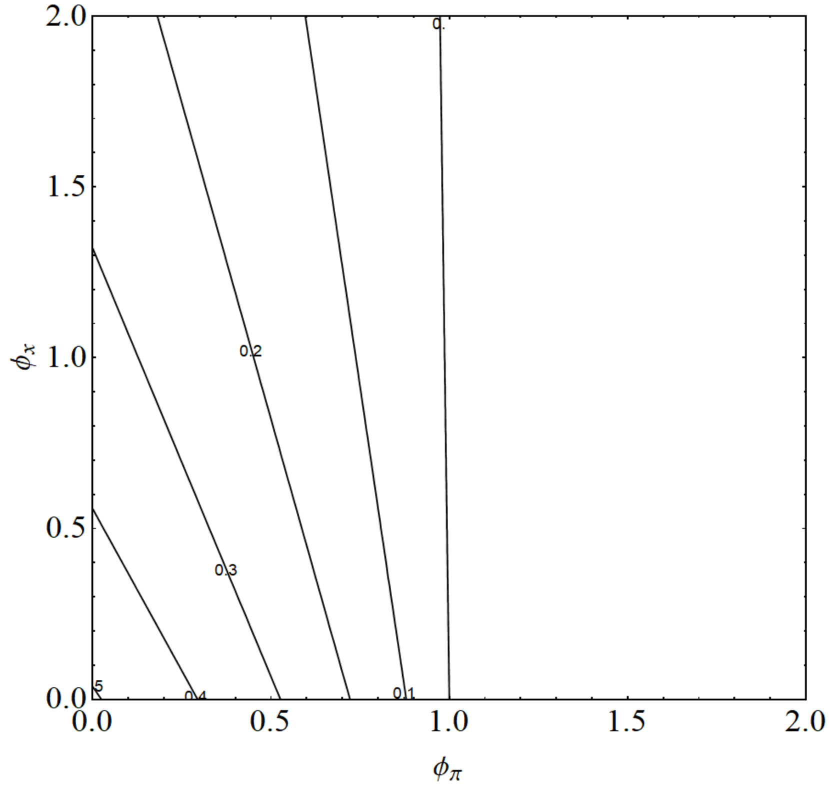

Figure 10: Boundary of the stability condition (15) with respect to the degree of credibility λ, under inflation targeting with the contemporaneous rule (3) and $(\alpha, \kappa, \beta) = (1/1.45, 0.77, 0.99)$ in Lubik and Schorfheide (2004, Table 3). The value on each boundary line represents the degree of credibility.

2% inflation rate. Using US and Swedish data after the 2008 global financial crisis, Bodenstein et al. (2012) estimate that with probability 0.5, the Federal Reserve and Riksbank are believed to keep their announced policy paths until the next quarter. Cole and Martínez-García (2021) find that 80% of professional forecasters believe the Federal Reserve's forward guidance announcements. In this sense, the stabilizing effect under perfect credibility is plausible even under the imperfect credibility of modern central banks.

These results further imply that central banks' efforts to improve their credibility might have supported recent stable economies near the zero lower bound. Over the last decade, central banks have introduced unconventional monetary policies to stimulate aggregate demand on the zero lower bound (see García-Schmidt and Woodford, 2019). These policies might have raised central banks' credibility as well. If this is the case, unconventional policies might have contributed to the stable period by raising the effect of inflation targeting.

While the contemporaneous rule (3) is adopted here, similar results are obtained under other policy rules. The boundary reaches the origin when $\lambda = 0.195$ under the forward-looking rule (4) with the calibrated parameters $(\alpha, \kappa, \beta) = (0.164, 0.3, 0.99)$ (McCallum and Nelson, 1999) and $\lambda = 0.416$ under the forward-looking variant rule (5) with the parameters $(4, 0.075, 0.99)$ (Clarida et al., 2000). The stabilizing effect might thus be more robust to imperfect credibility when the nominal interest rate responds to expected inflation.

7 Conclusion

This paper has investigated the impact of inflation targeting on agents' expectations formation and macroeconomic stability. If agents recognize the steady-state inflation rate under inflation targeting, they may fix the parameters or expectations of the steady-state inflation rate at or around the target. This simplifies agents' expectations formation.

