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Satohide MIZUTANI

RCSS

文部科学省私立大学学術フロンティア推進拠点
関西大学ソシオネットワーク戦略研究センター

Research Center of Socionetwork Strategies,
The Institute of Economic and Political Studies,
Kansai University
Suita, Osaka 564-8680 Japan
URL : http://www.rcss.kansai-u.ac.jp/
http://www.socionetwork.jp/
e-mail : keiseiken@jm.kansai-u.ac.jp
tel. 06-6368-1177
fax. 06-6330-3304

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Computational Models for Changes of Sentiments in Complete Connections Based on Heider's Balance Theory [1]

Satohide MIZUTANI

Research Center of Socionetwork Strategies, Kansai University[2]

Abstract

This study focuses on human relations as one of the factors determining how sentiments, positive or negative, toward persons, entities or issues change in the process of time. Changes of the sentiments toward persons in human relations have been simulated using computational models with a dyad rule and/or triad rules based on *pox* model which Heider (1946, 1958) proposed. In this paper, the computational models were extended to include complete connections between three or more persons and sentiments toward entities or issues, and the author showed behavior of the models at which sentiments were represented as directed relations with signed values under the conditions as follows. First of all, the changes of all relations are synchronously revised by triad rule or both dyad rule and triad rule. Secondly, all relations recognized by one person are equal to the relations recognized by the others. At the last, every relation doesn't change beyond the stable state. It was confirmed that all patterns converged, all relations became symmetry and all patterns became balanced states, when every relation in each individual cognitive representation turned into the balanced state, except for the triad rule at which some directed relations never have interactions with the others. The results are partially consistent with results of investigative studies. In addition, average convergence time was shown at each triad rule and it was also shown that convergence time increased more gently as number of persons increased. Furthermore, it has to be verified if the models are sufficient to explain human being's relations.

Keywords: sentiment, positive, negative, liking-disliking, simulation, Heider's *pox* model, balanced state, asymmetry, convergence, complete connections

1. Introduction

Human beings have positive sentiments and negative sentiments toward persons, entities or issues. The sentiments change dynamically, influenced by ones toward others. In this paper, changes of sentiments such as liking-disliking which is important at human relations are focused on. The changes have been simulated using computational models based on *pox* model which Heider (1946,1958) proposed, as described in section 1.2. The human relations are represented as directed relations with signed values in graph theory. It has been confirmed what patterns of directed relations

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[1] This paper is based on the report (Mizutani, 2004) presented by the author at 28th International Congress of Psychology in Beijing, China on 13 Aug, 2004. The author appreciates a comment for the chairman Erping Wang in Institute of Psychology, Chinese Academy of Sciences in the oral session.

[2] Research Assistant at Research Center of Socionetwork Strategies, Kansai University, Osaka, Japan.

with signed values converge, what patterns become balanced states and what relations become symmetry using the models in the case of three persons (Mizutani, Kosugi, & Fujisawa, 2002; Fujisawa, Mizutani, & Kosugi, 2003). In this paper, it will be shown how convergence time is related to each change's rule and the number of persons, and it will be emphasized that it has to be verified if the models are sufficient to explain human being's relations.

1.1 Changes of sentiments in human relations

This study focuses on human relations as one of a variety of factors which changes of sentiments such as liking and disliking in human beings consist of. Human relations change in the process of time, namely, the relations are not static but dynamic. It can be considered that changes of sentiments in human relations are composed of two factors described bellow.

A change of one person's sentiment toward another person will be focused on. First of all, we can consider that the change of one person's sentiment is influenced by another's sentiment toward one person (Aronson & Linder, 1956, etc). That is called dyad rule in this paper. It was insisted that the human relations might become symmetry, even if the initial relations had been asymmetric (Heider, 1958).

