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Public Information, Private Information, and the Price of Debt in a Creditor Coordination Game with Large and Small Creditors*

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Abstract

Large creditors can exert a disproportionate amount of influence on the likelihood of debt defaults due to coordination failure. Even if the fundamentals are sound, apprehension of premature foreclosure by a single large creditor may trigger preemptive actions by others, and the consequent liquidation of the distressed debtor's assets can result in self-fulfilling debt defaults. To examine the influence of large creditors on the coordination problem faced by creditors, we offer a model in which a large creditor and a group of small creditors independently decide whether to foreclose on a loan based on their private information on the fundamentals. In the absence of common knowledge of the fundamentals, the incidence of failure is uniquely determined. In the limit where both large and small creditors have very precise information, an increase in the size of the large creditor and in the precision of his information alleviates the debtor's vulnerability to premature foreclosure. However, results from numerical calculations show that an increase in the size and relative precision of information available to the large creditor may either increase or decrease the yield on the debt for general parameter values.

JEL-Classification: G33, G12, D82

Keywords: Large creditors, Creditor coordination, Debt pricing, Incomplete information, Global games

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1 Introduction

Conventional wisdom holds that large creditors can exert a disproportionate amount of influence on the likelihood of debt defaults due to coordination failure. Even if the fundamentals are sound, apprehension of premature foreclosure by a single large creditor may induce preemptive actions by others, and the consequent liquidation of the distressed debtor's assets can give rise to self-fulfilling debt defaults.

Until the early 1990s, the corporate finance literature had emphasized several distinguishing features of close bank-firm relationships in Japan, often referred to as main bank relationships. Notably, with main bank intervention, most financially distressed large firms in Japan efficaciously restructured troubled debt. If debtors are economically viable, creditors as a group will benefit from saving them. Saving such debtors, however, requires creditors to coordinate. By intervening, main banks can take the initiative in coordinating creditors, so that they can avoid inefficient liquidation. Main bank relationships have been considered to reduce financial constraints for firms in temporary difficulties.¹

Unfortunately, the burst of the economic bubble in the Japanese economy left banks with substantial bad loans, which significantly undermined the banks' financial standing. Recently, Japanese main banks are abandoning a number of their troubled debtors and are less likely to intervene to save them. In the absence of main bank intervention, what happens to creditor coordination problems? Do large, well-informed creditors still serve to prevent creditor coordination failure, even if they only act non-cooperatively and without close relationships to debtors? The model we develop, described below, is an attempt to articulate and investigate these points formally.

To examine the influence of large creditors in terms of their size and the precision of their information, we employ a theoretical framework building on the global games literature. Global games, pioneered by Carlsson and van Damme (1993), are games with incomplete information whose type space is determined by the players, each of whom observes a private noisy signal of the underlying state.² Morris and Shin (1999) applied the global game to the creditor coordination game and related it to the determinants of the value of defaultable debts.³

This paper extends the work of Morris and Shin (1999) by introducing a large creditor.⁴ The note by Takeda (2003) is the first step in this direction. By simply assuming uniform prior distribution of fundamentals, Takeda (2003) provided clear-

¹For discussions on this issue, see Hoshi *et al.* (1991), Rajan (1992), and Detragiache (1994).

²For global games, see an excellent survey by Morris and Shin (2002).

³Bruche (2003) extends the work of Morris and Shin (1999) and relates it to the continuous-time structural model of bond prices *à la* Merton (1974).

⁴Corsetti *et al.* (2001), Corsetti *et al.* (2002), Metz (2002), and Takeda (2000) use the technique of introducing a large trader into a currency crisis model based on the global game.

cut characterization of the role of large creditors in asymmetric global games with one large and many small creditors. This paper further extends and generalizes the work of Takeda (2003) by assuming normal prior distribution. This allows us to investigate the influence of large creditors on the price of debts. We are also able to examine how public and private information about fundamentals have distinctive impacts on the price. Thus, this paper provides an analytical framework with which to examine the role of large creditors in determining the price of debts under circumstances where self-fulfilling beliefs as well as fundamentals play an important role in determining outcomes.

In our model, the incidence of failure is uniquely determined in the absence of common knowledge of fundamentals. Comparative statics results show that a decrease in the size of the large creditor and in the precision of his information aggravates the debtor's vulnerability to premature foreclosure, when both large and small creditors have very precise information. However, results from numerical calculations show that a decrease in the size and relative precision of information available to a large creditor may either increase or decrease the yield for general parameter values.

The rest of this paper is organized as follows. In Section 2, we set up a creditor coordination game with large and small creditors. The unique equilibrium is then derived in Section 3. Section 4 provides comparative statics, and Section 5 presents numerical calculations in terms of the influence of the large creditor on the price of debt. Concluding remarks are provided in Section 6.

2 The model

Consider an economy in which a project is financed by a continuum of ex-ante identical small creditors and a single large creditor. Each small creditor has an infinitesimal portion of the whole stake. There are three periods, $t = 0, 1$, and 2 . A project matures at period 2 to yield a return v , which is uncertain at period 0. The proportion of loans financed by the large creditor is $\lambda \in (0, 1)$. The combined mass of loans financed by small creditors then amounts to $1 - \lambda$. The face value of repayment is $L > 0$. Each creditor can receive the face value at period 2 if the realized return v is large enough to cover repayment of debt. We assume that every creditor is fully rational and rationality is common knowledge.

At period 1, each creditor has the chance to decide whether to continue lending until the project matures, or to stop lending. Creditors make a decision simultaneously and independently. The loan is secured by collateral, whose liquidation value is $K^* \in (0, L)$ if it is liquidated at period 1. If the collateral is liquidated following the project's failure, it has a lower liquidation value $K_* \in [0, K^*)$. If a creditor continues lending, she will receive a payoff depending on the realized return v of

the project at maturity. The value of v depends on two factors: the underlying state θ , which is randomly determined, and the severity of disruption $z > 0$ of the project caused by the event of early liquidation by creditors. Using ℓ to denote the proportion of creditors who stop lending at period 1, if the underlying state θ is larger than $z\ell$, the firm remains in operation. Otherwise, the firm is forced into bankruptcy.

The realized value of the project v is given by

$$v(\theta, \ell) = \begin{cases} V & \text{if } z\ell < \theta, \\ K_* & \text{if } z\ell \geq \theta, \end{cases} \quad (1)$$

where $V > L$ is a constant representing the return when the project succeeds. By normalizing the payoffs so that $L = 1$ and $K_* = 0$, the payoffs to a creditor are given by the following matrix, where $\kappa \equiv (K^* - K_*)/(L - K_*)$, thereby $0 < \kappa < 1$:

	Project succeeds (if $z\ell < \theta$)	Project fails (if $z\ell \geq \theta$)
Continue lending	1	0
Stop lending	κ	κ

For simplicity, we assume that if continued lending yields the same expected payoff as stopped lending, then a creditor prefers to stop lending.

The creditors do not observe the realization of θ until period 2, but they receive private signals regarding it. The large creditor receives the signal with the noise η ,

$$x_L = \theta + \eta, \quad (2)$$

where η is normally distributed with mean 0 and variance $1/\gamma$. Similarly, a typical small creditor i receives the signal with the noise ε_i ,

$$x_i = \theta + \varepsilon_i, \quad (3)$$

where ε_i is normally distributed with mean 0 and variance $1/\beta$. ε_i is i.i.d. across creditors and each is independent of η and θ . The distributions of η and ε_i are assumed to be common knowledge among creditors.

The value of θ is normally distributed with mean y and variance α .⁵ Although θ is not common knowledge, both y and α are common knowledge and exogenously given. Hence, y is regarded as the creditors' expected value of fundamentals, which is based on public information. Upon receiving its respective signal, the representative creditor i can infer the value of θ and the distribution of signals received by the other creditors, as well as their estimates of θ . Likewise, all other creditors form their beliefs while relying only on their own information. This assumption of

⁵This specification follows the work of Morris and Shin (1999) and that of Metz (2002). In contrast, Corsetti *et al.* (2001), Corsetti *et al.* (2002), and Takeda (2000) assume that θ is uniformly distributed.

incomplete information is the key to deriving the unique equilibrium in the global games literature.