We find that inflation targeting not only anchors agents' long-run inflation expectations, but also stabilizes their expectations formation. Specifically, it significantly relaxes the conditions for the expectational stability of fundamental and sunspot equilibria, so that they are stable even if the nominal interest rate is pegged at the steady state. Hence, the Taylor principle becomes unnecessary for stabilizing the fundamental equilibrium, but necessary for preventing stable sunspot equilibria. In calibrated NK models, this stabilizing effect is unique to inflation targeting and robust under the calibrated credibility of the central bank. These results hold regardless of the type of nominal interest rate rule.

These findings are consistent with the empirical evidence. The presence of inflation targeting may describe expectations formation in economies staying near the zero lower bound or experiencing non-fundamental fluctuations. Unconventional monetary policies might have contributed to the recent stable period by enhancing the effect of inflation targeting. In this situation, the

Taylor principle is important for stabilizing fundamental business cycles and preventing non-fundamental cycles.

Future work should examine the stability of a *liquidity trap* in the presence of inflation targeting. This issue has attracted great attention since central banks began targeting low inflation rates with Taylor-type monetary policy rules. Benhabib et al. (2001) first demonstrate that Taylor-type monetary policy rules create the liquidity trap steady state in which the nominal interest rate is fixed at the zero lower bound. Aruoba et al. (2018) find that the Japanese economy shifted into this steady state in the late 1990s and remained there until the end of their sample (the mid-2010s). However, McCallum (2002) and Evans and Honkapohja (2005) show that the liquidity trap steady state is locally unstable under learning. While the present paper did not discuss the liquidity trap, we have found that inflation targeting can make equilibria stable under the interest-rate peg at the steady state. If the same mechanism works for the liquidity trap steady state, our mechanism could explain why the economy stays in the trap even if the central bank does not target it.

Appendix

A Positive feedback restrictions

Under the contemporaneous rule (3),

$$B \equiv \begin{bmatrix} 1+\alpha\phi_x & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix},$$

which provides

$$\begin{aligned} \det(B) &= \frac{\beta}{\alpha\phi_x+\alpha\kappa\phi_\pi+1} > 0, \\ \operatorname{tr}(B) &= \frac{\beta+\alpha\kappa+\alpha\beta\phi_x+1}{\alpha\phi_x+\alpha\kappa\phi_\pi+1} > 0. \end{aligned}$$

The model always has positive feedback.

Under the forward-looking rule (4),

$$B \equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1-\alpha\phi_x & -\alpha(\phi_\pi-1) \\ 0 & \beta \end{bmatrix},$$

which provides

$$\begin{aligned}\det(B) &= \beta(1-\alpha\phi_x),\\ \operatorname{tr}(B) &= 1+\beta(1-\alpha\phi_x)-\alpha(\kappa(\phi_\pi-1)+\phi_x(1-\beta)).\end{aligned}$$

$\det(B)>0$ and $\operatorname{tr}(B)>0$ if and only if

$$\begin{aligned}\phi_x &< \frac{1}{\alpha}, \qquad \text{(A.1)}\\ \kappa(\phi_\pi-1)+\phi_x(1-\beta) &< \frac{1+\beta(1-\alpha\phi_x)}{\alpha}.\end{aligned}$$

Under the forward-looking variant rule (5),

$$B \equiv \begin{bmatrix} 1+\alpha\phi_x & 0 \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -\alpha(\phi_\pi-1) \\ 0 & \beta \end{bmatrix},$$

which provides

$$\begin{aligned}\det(B) &= \frac{\beta}{1+\alpha\phi_x}>0,\\ \operatorname{tr}(B) &= \frac{1}{1+\alpha\phi_x}(1+\beta+\alpha\phi_x-\alpha(\kappa(\phi_\pi-1)+\phi_x(1-\beta))).\end{aligned}$$

$\det(B)>0$ and $\operatorname{tr}(B)>0$ if and only if

$$\kappa(\phi_\pi-1)+\phi_x(1-\beta)<\frac{1+\beta+\alpha\phi_x}{\alpha}.$$

B Proof of Proposition 1

Following Woodford (2003), both the eigenvalues of matrix B are inside the unit circle and the REE is determinate if and only if

$$\begin{aligned}|\det(B)| &< 1, \qquad \text{(B.1)}\\ |\operatorname{tr}(B)| &< 1+\det(B). \qquad \text{(B.2)}\end{aligned}$$

The non-generic boundary case ($|\det(B)|=1$ or $|\operatorname{tr}(B)|=1+\det(B)$) is omitted from our analysis.