Secondly, the change of one person's sentiment is influenced by sentiments between another person and the other and sentiments between one person and the other. In addition, the changes of sentiments are influenced by one person's sentiments toward an entity or an issue and another person's sentiments toward the entity or the issue. It was insisted that a pattern of the relations became a pattern with a balanced state in the relations between three persons or in the relations

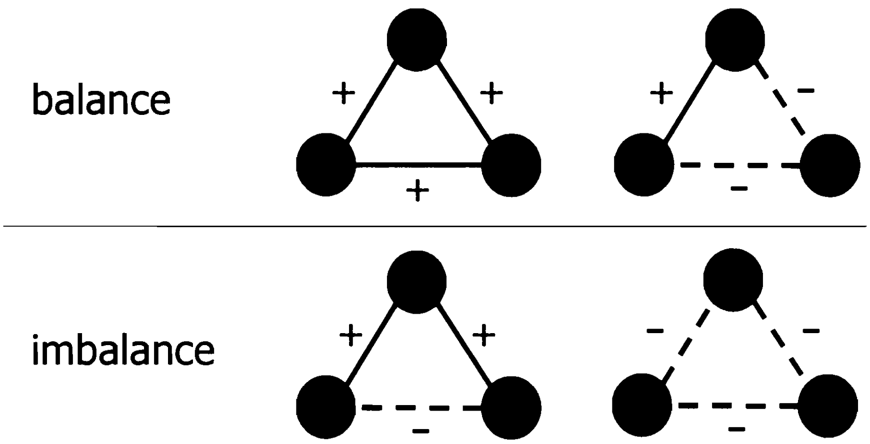

Figure 1: Three relations are focused on in these figures, in which each circle is a person, each solid line is a positive relation such as liking, and each dashed line is a negative relation such as disliking. Each state at the top of the figures is a balanced state, and each state at the bottom of the figures is an imbalanced state.

between two persons and the persons' relations toward an entity or an issue (Heider, 1958). Those are called triad rule. A state of a pattern of the relations is a balanced state, if the number of negative relations is even or zero at three relations. A state of a pattern of the relations is an imbalanced state, if the number of negative relations is odd at three relations. These are called Heider's balance model or *pox* model. Many sociological psychologists had conducted experimental or investigative studies, and insisted that the results of them supported the model partially (Jordan, 1953, Kogan & Tagiuri, 1958, etc).

When we consider three or more persons, the change of one person's sentiment is influenced by a combination of sentiments among another person and others and sentiments among one person and others. Cartwright & Harary (1956) described about directed relations with signed values, the state of balance in three or more persons, and the degrees of balances consisting of triads and cycles of four, five and more number of persons.

1.2 Researches for the dynamical approach

There are studies for dynamical changes of positive or negative sentiments at human relations as follow. Hunter (1978) focused on how each pattern with three or more relations turned into other patterns such as cliques and so on by differential equations. Kimura, Fujisawa, & Amemiya (1990), Fujisawa, Amemiya, & Kimura (1991) and Amemiya, Kimura, & Fujisawa (1993) described about changes of directed relations with signed values, showing a variety of examples. They had also simulated relations between two persons, etc and showed the results.

Mizutani, Kosugi, & Fujisawa (2002) constructed models about changes of sentiments liking-disliking between three persons, based on Heider's model and they showed about convergences, symmetry of relations, balanced states, initial patterns which an isolated person occurs from and influences by a dyad rule and a triad rule, using synchronous simulations, one of the models described bellow. Fujisawa, Mizutani, & Kosugi (2003) confirmed the number of iterations needed for convergence, symmetry of relations, balanced states and what patterns change particular patterns, using asynchronous simulations with three different triad rules and signs without values. Wang & Thorngate (2003) simulated the model of undirected relations among persons based on the model, and Nomura (2004) simulated the model of directed relations among persons based on Heider's model by a finite Markov chain.

In this paper, directed relations with signed values between three or more persons are treated, the results simulated with four different rules will be shown, and it will be shown how convergence time is related to each triad rule and the number of persons.

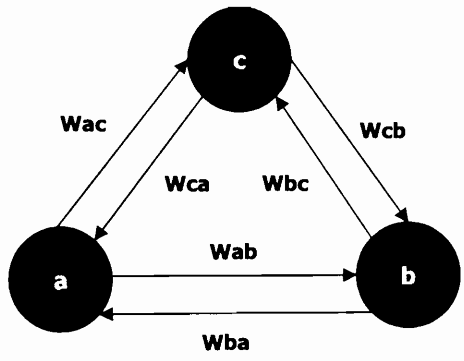

Figure 2: Each person named as "a", "b" and "c" is represented by a gray circle. There are six possible directed relations for sentiments in three persons.