The timing of the events is as follows:

- Period 0 The firm invests its debt-financed fund into the project.
 Nature chooses θ .
- Period 1 Signals are observed.
 Creditors decide whether to continue lending or to stop lending.
 Creditors who choose to stop lending receive κ .
- Period 2 Aggregate outcomes are realized.
 Creditors who choose to continue lending receive 1 or 0.

A strategy for a creditor is a decision rule that maps each realization of her signal to an action (i.e., to continue lending or to stop lending). An equilibrium is a profile of strategies that maximizes the creditor's expected payoff conditional on the creditor's signal, when all other creditors are following the strategies in the profile.

3 The equilibrium

Before solving the game outlined above, we briefly discuss the coordination problem under three special cases. The first is when θ is common knowledge among creditors, the second is when all creditors are small ($\lambda = 0$), and the third is when the only creditor is large ($\lambda = 1$). After discussing these special cases, we will solve for the equilibrium with large and small creditors.

3.1 The equilibria under complete information

Consider the optimal strategies for creditors in the case where all creditors know the true value of θ before deciding whether to continue lending or to stop lending. In this case, their optimal strategies can be described as follows. If $\theta > z$, then to continue lending is optimal, regardless of what the other creditors do, since the project will succeed even if all the other creditors stop lending. In contrast, if $\theta \leq 0$, then to stop lending is optimal irrespective of what the other creditors do, because the project will fail, even if all the other creditors continue lending.

When $\theta \in (0, z]$, there is a coordination problem among the creditors. Each creditor assumes that if all the other creditors continue lending, then the payoff for continued lending is 1, so that continued lending yields more than the early liquidation value κ . However, if they stop lending, the payoff is $0 < \kappa$, so that early liquidation is optimal. This type of coordination problem among creditors is similar to the bank run problem suggested by Diamond and Dybvig (1983). As in Diamond and Dybvig (1983), the complete information game where θ is common knowledge entails multiple equilibria. Two pure strategy Nash equilibria coexist in

this game: the Pareto-superior strategy of continued lending and the Pareto-inferior strategy of stopped lending. Each creditor faces indeterminacy of beliefs in the sense that she cannot choose one strategy by calculating the expected payoff of each equilibrium strategy based on the Bayes' theorem, because she cannot rationally assign subjective probabilities to possible outcomes.

3.2 The equilibrium with only small creditors

The case of $\lambda = 0$ is the symmetric game among small creditors, which is the same as that in Morris and Shin (1999). We solve for equilibria in which creditors use a simple *switching strategy*, in which they continue lending whenever the signal they receive, x_i , is higher than some given threshold value x^* , whereas they will otherwise stop lending.

The unique equilibrium is characterized by the critical value of fundamentals θ^* below which the project will always fail, and the critical signal x^* such that creditors receiving a signal below this value will always stop lending. To derive these two critical values, the two conditions described below are necessary.

When a small creditor i receives the realization of the signal x_i , its posterior distribution of θ is normally distributed with mean

$$\xi_i \equiv \frac{\alpha y + \beta x_i}{\alpha + \beta}, \quad (4)$$

and variance $1/(\alpha + \beta)$. When small creditors use a switching strategy, they have a threshold level ξ^* for their switching strategies, and continue lending if and only if the private signal x is greater than

$$x^*(\xi^*, y) \equiv \frac{\alpha + \beta}{\beta} \xi^* - \frac{\alpha}{\beta} y. \quad (5)$$

If the true state is θ , the probability that any particular creditor receives a signal below this level is

$$\Pr(x_i \leq x^* \mid \theta) = \Phi\left(\sqrt{\beta}(x^* - \theta)\right), \quad (6)$$

where Φ is the cumulative distribution function for a standard normal distribution. Creditors will stop lending whenever they receive a signal below x^* . Recalling that ℓ is the proportion of creditors who stop lending, ℓ equals the probability described in (6).

From (1), the condition for the project to fail is $\theta \leq z\ell$. When this holds with equality, θ takes the critical value θ^* , below which the project always fails given the

critical signal x^* . Thus the first equilibrium condition – a *critical mass* condition – is

$$\begin{aligned}\theta^* &= z\ell \\ &= z\Phi\left(\sqrt{\beta}(x^* - \theta^*)\right) \\ &= z\Phi\left(\frac{\alpha}{\sqrt{\beta}}(\xi^* - y) + \sqrt{\beta}(\xi^* - \theta^*)\right).\end{aligned}\quad (7)$$

Second, consider the optimal switching strategy for creditor i receiving a signal x_i , given θ^* . The creditor has the conditional probability of the project's success of

$$\begin{aligned}\Pr(\theta > \theta^* \mid x_i) &= 1 - \Phi\left(\sqrt{\alpha + \beta}(\theta^* - \xi_i)\right) \\ &= \Phi\left(\sqrt{\alpha + \beta}(\xi_i - \theta^*)\right).\end{aligned}\quad (8)$$

Likewise, the creditor has the conditional probability of the project's failure of $\Pr(\theta \leq \theta^* \mid x_i) = \Phi\left(\sqrt{\alpha + \beta}(\theta^* - \xi_i)\right)$. The creditor continues lending if and only if the expected payoff for continued lending is at least as high as that of early liquidation κ . Since the expected payoff for continued lending by the creditor who receives the critical signal x^* must be equal to that of early liquidation κ , the second equilibrium condition – an *optimal cutoff* condition – is

$$\Phi\left(\sqrt{\alpha + \beta}(\xi^* - \theta^*)\right) = \kappa. \quad (9)$$

This implies

$$\xi^* = \frac{\Phi^{-1}(\kappa)}{\sqrt{\alpha + \beta}} + \theta^*. \quad (10)$$

Solving for the equilibrium entails solving (7) and (9). Solving for θ^* , we obtain

$$\theta^* = z\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\theta^* - y + \Phi^{-1}(\kappa)\frac{\sqrt{\alpha + \beta}}{\alpha}\right)\right). \quad (11)$$

The interval $(0, \theta^*)$ represents those states where the creditors could make the project successful, but the disruption caused by creditors' early liquidation is so substantial that the project eventually fails. In other words, the debtor is solvent, but illiquid. This equilibrium is Pareto-inferior to the equilibrium at which the project succeeds.

Denoting ϕ the standard normal distribution function, (11) has a unique solution when $z\phi(\alpha/\sqrt{\beta}) < 1$. Since $\phi \leq 1/\sqrt{2\pi}$, a sufficient condition for a unique solution θ^* is given by

$$\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{z}. \quad (12)$$

Since α is the precision of the prior distribution of θ , and β is the precision of the creditors' signals, (12) is satisfied whenever the private signals are sufficiently precise relative to the prior information.⁶

Next, we examine how uncertainty of the fundamentals affects uncertainty of other creditors' actions. Let us consider the case in which the private signals of the creditors become very precise, and noise becomes negligible; that is, $\beta \rightarrow \infty$. From (11) the critical value θ^* satisfies

$$\theta^* \rightarrow z\Phi(\Phi^{-1}(\kappa)) = z\kappa. \quad (13)$$

Even though information on fundamentals becomes very precise ($\beta \rightarrow \infty$), θ^* remains finite. That is, the strategic uncertainty of the actions of other creditors remains, and so does the positive probability of the inefficient failure of the project. In addition, if either z or κ is large, the efficiency loss is sizeable.

3.3 The equilibrium with a single large creditor

We next consider the case of $\lambda = 1$, where there is a single large creditor. This simplifies the game to the decision problem by a large creditor alone. In this case, only the fundamental risk matters in the absence of coordination risk. This situation is analogous to that in the classic defaultable debt pricing model of Merton (1974), where it was assumed that bankruptcy occurs when asset value falls short of some given level relative to liabilities.