In the case of the contemporaneous rule (3), Eq. (B.1) is always satisfied and Eq. (B.2) provides Eq. (10). If Eq. (10) is violated, the REE is indeterminate and the eigenvalues of matrix B are all real as

$$\frac{1}{2(\alpha\phi_x+\alpha\kappa\phi_\pi+1)} \times\left((\alpha\kappa+\alpha\beta\phi_x+1+\beta)\pm\sqrt{(\alpha\kappa+\alpha\beta\phi_x+1-\beta)^2-4\alpha\beta(\kappa(\phi_\pi-1)+\phi_x(1-\beta))}\right).$$

In the case of the forward-looking rule (4), Eq. (B.1) is satisfied and Eq. (B.2) yields Eq. (10) under positive feedback (7). If Eq. (10) is violated, the REE is indeterminate and the eigenvalues of matrix B are all real as

$$\frac{1}{2}\left(1+\beta-\alpha\kappa\left(\phi_\pi-1\right)-\alpha\phi_x\right)$$
$$\pm\frac{1}{2}\sqrt{\left(1-\beta+\alpha\kappa\left(\phi_\pi-1\right)+\alpha\phi_x\right)^2-4\alpha\left(\kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right)\right)}.$$

In the case of the forward-looking variant rule (5), Eq. (B.1) is always satisfied, and Eq. (B.2) provides Eq. (10) under positive feedback (7). If Eq. (10) is violated, the REE is indeterminate and the eigenvalues of matrix B are all real as

$$\frac{1}{2\alpha\phi_x+2}\left(\left(1+\beta+\alpha\phi_x\right)-\alpha\left(\kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right)\right)\right)$$
$$\pm\frac{1}{2\alpha\phi_x+2}\sqrt{\left(\left(1+\beta+\alpha\phi_x\right)-\alpha\left(\kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right)\right)\right)^2-4\beta\left(1+\alpha\phi_x\right)}.$$

Therefore, regardless of the type of policy rule, Eq. (10) is the sufficient and necessary condition for the determinacy of the equilibrium. If and only if Eq. (10) is violated, there exist sunspot REEs of the form (9). The proof is complete.

C Proof of Proposition 2

If and only if the Jacobian, $D\left(Ba-a\right)=B-I_2$, has all eigenvalues with negative real parts, then

$$\det\left(B-I_2\right)>0, \quad \operatorname{tr}\left(B-I_2\right)<0.$$

This provides

$$\begin{aligned}\kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right) &> 0,\\ \kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right) &> -\frac{\left(1-\beta\right)+\alpha\left(\kappa+\beta\phi_x\right)}{2\alpha}\end{aligned}$$

under the contemporaneous rule (3);

$$\begin{aligned}\kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right) &> 0,\\ \kappa\left(\phi_\pi-1\right)+\phi_x\left(1-\beta\right) &> -\frac{\left(1-\beta\right)+\alpha\beta\phi_x}{\alpha}\end{aligned}$$

under the forward-looking rule (4);

$$\begin{aligned}\kappa(\phi_\pi - 1) + \phi_x(1-\beta) &> 0, \\ \kappa(\phi_\pi - 1) + \phi_x(1-\beta) &> -\frac{(1-\beta)+\alpha\phi_x}{\alpha}\end{aligned}$$

under the forward-looking variant rule (5).

That is, under any of the policy rules, the Taylor principle (10) is the sufficient and necessary condition for stability. The proof is complete.