2. Constructions of models

Some models in which changes of liking-disliking toward persons occur have been constructed from particular factors of the human relations in previous studies. The models and additionally some models will be shown bellow. One person "a" will be focused on as incidents at that time.

In this paper, it is considered that a change of a sentiment such as liking and disliking from one person to another person consists of two kinds of factors, a dyad rule and triad rules, in human relations between three or more persons, as well as prior study (Mizutani, Kosugi, & Fujisawa, 2002). Here, the character "W" is a notation for a directed relation of sentiment. Two persons are marked beside "W", such as "Wab", which is sentiment from a person "a" to another person "b".

2.1 Four triad rules

When directed relations between three persons are considered by Heider's *pox* model, four triad rules can be derived. As one of four triad rules, it can be considered that Wab is influenced by Wac and Wbc. This rule is called rule 1 in this paper. As the other triad rules, it can be considered that there are rule 2 at which Wab is influenced by Wac and Wcb, rule 3 at which Wab is influenced by Wca and Wcb and rule 4 at which Wab is influenced by Wbc and Wca.

Instead of a person, it is possible to regard a node c connected with a and b as an object, an event, an issue or a group of them, when a triad rule which Wab is influenced by is rule 1, or if they were anthropomorphic, rule 2, rule 3 or rule 4 can be adopted as a triad rule in individual cognitive representations at persons.

2.2 Variables and the values

The variable W_{ij} indicates a weight of a directed relation of a sentiment liking-disliking from i to j, where i and j are one person and another person, respectively. In this study, it is assumed that human

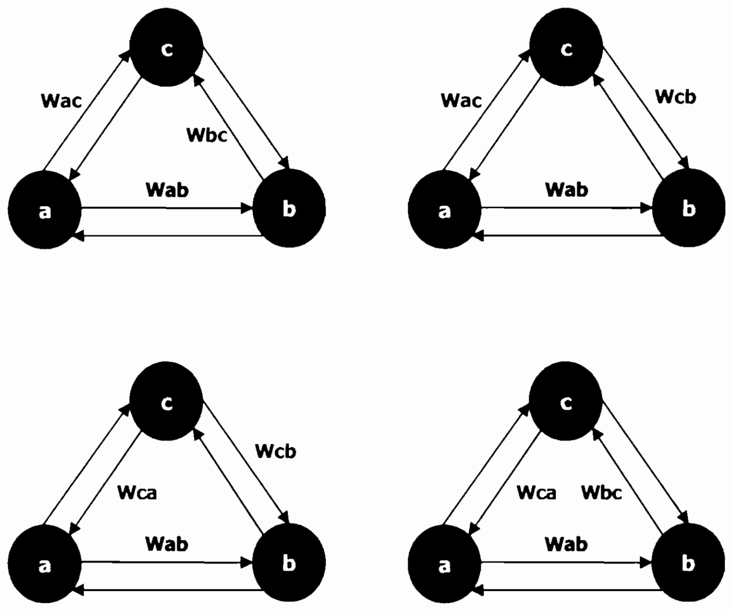

Figure 3: Four triad rules at which a sentiment, Wab, changes are shown in these figures. Two signs which influence into Wab are put beside the lines.

relations in one person's cognitive representation are equal to the relations in others' cognitive representations completely. The variable W_{ij} represents both the relation in individual representations and the relation in the real world. When a value of W_{ij} is bigger than zero, $W_{ij} > 0$, the value of the variable represents a positive sentiment such as a kind of liking. When a value of W_{ij} is smaller than zero, $W_{ij} < 0$, the value of the variable represents a negative sentiment such as a kind of disliking. When a value of W_{ij} is zero, $W_{ij} = 0$, the value of the variable does a sentiment such as neither positive nor negative.