Since there is no coordination problem among creditors, a critical mass condition like (7) is not necessary. The only relevant condition is the optimal cutoff condition. That is, the large creditor will continue lending if and only if the expected payoff for continued lending is higher than that of early liquidation.

When a large creditor receives the realization of the signal x_L , its posterior distribution of θ is normally distributed with mean

$$\xi_L \equiv \frac{\alpha y + \gamma x_L}{\alpha + \gamma},$$

and variance $1/(\alpha + \gamma)$. The large creditor will continue lending if and only if the expected payoff for continued lending is higher than that of early liquidation; that is,

⁶For a proof of the existence of the unique equilibrium in (11) under the condition (12), see Section 4 of Morris and Shin (2001).

$$\begin{aligned}
\Pr(\theta > 0 \mid x_L) &= 1 - \Phi(\sqrt{\alpha + \gamma}(0 - \xi_L)) \\
&= \Phi(\sqrt{\alpha + \gamma}\xi_L) \\
&> \kappa.
\end{aligned}$$

This implies

$$\xi_L > \frac{\Phi^{-1}(\kappa)}{\sqrt{\alpha + \gamma}}. \quad (14)$$

Thus, the cutoff point of the optimal switching strategy for the large creditor is given by $\xi_L^* = \Phi^{-1}(\kappa)/\sqrt{\alpha + \gamma}$. After receiving the signal x_L , the large creditor will stop lending if and only if $\xi_L \leq \xi_L^*$, and continue lending if $\xi_L > \xi_L^*$. Note that ξ_L^* tends to zero as α or γ goes to ∞ ; that is, the uncertainty of the fundamentals becomes small.

3.4 The equilibrium with large and small creditors

We now show that there is a unique, dominance-solvable equilibrium in which both types of creditors follow their respective switching strategies around the critical signals x^* and x_L^* .

To derive the unique equilibrium, we first focus on solving for an equilibrium in switching strategies. Assume that the small creditors follow the switching strategy around x^* . Since there is a continuum of small creditors, there is no aggregate uncertainty of the proportion of small creditors who continue lending. Let us consider the case in which the large creditor stops lending. Since, given θ , $\Pr(x_i > x^* \mid \theta) = (1 - \lambda)(1 - \Phi(\sqrt{\beta}(x^* - \theta))) = (1 - \lambda)\Phi(\sqrt{\beta}(\theta - x^*))$ is the proportion of small creditors receiving the signal higher than x^* , small creditors' lending alone is sufficient for the project to succeed at θ if $z(1 - (1 - \lambda)\Phi(\sqrt{\beta}(\theta - x^*))) < \theta$. Hence, we can define a critical value of the fundamentals $\bar{\theta}$ above which lending by small creditors alone is sufficient for the project to succeed. $\bar{\theta}$ is given by

$$\bar{\theta} = z \left(1 - (1 - \lambda)\Phi \left(\sqrt{\beta}(\bar{\theta} - x^*) \right) \right). \quad (15)$$

Whenever θ is below $\bar{\theta}$, the project is successful irrespective of the action of the large creditor. Note that $\bar{\theta}$ lies between $z\lambda$ and z .

Next we consider the case in which the large creditor continues lending. The proportion of small creditors who continue lending at θ is given by $(1 - \lambda)\Phi(\sqrt{\beta}(\theta - x^*))$. If the large creditor also chooses to continue lending, there is an additional λ to the proportion of creditors who continue lending. Hence, if the large creditor also continues lending, the project succeeds whenever $z(1 - \lambda + (1 - \lambda)\Phi(\sqrt{\beta}(\theta - x^*))) < \theta$.

Thus we can define the critical value of the fundamentals $\underline{\theta}$ above which the project succeeds if and only if both creditors continue lending. $\underline{\theta}$ is defined by

$$\underline{\theta} = z \left(1 - \lambda - (1 - \lambda) \Phi \left(\sqrt{\beta}(\underline{\theta} - x^*) \right) \right). \quad (16)$$

Note that $\underline{\theta}$ lies between 0 and $z(1 - \lambda)$.

Since both $\bar{\theta}$ and $\underline{\theta}$ are functions of the switching point x^* , which depends on the large creditor's switching point x_L^* , we need to solve these two switching points simultaneously from the viewpoint of the respective optimization problems of creditors. First, consider the large creditor's problem. A large creditor receiving signal x_L assigns probability $\Pr(\theta > \underline{\theta} | x_L) = 1 - \Phi(\sqrt{\alpha + \gamma}(\underline{\theta} - \xi_L)) = \Phi(\sqrt{\alpha + \gamma}(\xi_L - \underline{\theta}))$ to the event $\theta \geq \underline{\theta}$. Thus, his optimal strategy is to continue lending if and only if his expected payoff for continued lending conditional on x_L , which is given by $\Phi(\sqrt{\alpha + \gamma}(\xi_L - \underline{\theta}))$, exceeds its payoff for stopped lending, which equals κ . Hence, the switching point $x_L^*(\xi_L^*, y)$ is defined by:

$$\Phi(\sqrt{\alpha + \gamma}(\xi_L^* - \underline{\theta})) = \kappa. \quad (17)$$

We now consider a small creditor's problem. Conditional on signal x , the posterior density over θ for this creditor is given by

$$\phi(\sqrt{\alpha + \beta}(\theta - \xi)). \quad (18)$$

If $\theta > \bar{\theta}$, the continuation of lending can be successful irrespective of the action of the large creditor. If $\underline{\theta} < \theta \leq \bar{\theta}$, the project succeeds if and only if the large creditor continues lending. If $\theta \leq \underline{\theta}$, the project fails even if the large creditor continues lending. Hence, the expected payoff to continuing lending conditional on signal x can be given by

$$\begin{aligned} & \Pr(\underline{\theta} < \theta \leq \bar{\theta}, x_L > x_L^* | x) + \Pr(\theta > \bar{\theta} | x) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \phi(\sqrt{\alpha + \beta}(\theta - \xi)) \Phi(\sqrt{\gamma}(\theta - x_L^*)) d\theta + \int_{\bar{\theta}}^{\infty} \phi(\sqrt{\alpha + \beta}(\theta - \xi)) d\theta. \end{aligned} \quad (19)$$

The first term of (19) is the portion of expected payoff attributable to the region of $\theta \in (\underline{\theta}, \bar{\theta}]$. The second term is the portion of expected payoff that is attributable to the interval of θ where $\theta > \bar{\theta}$. In the first region, the project is successful if and only if both large and small creditors continue lending. The probability that the large creditor continues lending at θ , given his switching strategy around x_L^* , is given by $\Pr(x_L > x_L^* | \theta) = \Phi(\sqrt{\gamma}(\theta - x_L^*))$, so that the payoffs in the first term are weighted by this value.