D Proof of Corollary 1

Suppose that the proportion $\omega \in [0,1]$ of agents are informed of the steady-state output gap ($x = 0$), whereas the other agents are not. Then, informed agents fix the constant term a_x in vector a at the steady-state output gap, while uninformed agents estimate a_x. When the fundamental REE is learned, informed agents form the PLM $y_t = (0,0)' + cv_t + e_t$ and the forecast $E^*_{it}y_{t+1} = 0$, while uninformed agents form the PLM $y_t = (a_x, 0)' + cv_t + e_t$ and the forecast $E^*_{ut}y_{t+1} = (a_x, 0)'$. E^*_{it} and E^*_{ut} are the operators of the expectations of informed and uninformed agents, respectively. Hence, the average of the forecasts of all agents is $E^*_t y_{t+1} = \omega E^*_{it}y_{t+1} + (1-\omega) E^*_{ut}y_{t+1} = (1-\omega)(a_x, 0)'$, and the ALM is $y_t = (1-\omega) B (a_x, 0)' + Cv_t$.

As v_t follows an iid process, the parameter a_x is estimated as the sample mean of x_t, which is shown by the following recursive algorithm:

$$a_{xt} = a_{x,t-1} + t^{-1}\left((1-\omega) b_{11} a_{x,t-1} - a_{x,t-1}\right),$$

where a_{xt} is the estimate of a_x at time t. Hence, the convergence of a_x is determined by the following ODE:

$$\frac{\mathrm{d}a_x}{\mathrm{d}\tau} = (1-\omega) b_{11} a_x - a_x.$$

The same ODE is obtained when sunspot REEs are learned.

This ODE is locally stable if and only if the eigenvalue of the Jacobian is negative:

$$D\left((1-\omega) b_{11} a_x - a_x\right) = (1-\omega) b_{11} - 1 < 0.$$

If $\omega > 0$, then this condition holds for any $\phi_\pi, \phi_x \geq 0$ under any of the policy rules as $b_{11} = \frac{1}{\alpha\phi_x + \alpha\kappa\phi_\pi + 1}$ under the contemporaneous rule (3), $b_{11} = 1 - \alpha\phi_x$ under the forward-looking rule (4), and $b_{11} = \frac{1}{\alpha\phi_x + 1}$ under the forward-looking variant rule (5). The proof is complete.

E Proof of Proposition 6

Suppose that the output gap target, instead of the inflation target, is announced. Then, agents fix the constant term a_x in vector a at the output gap target. When the fundamental REE is learned, agents form the PLM $y_t = (0, a_\pi)' + cv_t$ and the forecast $E_t^* y_{t+1} = (0, a_\pi)'$. Then, the ALM is determined as $y_t = B(0, a_\pi)' + Cv_t$.

As v_t follows an iid process, the parameter a_π is estimated by the sample mean of π_t, which is represented by a recursive algorithm:

$$a_{\pi t} = a_{\pi,t-1} + t^{-1}\left(b_{22}a_{\pi,t-1} - a_{\pi,t-1}\right),$$

where $a_{\pi t}$ is the estimate of a_π at time t and b_{22} is the element in the second row and second column of matrix B. Hence, the convergence of a_π is governed by the following ODE:

$$\frac{\mathrm{d}a_\pi}{\mathrm{d}\tau} = b_{22}a_\pi - a_\pi.$$

The same ODE is obtained when sunspot REEs are learned.

This ODE is locally stable if and only if its Jacobian is negative:

$$D\left(b_{22}a_\pi - a_\pi\right) = b_{22} - 1 < 0.$$

This provides $\kappa(\phi_\pi - 1) + \phi_x(1-\beta) > -\frac{1-\beta}{\alpha}$ under the contemporaneous rule (3) as $b_{22} = \frac{\beta+\alpha\kappa+\alpha\beta\phi_x}{\alpha\phi_x+\alpha\kappa\phi_\pi+1}$, $\phi_\pi > 1 - \frac{1-\beta}{\alpha\kappa}$ under the forward-looking rule (4) as $b_{22} = \beta - \alpha\kappa(\phi_\pi - 1)$, and $\kappa(\phi_\pi - 1) + \phi_x(1-\beta) > -\frac{1-\beta}{\alpha}$ under the forward-looking variant rule (5) as $b_{22} = \frac{\beta+\alpha\kappa-\alpha\kappa\phi_\pi+\alpha\beta\phi_x}{\alpha\phi_x+1}$. The proof is complete.