2.3 Change equations

It is considered that there are four triad rules in which a value of W_{ij} changes. At rule 1, a product of two signs of Wac and Wbc is considered as the balanced state from Heider (1958)'s *pox* model, and in addition a geometric average of a value of Wac and Wbc without signs is used as one element of the balanced state. Each equation bellow consists of a dyad rule and one of triad rules. The variation of W_{ij} is represented as

$$\Delta W_{ij} = \alpha (W_{ji} - W_{ij}) + \beta \{ \phi (W_{ik} W_{jk}) \cdot |W_{ik} W_{jk}|^{1/2} - W_{ij} \}$$

where α indicates a coefficient of a dyad rule, β does a coefficient of a triad rule, and the function ϕ (x) operates x as follows:

$$\phi(x) = \begin{cases} +1 \text{ if}, x > 0 \\ +0 \text{ if}, x = 0 \\ -1 \text{ if}, x < 0 \end{cases}$$

from rule 1 to rule 5 and an equation of three or more persons.

As the other rules, the variation of W_{ij} is represented as follows:

$$\Delta W_{ij} = \alpha\,(W_{ji} - W_{ij}) + \beta\{\ \phi\,(W_{ik}W_{kj})\cdot|W_{ik}W_{kj}|^{1/2} - W_{ij}\ \}$$

at rule 2,

$$\Delta W_{ij} = \alpha\,(W_{ji} - W_{ij}) + \beta\{\ \phi\,(W_{ki}W_{kj})\cdot|W_{ki}W_{kj}|^{1/2} - W_{ij}\ \}$$

at rule 3 and

$$\Delta W_{ij} = \alpha\,(W_{ji} - W_{ij}) + \beta\{\ \phi\,(W_{jk}W_{ki})\cdot|W_{jk}W_{ki}|^{1/2} - W_{ij}\ \}$$

at rule 4. Another change's equation named as rule 5 in which a sum of rule 1 to rule 4 for a triad rule is used is considered as follow:

$$\Delta W_{ij} = \alpha\,(W_{ji} - W_{ij}) + \beta\{\ \phi\,(W_{ik}W_{jk})\cdot|W_{ik}W_{jk}|^{1/2} + \phi\,(W_{ik}W_{kj})\cdot|W_{ik}W_{kj}|^{1/2} + \phi\,(W_{ki}W_{kj})\cdot|W_{ki}W_{kj}|^{1/2} + \phi\,(W_{jk}W_{ki})\cdot|W_{jk}W_{ki}|^{1/2} - 4W_{ij}\ \}/4$$

When the number of persons is three or more, change's equation called rule 6 is considered as follow:

$$\Delta W_{ij} = \alpha\,(W_{ji} - W_{ij}) + \beta\,/\,(N\text{-}2) \sum_{k \neq i,\, k \neq j} triad(i,j,k)$$

where N is the number of persons in human relations at which each person is connected with the others, and $triad(i,j,k)$ represents either the triad's term of rule 1, rule 2, rule 3, rule 4 or rule 5, where k is one of the others, in other words, k is neither i nor j.

3. Computer simulations[3]

First of all, it will be shown that changes of sentiments liking-disliking between three persons were

[3] The author wrote sources about them with C language and ran the executed programs compiled by Borland C++ Builder on Microsoft Windows OS.

simulated by a synchronous revise of W_{ij}, by each rule, rule 1, rule 2, rule 3 or rule 4 as a triad rule. An initial value of W_{ij} was either +1 or -1 and a value from 0.0 to 0.9 by 0.1 was used as each value of α and β. Every possible pattern by six initial values for W_{ij} was used when it was simulated within every possible combination of values for two coefficients at each rule. However values of α and β were used in the limit as a value of $\alpha+\beta$ was bigger than 0.0 and smaller than 1.0[4]..

An index of the convergence is sum of every absolute difference of a relation's value at a time t and the relation's value at a time t+1 in each relation as follow:

$$E^{(t)} = \sum_{i \neq j} \sum_{j \neq i} | W_{ij}{}^{(t)} - W_{ij}{}^{(t+1)} |$$

where values of W_{ij} are rounded bellow 0.01. It was justified that patterns which changed by each rule converged, when a value of E has been smaller than 0. And it was also confirmed whether the sum of $E^{(t)}$ from t to t-10 was smaller than 0. Next, an index of the balanced state with each relation is considered as follow:

$$B_{ij}^{(t)} = \sum_{k \neq i,\, k \neq j} triad(i,j,k)^{(t)}$$

where $W_{ij}{}^{(t)}$ is the balanced state, when the value at $B_{ij}^{(t)}$ is 0. At the end, an index for the symmetry of relations is considered as follow:

$$S_{ij}^{(t)} = W_{ji}{}^{(t)} - W_{ij}{}^{(t)}$$

where a relation of $W_{ij}{}^{(t)}$ and $W_{ji}{}^{(t)}$ is symmetric, when the value at $S_{ij}^{(t)}$ is 0.