Since a small creditor's optimal strategy is to continue lending if and only if its expected payoff to continued lending conditional on signal x exceeds its payoff for

early liquidation κ , from (19) the switching point $x^*(\xi^*, y)$ for the small creditor is given by:

$$\int_{\underline{\theta}}^{\bar{\theta}} \phi\left(\sqrt{\alpha+\beta}(\theta-\xi^*)\right) \Phi\left(\sqrt{\gamma}(\theta-x_L^*)\right) d\theta + \int_{\bar{\theta}}^{\infty} \phi\left(\sqrt{\alpha+\beta}(\theta-\xi^*)\right) d\theta = \kappa. \quad (20)$$

There is a unique $x^*(\xi^*, y)$ that solves (20). To see this, let us change the variables in the integrals, as follows:

$$s \equiv \sqrt{\alpha+\beta}(\theta-\xi^*), \quad \underline{\delta} \equiv \sqrt{\alpha+\beta}(\underline{\theta}-\xi^*), \quad \text{and} \quad \bar{\delta} \equiv \sqrt{\alpha+\beta}(\bar{\theta}-\xi^*). \quad (21)$$

From (17), using these variables, the critical point of the posterior distribution of θ for the large creditor can be written as

$$\begin{aligned} \xi_L^* &= \underline{\theta} + \frac{\Phi^{-1}(\kappa)}{\sqrt{\alpha+\gamma}} \\ &= \frac{\underline{\delta}}{\sqrt{\alpha+\beta}} + \xi^* + \frac{\Phi^{-1}(\kappa)}{\sqrt{\alpha+\gamma}}. \end{aligned}$$

Hence,

$$\begin{aligned} x_L^* &= \frac{\alpha+\gamma}{\gamma} \xi_L^* - \frac{\alpha}{\gamma} y \\ &= \frac{\alpha+\gamma}{\gamma\sqrt{\alpha+\beta}} \underline{\delta} + \frac{\alpha+\gamma}{\gamma} \xi^* + \frac{\sqrt{\alpha+\gamma}}{\gamma} \Phi^{-1}(\kappa) - \frac{\alpha}{\gamma} y, \end{aligned}$$

and then,

$$\begin{aligned} \sqrt{\gamma}(\theta-x_L^*) &= \sqrt{\gamma} \left(\frac{s}{\sqrt{\alpha+\beta}} + \xi^* - \frac{\alpha+\gamma}{\gamma\sqrt{\alpha+\beta}} \underline{\delta} - \frac{\alpha+\gamma}{\gamma} \xi^* - \frac{\sqrt{\alpha+\gamma}}{\gamma} \Phi^{-1}(\kappa) + \frac{\alpha}{\gamma} y \right) \\ &= \sqrt{\frac{\gamma}{\alpha+\beta}} \left(s - \frac{\alpha+\gamma}{\gamma} \underline{\delta} \right) - \frac{\alpha}{\sqrt{\gamma}} \xi^* - \sqrt{\frac{\alpha+\gamma}{\gamma}} \Phi^{-1}(\kappa) + \frac{\alpha}{\sqrt{\gamma}} y. \end{aligned}$$

Using these variables, the conditional expected payoff of continuing lending for the small creditor given critical signal x^* in (19) is

$$\int_{\underline{\delta}}^{\bar{\delta}} \phi(s) \Phi(A) ds + \int_{\bar{\delta}}^{\infty} \phi(s) ds,$$

where

$$A = \sqrt{\frac{\gamma}{\alpha+\beta}} \left(s - \frac{\alpha+\gamma}{\gamma} \underline{\delta} \right) - \frac{\alpha}{\sqrt{\gamma}} \xi^* - \sqrt{\frac{\alpha+\gamma}{\gamma}} \Phi^{-1}(\kappa) + \frac{\alpha}{\sqrt{\gamma}} y. \quad (22)$$

From (20), the following equation holds at the equilibrium:

$$\int_{\underline{\delta}}^{\bar{\delta}} \phi(s) \Phi(A) ds + \int_{\bar{\delta}}^{\infty} \phi(s) ds - \kappa = 0. \quad (23)$$

Note, however, that both $\underline{\delta}$ and $\bar{\delta}$ are strictly decreasing in ξ^* , since

$$\frac{d\underline{\delta}}{d\xi^*} = -\frac{1}{z(1-\lambda)\phi(\sqrt{\beta}\underline{\delta}) + 1/\sqrt{\beta}} < 0, \text{ and}$$

$$\frac{d\bar{\delta}}{d\xi^*} = -\frac{1}{z(1-\lambda)\phi(\sqrt{\beta}\bar{\delta}) + 1/\sqrt{\beta}} < 0.$$

Given that the left hand side of (23) is strictly decreasing in both $\underline{\delta}$ and $\bar{\delta}$, it is strictly increasing in ξ^* . The left hand side of (23) is negative for sufficiently small ξ^* , and positive for sufficiently large ξ^* . Given that the left hand side of (23) is continuous in ξ^* , there is a unique solution to (23). Once ξ^* is determined, the large creditor's threshold ξ_L^* is determined from (17).

Thus far, we have shown that there is a unique equilibrium within switching strategies. We can complete the argument by showing that the switching equilibrium identified above is the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies. The proof is relegated to the appendix. The following proposition summarizes what we find in this subsection:

Proposition 1 *There is a unique, dominance-solvable equilibrium in which the large creditor uses the switching strategy around x_L^* and the small creditors use the switching strategy around x^* .⁷*

The unique equilibrium is determined by the four key equations (15), (16), (17), and (20). These equations jointly determine the critical states $\bar{\theta}$ and $\underline{\theta}$ and the switching points x^* and x_L^* .

4 Comparative statics

Having established the uniqueness of the equilibrium, we proceed to conduct comparative statics exercises to analyze the equilibrium with large and small creditors. We examine the effect on the critical fundamentals of the change in the following variables: (i) the precision of information of the large creditor relative to the small

⁷The general structure of our model conforms to a class of supermodular games in which the iterative elimination of strictly dominated strategies yields a unique equilibrium. See Milgrom and Roberts (1990) and Vives (1990) for the class of supermodular games. Besides, the equilibrium of our model is not robust to incomplete information. See Kajii and Morris (1997) for robustness of equilibria to incomplete information.

creditors, (ii) the size of the large creditor, (iii) the ex-ante mean of fundamentals, and (iv) the severity of disruption caused by early liquidation.

Contrary to the equilibrium with small creditors alone, the equilibrium with large and small creditors cannot be solved explicitly. Before conducting comparative statics, we restate the four key equations (15), (16), (17), and (23), which characterize the equilibrium, using the expression of (21):

$$\bar{\theta} = z \left(1 - (1 - \lambda) \Phi \left(\sqrt{\frac{\beta}{\alpha + \beta}} \bar{\delta} \right) \right), \quad (24)$$

$$\underline{\theta} = z \left(1 - \lambda - (1 - \lambda) \Phi \left(\sqrt{\frac{\beta}{\alpha + \beta}} \underline{\delta} \right) \right), \quad (25)$$

$$\Phi \left(\sqrt{\alpha + \gamma} (\xi_L^* - \underline{\theta}) \right) = \kappa, \text{ and} \quad (26)$$

$$\int_{\underline{\delta}}^{\bar{\delta}} \phi(s) \Phi(A) ds + \int_{\bar{\delta}}^{\infty} \phi(s) ds - \kappa = 0. \quad (27)$$

These four equations jointly determine the critical states $\bar{\theta}$ and $\underline{\theta}$, and the switching points $x^*(\xi^*, y)$ and $x_L^*(\xi_L^*, y)$. It is difficult to obtain definitive answers to the comparative statics for general parameter values. In contrast, the limiting case where both types of creditors have very precise information gives us tidy results. Even though fundamental uncertainty of the payoff relevant state θ becomes smaller and smaller, strategic uncertainty of the action of others remains as large as ever. Thus, we can highlight the strategic influence of the large creditor.

Let us explore the limiting case where both types of creditors have arbitrarily precise information, while the precision of the large creditor's signals relative to the small creditors' tends to r :

$$\beta \rightarrow \infty, \gamma \rightarrow \infty, \text{ and } \frac{\gamma}{\beta} \rightarrow r.$$

Compared to the general case, the limiting case is much more tractable. This is because the critical signals for both creditors tend to $\underline{\theta}$, so that we can identify $\underline{\theta}$ as the critical state below which the project fails. It follows from (26) that as $\gamma \rightarrow \infty$, ξ_L^* has to converge to $\underline{\theta}$, or, otherwise, the left hand side of (26) converges to either zero or one but is not equal to κ . Therefore, in the limit, the large creditor always continues lending at states better than $\underline{\theta}$, while he stops lending at states equal to or worse than $\underline{\theta}$. Since small creditors also have very precise information ($\beta \rightarrow \infty$), (24) and (25) can be written as follows:

$$\bar{\theta} = z \left(1 - (1 - \lambda) \Phi(\bar{\delta}) \right), \text{ and} \quad (28)$$

$$\underline{\theta} = z(1 - \lambda - (1 - \lambda)\Phi(\underline{\delta})). \quad (29)$$

From (28) and (29), as $\beta \rightarrow \infty$ the same argument holds for the small creditors, so that they follow the switching strategy around $\underline{\theta}$. Thus, in the limit we have $\xi^* = \xi_L^* = \underline{\theta}$.