F Proof of Proposition 7

Suppose that agents of proportion $\lambda \in [0,1)$ (type 1) believe the inflation target, whereas the other agents (type 2) do not. Then, type 1 fix the constant term a_π at the inflation target, while type 2 estimate a_π. When the fundamental REE is learned, both types form the PLMs $y_t = a_1 + c_1v_t + e_{1t}$ and $y_t = a_2 + c_2v_t + e_{2t}$ where $a_1 \equiv (a_{1x}, 0)'$ and $a_2 \equiv (a_{2x}, a_{2\pi})'$, and they form the forecasts $E_{1t}^* y_{t+1} = a_1$ and $E_{2t}^* y_{t+1} = a_2$. E_{1t}^* and E_{2t}^* are the operators of both types' expectations. Hence, the average forecast is $E^* y_{t+1} = \lambda E_{1t}^* y_{t+1} + (1-\lambda) E_{2t}^* y_{t+1} = (\lambda a_{1x} + (1-\lambda) a_{2x}, (1-\lambda) a_{2\pi})'$, and the ALM is $y_t = B(\lambda a_{1x} + (1-\lambda) a_{2x}, (1-\lambda) a_{2\pi})' + Cv_t$.

The local dynamics of (a_1, c_1) are governed by the stability of the ODE for a_1, $\frac{\mathrm{d}a_1}{\mathrm{d}\tau} = B\left(\lambda a_{1x} + (1-\lambda)\, a_{2x}, (1-\lambda)\, a_{2\pi}\right)' - (a_{1x}, 0)'$, which is reduced to the ODE for a_{1x}, $\frac{\mathrm{d}a_{1x}}{\mathrm{d}\tau} = (\lambda b_{11} - 1)\, a_{1x} + (1-\lambda)\,(b_{11} a_{2x} + b_{12} a_{2\pi})$. The ODE is locally stable if and only if its Jacobian has an eigenvalue with a negative real part:

$$D\left((\lambda b_{11} - 1)\, a_{1x}\right) = \lambda b_{11} - 1 < 0. \tag{F.1}$$

The local dynamics of (a_2, c_2) are governed by the stability of the ODE for a_2, $\frac{\mathrm{d}a_2}{\mathrm{d}\tau} = B\left(\lambda a_{1x}, 0\right)' + (1-\lambda)\, B a_2 - a_2$. The ODE is locally stable if and only if its Jacobian has all eigenvalues with negative real parts:

$$D\left((1-\lambda)\, B a_2 - a_2\right) = (1-\lambda)\, B - I_2. \tag{F.2}$$

The same ODEs are obtained when sunspot REEs are learned.

Under the contemporaneous rule (3), Eq. (F.1) is always satisfied and Eq. (F.2) represents the stability condition of REEs:

$$\kappa\,(\phi_\pi - 1) + \phi_x\,(1-\beta) > -\frac{\lambda}{\alpha}\left(1 - \beta\,(1-\lambda) + \alpha\,(\kappa + \beta\phi_x)\right).$$

Under the forward-looking rule (4), Eq. (F.1) is always satisfied and Eq. (F.2) represents the stability condition of REEs under the positive feedback restriction (A.1):

$$\kappa\,(\phi_\pi - 1) + \phi_x\,(1-\beta) > -\frac{\lambda}{\alpha\,(1-\lambda)}\left(1 - \beta\,(1-\lambda)\,(1 - \alpha\phi_x)\right).$$

Under the forward-looking variant rule (5), Eq. (F.1) is always satisfied and Eq. (F.2) represents the stability condition of REEs:

$$\kappa\,(\phi_\pi - 1) + \phi_x\,(1-\beta) > -\frac{\lambda}{\alpha\,(1-\lambda)}\left(1 - \beta\,(1-\lambda) + \alpha\phi_x\right).$$

The proof is complete.

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