As the result, it was confirmed that the all patterns converged, each relation in all patterns became symmetry and all patterns became balanced states, except that each relation in some patterns didn't become symmetry in the simulations with rule 4 without a term of the dyad. It was shown in Table 1 that averaged time to converge at rule 2 was the longest. The results above is consistent with Fujisawa, Mizutani, & Kosugi (2003)'s results which were gotten from asynchronous simulations of each triad rule from rule 1 to rule 3 with transitivity in the case of signed directed relations without value.

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[4] Periodicities occur at some patterns, when $\alpha + \beta = 1$. Divergences occur at some patterns, when $\alpha + \beta > 1$. That means that one person's sentiment would be extremely at some patterns if one person's sentiment toward another person changed rapidly. We have to use an attenuation function to see extreme changes.

Table 1: Means and standard deviations about convergence times

	Dyad & Triad ($\alpha \neq 0, \beta \neq 0$)		Triad ($\alpha = 0, \beta \neq 0$)	
	M	*SD*	*M*	*SD*
Rule 1	10.5	7.47	12.0	10.81
Rule 2	17.8	12.87	17.7	12.72
Rule 3	10.5	7.47	12.0	10.81
Rule 4	15.2	9.42	3.8	5.75

Secondly, it will be shown that changes of sentiments liking-disliking between three or more persons were simulated, in which W_{ij} was revised synchronously by rule 6 at which rule 5 was used as a triad rule. The number of persons was used from 3 to 8 by 1. A value of W_{ij} was one of +1, 0 or -1 as initial values of W_{ij}. The values from 0.0 to 0.9 by 0.1 were used as a value of α and a value of β in each simulation. Combinations of 100 patterns selected randomly as initial values of W_{ij} within all possible patterns led from values of two coefficients were used at the simulations. However values of α and β were used in the limit as a value of $\alpha+\beta$ was bigger than 0 and smaller than 1. The index of convergence is as same equation as first simulations.

As the result, it was confirmed that the all patterns converged, relations in all patterns became symmetry and all patterns became balanced states. In addition, it was confirmed that convergence time increased more gently as the number of persons, N, increased, as shown at Figure 4, even though the number of relations increased rapidly. The number of relations are derived from $N(N-1)$. Note that the convergence time depended on initial values and two coefficients.

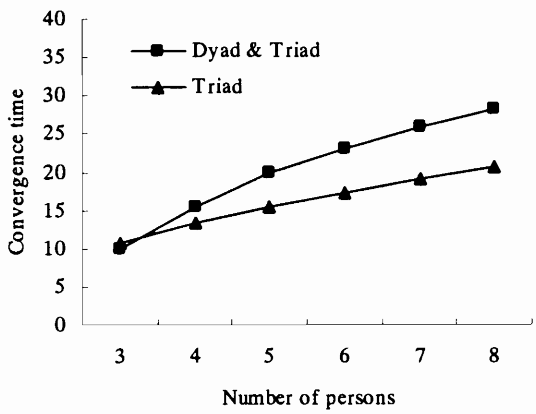
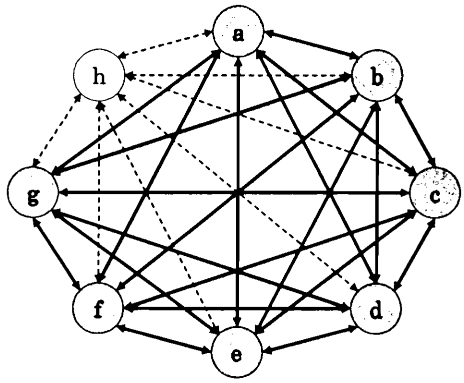