In addition, (5) gives $x^* \equiv \xi^* + (\xi^* - y)\alpha/\beta$. Thus, when small creditors have very precise information ($\beta \rightarrow \infty$), x^* converges to ξ^* . Likewise, for the large creditor, when $\gamma \rightarrow \infty$, x_L^* converges to ξ_L^* . Therefore, combining all the relationships among variables mentioned above, in the limit as $\beta \rightarrow \infty$ and $\gamma \rightarrow \infty$, we must have $x^* = x_L^* = \xi^* = \xi_L^* = \underline{\theta}$. In other words, the switching points of both types of creditors converge to the critical state $\underline{\theta}$, at which the project switches from being failed to being completed successfully in the limit.

As Corsetti *et al.* (2001) point out in their currency crisis model, in solving for the critical state in the limit, it is important to distinguish the case where the large player is sufficiently large from the case where he is not. We know from (28) and (29) that it is only when $\underline{\theta} \geq z\lambda$ that $\underline{\theta} = \bar{\theta}$ holds, while $\bar{\theta}$ is larger than $\underline{\theta}$ when $\underline{\theta} < z\lambda$. We can characterize the equilibrium value of $\underline{\theta}$ in the limit as follows:

Proposition 2 *In the limit as $\beta \rightarrow \infty$, $\gamma \rightarrow \infty$, and $\gamma/\beta \rightarrow r$, all of x^* , x_L^* , ξ^* , ξ_L^* , and $\underline{\theta}$ converge to $z(1 - \lambda - (1 - \lambda)\Phi(\underline{\delta}))$. If $\underline{\theta} < z\lambda$, then $\underline{\delta}$ is the unique solution to*

$$\int_{\underline{\delta}}^{\infty} \phi(s) \Phi(A') ds = \kappa. \quad (30)$$

If $\underline{\theta} \geq z\lambda$, then $\underline{\delta}$ is the unique solution to

$$\int_{\underline{\delta}}^M \phi(s) \Phi(A') ds + \int_M^{\infty} \phi(s) ds = \kappa, \quad (31)$$

where

$$A' = \sqrt{r}(s - \underline{\delta}) - \Phi^{-1}(\kappa), \text{ and}$$

$$M = \Phi^{-1}\left(\Phi(\underline{\delta}) + \frac{\lambda}{1 - \lambda}\right).$$

The proof of Proposition 2 is as follows. Suppose that $\lim \underline{\theta} < z\lambda$, so that $\lim \underline{\theta} < \lim \bar{\theta}$. Since $\xi^* \rightarrow \underline{\theta}$, $\bar{\delta} = \sqrt{\alpha + \beta}(\bar{\theta} - \xi^*) \rightarrow \infty$ must hold. In addition, as $\beta \rightarrow \infty$, $\gamma \rightarrow \infty$, and $\frac{\gamma}{\beta} \rightarrow r$, A in (22) can be written as $A' = \sqrt{r}(s - \underline{\delta}) - \Phi^{-1}(\kappa)$. Thus, in the limit, (27) is expressed as (30).

Conversely, when $\lim \underline{\theta} \geq z\lambda$, we have $\lim \underline{\theta} = \lim \bar{\theta}$. Since $\xi^* \rightarrow \underline{\theta}$, $\bar{\delta}$ becomes a finite number $\sqrt{\alpha + \beta}(\underline{\theta} - \xi^*)$. From (28) and (29) we have $1 - (1 - \lambda)\Phi(\bar{\delta}) = 1 - \lambda - (1 - \lambda)\Phi(\underline{\delta})$. Thus, we obtain

$$\bar{\delta} = \Phi^{-1}\left(\Phi(\underline{\delta}) + \frac{\lambda}{1 - \lambda}\right).$$

Therefore, in the limit, (27) is expressed as (31). Since both the left hand side of (30) and that of (31) are strictly increasing in $\underline{\delta}$, there exists a unique $\underline{\delta}$ that solves both equations. Accordingly, Proposition 2 follows from (29).

4.1 Precision of information available to the large creditor

Now we examine how a change in precision of the large creditor's signal relative to the small creditors' affects the critical state from Proposition 2. The left hand side of (30) and (31) is strictly increasing in r , while it is strictly decreasing in $\underline{\delta}$. Therefore, $\underline{\delta}$ must be strictly increasing in r . It follows from (29) that as $\underline{\delta}$ increases, $\underline{\theta}$ must decrease:

$$\frac{d\underline{\theta}}{d\underline{\delta}} = -z(1 - \lambda)\phi(\underline{\delta}) < 0. \quad (32)$$

This gives rise to the following proposition:

Proposition 3 *In the limit as $\beta \rightarrow \infty$, $\gamma \rightarrow \infty$, and $\gamma/\beta \rightarrow r$, the critical state $\underline{\theta}$ is strictly decreasing in r .⁸*

In other words, a decrease in precision of the large creditor's signal relative to the small creditors' reduces the range of fundamentals according to which creditors continue lending, and raises the probability of the project's failure.

4.2 Size of the large creditor

Next, we consider how a change in the size of the large creditor affects the probability of failure. It follows from Proposition 2 that as λ increases, $\underline{\theta}$ must decrease. Before showing this, we first investigate the relationship between $\underline{\delta}$ and λ . When $\underline{\theta} < z\lambda$, (31) holds. The left hand side of (31) is strictly decreasing in λ , through the effect of λ on M . Thus $\underline{\delta}$ is strictly increasing in λ . When $\underline{\theta} \geq z\lambda$, (30) holds. Since the left hand side of (30) does not depend on λ , $\underline{\delta}$ does not depend on λ . Thus, in the limit, irrespective of the size of λ , $\underline{\delta}$ is non-decreasing in λ . It follows from (25) that the overall effect of λ on $\underline{\theta}$ is given by

$$\frac{d\underline{\theta}}{d\lambda} = -z \left(1 - \Phi(\underline{\delta}) + (1 - \lambda)\phi(\underline{\delta}) \frac{d\underline{\delta}}{d\lambda} \right). \quad (33)$$

Because $d\underline{\delta}/d\lambda \geq 0$, (33) is always negative. Thus, the following proposition holds:

⁸In a symmetric global game *à la* that discussed by Morris and Shin (1999), the equilibrium in the limit does not depend on the structure of the noise. Proposition 3 implies that noise-independent equilibrium selection fails in our model with payoff asymmetry among creditors. See Frankel *et al.* (2003) for noise-independent equilibrium selection.

Proposition 4 *In the limit as $\beta \rightarrow \infty$, $\gamma \rightarrow \infty$, and $\gamma/\beta \rightarrow r$, $\underline{\theta}$ is strictly decreasing in λ .*

In other words, a decrease in the size of the large creditor reduces the range of fundamentals according to which creditors continue lending, and raises the probability of the project's failure.

In their currency crisis model, Corsetti *et al.* (2001) reveal that the *existence* of a large trader always has a non-negative effect on the incidence of attack on the currency in the limit where traders have very precise information. They do not, however, provide a definitive conclusion on the *size* effect of the large trader.⁹ In contrast, our model shows that within the class of asymmetric global games with a large player, a case exists where we can unambiguously identify the global sign of the effect of the large player's size on the equilibrium, regardless of the large player's size itself.

Our results show that increasing the size of the large creditor, as well as increasing the relative precision of information available to it, increases the likelihood of project success, which is at the Pareto superior equilibrium in our model. On the contrary, Corsetti *et al.* (2001) argue that when the large trader is sufficiently large, increasing the size of it, as well as increasing the relative precision available to it, increases the likelihood of peg failure, which is at the Pareto inferior equilibrium in their currency crisis model. The contrastive results come from the difference in the payoff structure between the two models. The large creditor in our model receives the better payoff when the project succeeds, whereas the large trader in their model receives the better payoff when the peg fails. If we consider a creditor coordination game where a large creditor can make a profit from a debtor's bankruptcy, the comparative statics will likely provide quite different results from those in this paper.

4.3 Ex-ante mean of fundamentals

Then we examine the effect of a change in the ex-ante mean of fundamentals y on the critical state $\underline{\theta}$. We can show that an increase in y decreases in $\underline{\theta}$. The left hand side of (30) and (31) is strictly increasing in y , while it is decreasing in $\underline{\delta}$. Thus, in equilibrium, an increase in y decreases $\underline{\delta}$. Since (29) holds in equilibrium, an increase in $\underline{\delta}$ raises $\underline{\theta}$. Therefore, the following proposition holds:

Proposition 5 *In the limit as $\beta \rightarrow \infty$, $\gamma \rightarrow \infty$, and $\gamma/\beta \rightarrow r$, $\underline{\theta}$ is strictly decreasing in y .*

⁹To be more precise, they suggest that it is only when the size of the large trader is sufficiently large that the size effect has an unambiguously positive effect on the incidence of attack on the currency in the limiting case. When the size is small, the size effect is ambiguous in their model.

In other words, an increase in the ex-ante expected value of fundamentals reduces the upper bound of the range of fundamentals according to which creditors stop lending, and raises the probability of the project's success.

4.4 Severity of disruption caused by early liquidation

Finally, we consider how a change in z , severity of disruption of the project caused by early liquidation, affects the probability of the project's failure. We can show that an increase in z always increases $\underline{\theta}$. Because the left hand side of (30) and (31) does not depend on z , we have $d\underline{\delta}/dz = 0$. Accordingly, it follows from (29) that:

$$\begin{aligned}\frac{d\underline{\theta}}{dz} &= 1 - \lambda - (1 - \lambda)\Phi(\underline{\delta}) \\ &= (1 - \lambda)(1 - \Phi(\underline{\delta})) \\ &> 0.\end{aligned}$$

Proposition 6 *In the limit as $\beta \rightarrow \infty$, $\gamma \rightarrow \infty$, and $\gamma/\beta \rightarrow r$, $\underline{\theta}$ is strictly increasing in z .*

In other words, an increase in severity of disruption to the project caused by early liquidation reduces the range of fundamentals according to which creditors continue lending, and increases the probability of the project's failure.

4.5 Comparative statics away from the limit

In contrast to the limiting case in which both large and small creditors have arbitrarily precise information, the results away from the limit are not so definitive. For example, consider the probability of the project's success at a certain state θ . From the definition of the critical states, the project fails whenever $\theta \leq \underline{\theta}$, whereas it succeeds whenever $\theta > \bar{\theta}$. When θ lies in the interval $(\underline{\theta}, \bar{\theta}]$, the success depends on whether the large creditor continues lending. In comparison, in the case without a large creditor ($\lambda = 0$), the project fails if and only if $\theta^* \leq \theta$ in (11). Thus, whether the project is more likely to succeed with the large creditor depends on the relative size of $\underline{\theta}$ and $\bar{\theta}$ compared to θ^* . When $\bar{\theta} < \theta^*$, the presence of the large creditor increases the probability of the project's success, whereas it increases the probability of the project's failure when $\theta^* < \underline{\theta}$. When $\underline{\theta} < \theta^* < \bar{\theta}$, however, we cannot give a definitive answer as to whether the presence of the large creditor increases the probability of the project's success. The probability of success increases by the presence of the large creditor for states $\theta \in (\underline{\theta}, \theta^*)$, whereas it decreases for states $\theta \in (\theta^*, \bar{\theta})$.

5 The price of debt

We now turn to the question of how a change in the size and the precision of information available to the large creditor, and how the precision and the level of prior information, affect the value of debts. To address this question, we provide an argument about the pricing of defaultable debts. Following the work of Morris and Shin (1999), we consider the value of a long-term unsecured loan with face value 1 to the project. The creditor receives 1 when the project succeeds, and receives zero when it fails.¹⁰ Given ex-ante mean of fundamentals y , the ex-ante price of such a loan is

$$W(y) = \int_{\underline{\theta}}^{\bar{\theta}} \phi(\sqrt{\alpha}(\theta - y)) \Phi(\sqrt{\gamma}(\theta - x_L^*)) d\theta + \int_{\bar{\theta}}^{\infty} \phi(\sqrt{\alpha}(\theta - y)) d\theta.$$

For simplicity, we define the yield on the unsecured loan as

$$\text{Yield} = \frac{\text{Par} - \text{Price}}{\text{Price}}.$$

Based on the equations above, we can examine how the (i) size of the large creditor λ , (ii) precision of the large creditor's signal relative to the small creditors' signals $r \equiv \gamma/\beta$, (iii) severity of disruption caused by early liquidation, (iv) accuracy of prior information α , and (v) ex-ante mean of the fundamentals y affect the yield.¹¹ We use the following set of variables as a benchmark case for numerical calculations:

$$\alpha = 0.1, \beta = 5, \gamma = 5, z = 1, \kappa = 0.5, \lambda = 0.5, \text{ and } y = 2.$$

That is, we conduct numerical simulations to examine the effect of a change in a particular variable on the yield, maintaining the other variables in the benchmark case. Note that the recovery rate of the collateralized debt with collateral κ affects the price and the yield of the defaultable debt only through the equilibrium strategies of the other creditors who hold the collateralized debt, but not through the payoff for early liquidation of the unsecured loan. Unlike creditors who hold collateralized debt, the creditor who holds the unsecured loan cannot obtain the payoff κ from early liquidation, since her payoff is zero, unless the project succeeds.

¹⁰For simplicity, we focus on the price of unsecured debt. If we consider the price of collateralized debt, we need to calculate the price that depends not only on the probability of the project's failure but also on the creditors' decision to stop lending, taking into consideration the recovery of collateral.

¹¹We use the numerical optimization method to calculate the equilibrium of the model based on the parameters given below. We adopt the Gauss-Newton algorithm for actual computation.

We first analyze the effect of the size of the large creditor on the yield. In the previous section, which focuses on the limiting case in which both types of creditors have very precise information, we found that a decrease in the size of the large creditor reduces the probability of the project's failure. In contrast, our numerical examples in a more general case indicate that the decrease in the size of the large creditor may either reduce or raise the probability of project failure. In particular, the effect of the size of the large creditor is affected by the precision of prior information α . Figure 1 shows the relationship between λ and the yield, when varying α . When α is small, the larger the size of the large creditor, the lower the yield. When α is large, however, the larger the size of the large creditor, the higher the yield. These results indicate that when the prior information is sufficiently accurate, an increase in the size of the large creditor raises the probability of the project's success and lowers the yield, whereas, when the prior information is not accurate, an increase in the size of the large creditor raises the probability of the project's failure and raises the yield.