Figure 4: The x-axis is the number of persons, N, and the y-axis is the number of average convergence times in the second simulations about three or more persons. The line with Dyad & Triad shows the times in the limit as $\alpha \neq 0$ and $\beta \neq 0$. The line with Triad shows them in the limit as $\alpha = 0$ and $\beta \neq 0$.

one person	another person: a	b	c	d	e	f	g	h
a		1	-1	0	-1	1	0	-1
b	1		0	1	0	1	1	1
c	1	0		-1	0	-1	-1	0
d	0	1	1		1	0	-1	0
e	0	0	-1	1		1	1	0
f	0	-1	1	0	1		1	1
g	1	0	1	0	0	-1		-1
h	-1	1	0	1	0	-1	0	

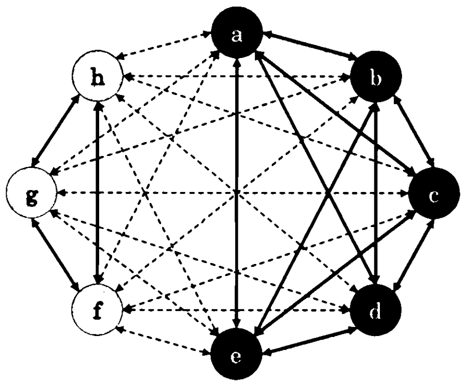
(a) Initial values 1



(b) Isolation with negative sentiments

one person	another person: a	b	c	d	e	f	g	h
a		-1	0	0	0	-1	1	0
b	1		1	1	0	-1	-1	1
c	0	-1		0	-1	0	-1	-1
d	1	-1	0		0	0	1	-1
e	0	0	1	0		0	1	-1
f	-1	1	1	0	0		0	0
g	-1	0	-1	1	-1	-1		1
h	-1	-1	1	-1	-1	1	0	

(c) Initial values 2



(d) Cliques

Figure 5: The left figures are adjacency matrices of initial values about relations. The right figures are states of relations after convergence. Interaction of triad rule brings out isolation with negative sentiments and cliques.

Representative patterns after convergence will be shown bellow. For instance, human relations were simulated using initial values (a) illustrated in figure 5, $\alpha = 0.1$ and $\beta = 0.8$, so that the pattern of the relations became the state (b) with an isolated person who had negative sentiments from the others. They were simulated using initial values (c), $\alpha = 0.7$ and $\beta = 0.1$, so that the pattern of them became the state (d) with cliques. It is confirmed that isolation with negative sentiments, cliques and a group with all positive relations among all persons occurs when both triad rule and dyad rule operate changes of relations and triad rule operates them.

4. Conclusions and Discussion

In the simulations with both a dyad rule and each triad rule in three persons, changes of all patterns converged, all relations become symmetry, and all patterns become balanced states. In the simulations with each only triad rule in three persons, the results are same as above, except that some patterns don't become symmetry in the simulations with rule 4 without a term of the dyad rule. Note that all patterns could become not only balanced states but also all relations could become symmetry, even if changes of sentiments were influenced by only triad rules, exception for rule 4 without the dyad rule. The results are in the limit as $0.0 < \alpha + \beta < 1.0$ and $\beta \neq 0$. Some relations between W_{ij} and W_{ji} in some patterns of initial values don't become symmetry at rule 4 without dyad rule. Note that transitivity exists at each triad term from rule 1 to rule 3, but transitivity doesn't exist at the triad term of rule 4 (Fujisawa, Mizutani, & Kosugi, 2003). The reason is that two different sets of relations like chains in which some directed relations interact within themselves and they don't interact with the others can be taken out from six relations at rule 4 without dyad rule.

In simulations of three or more persons, all results are the same as results above without exceptions, in the limit as $0.0 < \alpha + \beta < 1.0$ and $\beta \neq 0$. As an interesting point, it has been confirmed that convergence time increased more gently as number of persons increased and isolation with negative sentiments and cliques were brought out by interaction of triad rule.

These results can support that changes of sentiments become smaller, relations between pairs in people become symmetry and relations become balanced states, while each relation in every person turns into the balanced state, under some conditions. We can consider the conditions as follows. First of all, every relation doesn't change beyond the stable state. Secondly, a triad rule isn't the rule at which some directed relations never have interactions with the others' directed relations. At the last, all relations recognized by one person are equal to the relations recognized by the others, in other words, every person expresses sentiments toward people appropriately without deceptions and every person recognizes all relations correctly.