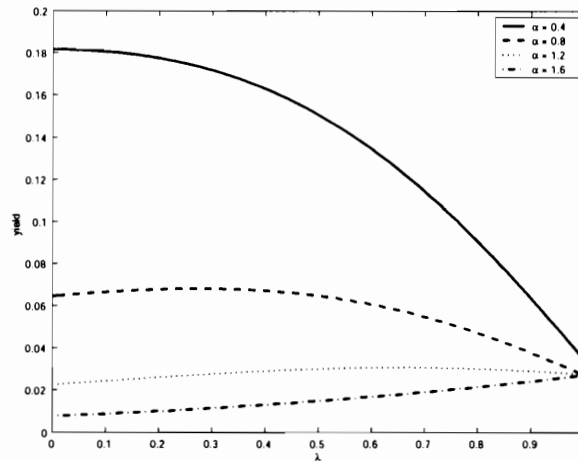


Figure 1 The Effect of λ on the Yield

We next examine the relationship between the size of the large creditor and the relative precision of information. Figure 2 shows the relationship between λ and the yield in two cases; one in which the large creditor has more precise information than the small creditors ($r = 10$), the other in which the small creditors have more precise information ($r = 0.1$). In both cases, the larger the size of the large creditor, the lower the yield. In addition, the yield in the former case ($r = 0.1$) is always lower than the yield in the latter case ($r = 10$), irrespective of the size of the large creditor. These results indicate that the more precise the information of the large creditor relative to that of the small creditors, the higher the probability of the project's success. In contrast, Figure 3 presents the relationship between λ and the

yield when $\alpha = 2$. In this case, the larger the size of the large creditor, the higher the yield. The yield in the case of $r = 0.1$ is always higher than the yield in the case of $r = 10$, irrespective of the size of the large creditor. These results indicate that the more precise the information of the large creditor relative to that of the small creditors, the higher the probability of the project's failure.

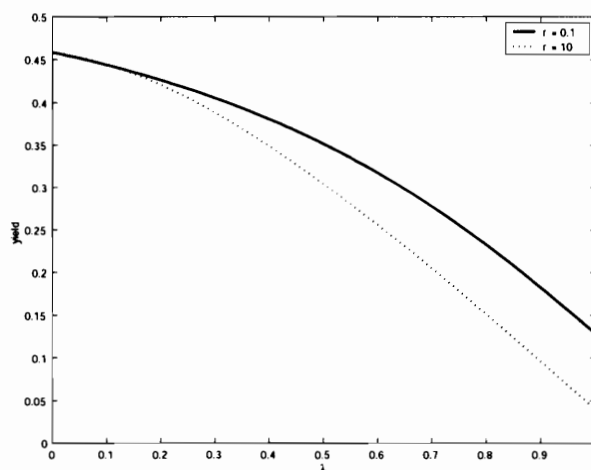


Figure 2 The Effect of λ on the Yield ($\alpha = 0.1$)

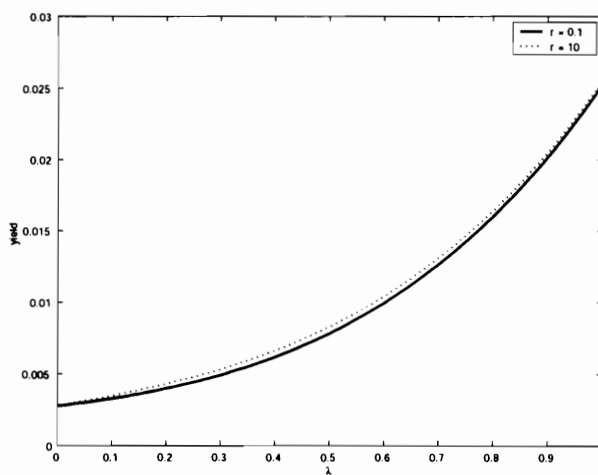


Figure 3 The Effect of λ on the Yield ($\alpha = 2$)

Figure 4 compares the case of large and small creditors ($\lambda = 0.5$), with that of only small creditors ($\lambda = 0$) in order to show the effect of z , the severity of disruption caused by early liquidation, on the yield. In the case of only small creditors, the yield has a upper slope, indicating that the yield increases as z increases. In contrast, in

the case with large and small creditors, the yield has a U-shaped curve, meaning that for smaller z the yield decreases as z increases, whereas for larger z the yield increases together with z . In other words, in the case of only small creditors, the probability of the project's success becomes higher as z approaches 0, whereas in the case with large and small creditors, the probability of the project's success hits a peak in the range of mid-sized z . We shall call z *optimal* at the point at which the probability of the project's success hits maximum and the yield reaches minimum.

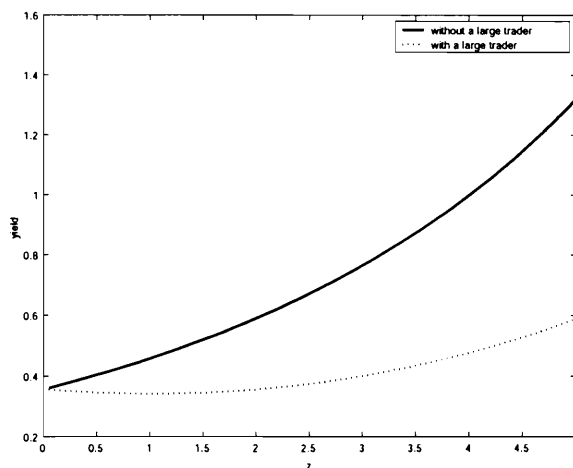


Figure 4 The Effect of z on the Yield

To show the value of optimal z , Figure 5 illustrates the combination of λ and the optimal z . For smaller λ , the optimal z is zero, whereas for larger λ , the optimal z increases together with λ . This implies that when the size of the large creditor is sufficiently small, an increase in z always raises the probability of the project's failure. In contrast, when the size of the large creditor is sufficiently large, for smaller z an increase in z raises the probability of the project's success, and, further, the larger the size of the large creditor becomes, the wider the range of z , over which the probability of the project's success, together with z , increases, becomes.

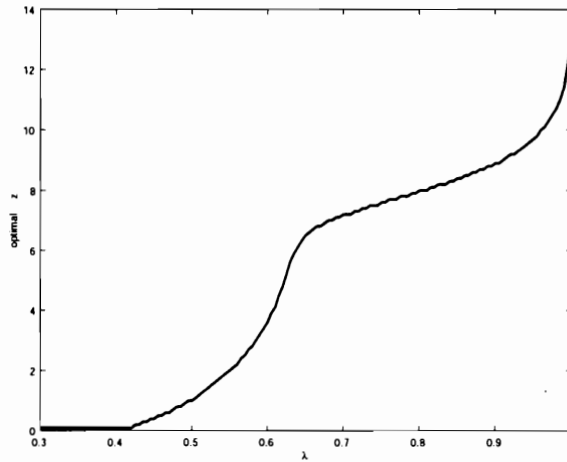


Figure 5 Optimal z

Now let us investigate the effect of the precision of prior information α on the yield. Figure 6 shows that an increase in α is associated with a decrease in the yield. This means that the more precise prior information becomes, the lower the yield becomes. Comparison between the case with large and small creditors ($\lambda = 0.5$) and the case with only small creditors ($\lambda = 0$) reveals that for smaller α the yield is lower in the former case than in the latter case, whereas for larger α it is higher. In other words, when prior information is not sufficiently precise, the presence of the large creditor increases the probability of the project's success, whereas the presence of the large creditor raises the probability of the project's failure when prior information is sufficiently precise.

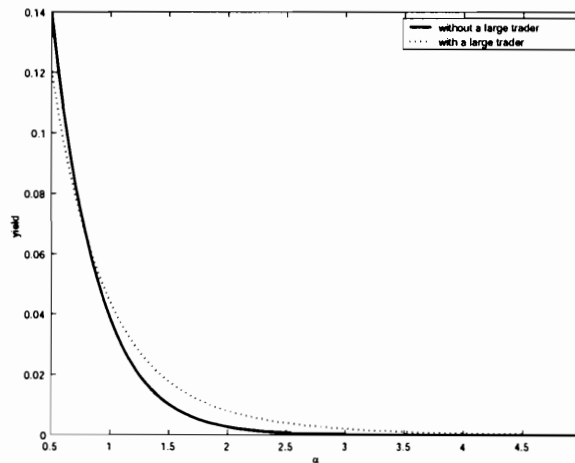


Figure 6 The Effect of α on the Yield

For comparison, Figure 7 presents the case in which the average level of prior information is smaller ($y = 0.5$), where an increase in α corresponds to an increase in the yield in the case with large and small creditors. That is, the more precise the prior information, the greater is the chance of the project's failure. Note that in the case with only small creditors, the yield does not depend on α , and is constant in Figure 7. This result is sensitive to changes in other parameters. To show this, in Figure 8 ($y = 0.6$) the yield decreases as α increases in the case with only small creditors, whereas in Figure 9 ($y = 0.4$) the yield increases as α increases.