In the real groups, it has been shown to change into the balanced states in the process of time by investigations of human relations (Kosugi, Fujisawa, & Fujihara, 2004). And it was also shown that changes of relations decreased as time changes (Newcomb, 1961). Their results are consistent with results of simulations above.

In the simulations, a node k was a person. The node k can be regard as an object, an event, an issue or their group, if they are anthropomorphic, when we interpret the simulations. If the condition doesn't consist in, it is necessary to simulate leaving the relations from node k. In the case, rule 1 has to be used as the rule for changes of W_{ij} and W_{ji}. The relations between W_{ij} and W_{ji} also become symmetry and balanced state. And either rule 2 or rule 3 has to be used as a rule for changes of W_{ik} and W_{jk}.

Furthermore, it is necessary to verify how much range the results of the simulations can explain in the real world and to consider to what degree the models can be extended.

References

Amemiya, T., Kimura, Y., & Fujisawa, H. 1993 Theory of socion (3): general theory of socion. *Bulletin of the Faculty of Sociology, Kansai University*, 25(1), 63-163 (*in Japanese*).

Aronson, E., & Linder, D. 1965 Gain and loss of esteem as determinants of interpersonal attractiveness. *Journal of Experimental Social Psychology*, 1, 156-171.

Cartwright, D., & Harary, F. 1956 Structural balance: a generalization of Heider's theory. *Psychological Review*, 63, 277-293.

Fujisawa, H., & Amemiya, T., & Kimura, Y. 1991 Theory of socion (2): from dyad to triad. *Bulletin of the Faculty of Sociology, Kansai University*, 22(2), 165-221 (*in Japanese*).

Fujisawa, T., Mizutani, S., & Kosugi, K. 2003 Simulations of computational socion models (2): examinations of transitive operation and weight balance on triad socions. *Human Science, the Graduate Course of Kansai University*, 58, 133-147 (*in Japanese*).

Heider, F. 1946. Attitudes and cognitive organization. *Journal of Psychology*, 21, 107–112.

Heider, F. 1958 *The Psychology of Interpersonal Relations*. John Wiley & Sons.

Hunter, E. 1978 Dynamic sociometry. *Journal of Mathematical Sociolgy*, 6, 87-138.

Jordan, N. 1953 Behavioral forces that are a function of attitudes and of cognitive organization. *Human Relations*, 6, 273-287.

Kimura, Y., Fujisawa, H., & Amemiya, T. 1990 Introduction to socion theory: a system dynamic approach to social network. *Bulletin of the Faculty of Sociology, Kansai University*, 21(2), 67-143 (*in Japanese*).

Kogan, N., & Tagiuri, R. 1958 Interpersonal preference and cognitive organization. *Journal of Abnormal and Social Psycholog*, 56, 113-116.

Kosugi, K., Fujisawa, T., & Fujihara, T. 2004 Isomorphism of balance theory and eigen-decomposition, *Sociological Theory and Methods*, 19(1), 87-100 (*in Japanese*).

Mizutani, S., Kosugi, K., & Fujisawa, T. 2002 Simulations of the computational socion models: changes of positive and negative potentials using dyad and triad rules. *Human Science, the Graduate Course of Kansai University*, 57, 109-133 (*in Japanese*).

Mizutani, S. 2004 Computer simulations of relations between three or more persons based on a balance theory. *Abstract Book on 28th International Congress of Psychology(CD-ROM)*, 1372.

Newcomb, T. M. 1961 *The acquaintance process*. New York: Holt. Rinehart and Winston.

Nomura, T. 2004 Analysis and simulation of group dynamics based on Heider's balance theory and a finite Markov chain. *Proceedings of the 2nd International Conference of the European Social Simulation Association*. (http://rikou.st.ryukoku.ac.jp/~nomura/docs/Nomura_ESSA04.pdf).

Wang, Z. & Thorngate, W. 2003 Sentiment and social mitosis: implications of Heider's balance theory.

Journal of Artificial Societies and Social Simulation, 6(3). (http://jasss.soc.surrey.ac.uk/6/3/2.html).