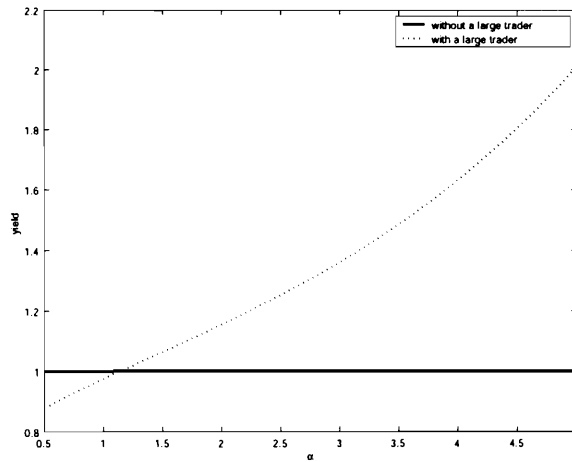


Figure 7 The Effect of α on the Yield ($y = 0.5$)

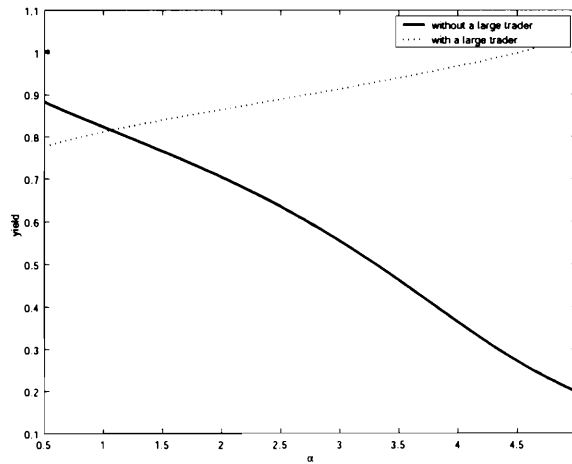


Figure 8 The Effect of α on the Yield ($y = 0.6$)

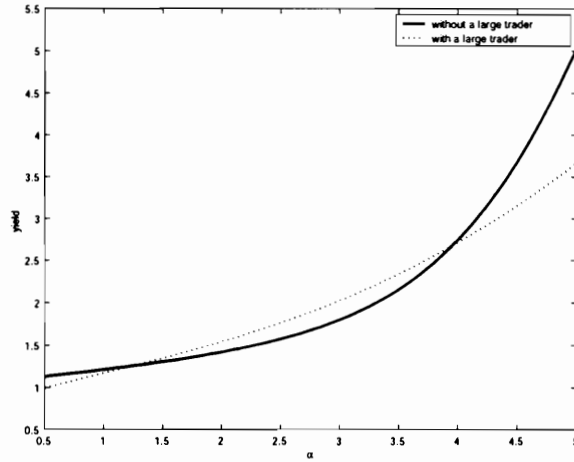


Figure 9 The Effect of α on the Yield ($y = 0.4$)

Finally, we consider the effect on the yield of a shift in the ex-ante mean of fundamentals y . Figure 10 shows that the yield falls as y increases. This indicates that the higher the ex-ante mean of fundamentals, the higher the probability of the project's success. Comparison between the case with large and small creditors ($\lambda = 0.5$) and the case only with small creditors ($\lambda = 0$) reveals that the yield is lower in the former case than in the latter case.

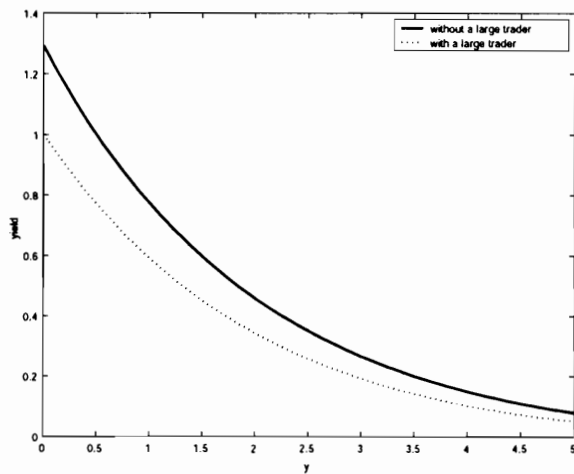


Figure 10 The Effect of y on the Yield

However, in Figure 11 ($\alpha = 2$), for smaller y the yield is lower in the former case, and for larger y the yield is higher in the former case. Figure 11 also presents

the yield from the naive model that does not take coordination risk into account.¹² Comparing the yield from the true model to that from the naive model, the yield difference becomes large as y falls, indicating an increase in the size of inefficient liquidation due to coordination failure. These results are consistent with those in Morris and Shin (1999).

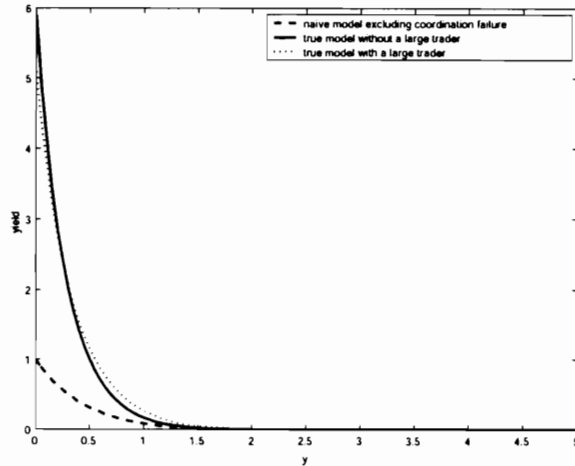


Figure 11 The Effect of y on the Yield ($\alpha = 2$)

To conclude this section, the size and the precision of information of the large creditor have respective significant influence on the incidence of project's success, and on the resulting yield on the debt. When both large and small creditors have very precise information, the direction of the influence is very clear. In such a case, an increase in the size of the large creditor and in the precision of his information always alleviates the debtor's vulnerability to premature foreclosure. However, in the case away from the limit, an increase in the size and the precision of information of the large creditor increases either the project's success or its failure. Nevertheless, an increase in the ex-ante expected value of fundamentals always raises the probability of the project's success and decreases the yield.

6 Concluding remarks

We have studied a model of a coordination game played by one large and many small creditors in the framework of global games. In the absence of common knowledge of the fundamentals, the incidence of failure is uniquely determined. In the limit where both large and small creditors have arbitrarily precise information, a decrease

¹²The naive model applies to the case of subsection 3.3, in which there is a single large creditor and only the fundamental risk matters in the absence of coordination risk.

in the size of the large creditor and in the precision of his information aggravates the debtor's vulnerability to premature foreclosure. In addition, the larger the severity of disruption caused by early liquidation becomes, the more likely creditors are to stop lending despite sound fundamentals. However, in the case away from the limit, an increase in the size and relative precision of information of a large creditor can either increase or decrease the yield on the debt. In contrast, it has been shown that the higher the ex-ante expected value of fundamentals, the lower the yield.

In sum, we find that when creditors have very precise information, increasing the size of the large creditor, as well as increasing the relative precision of information available to him, raises the value of debts. However, these two factors cannot be disentangled into separable determinants of the price of debts for general parameter values. The interplay between them is complicated. These findings shed light on the importance of close monitoring of the actions of large creditors in evaluating the value of debts. A more profound understanding of the role of large creditors in determining the price of debts awaits further refinement of our current method or reliance on alternative methods of modeling large creditors in creditor coordination problems.

Appendix

Proof of Proposition 1. We complete the proof for proposition 1 by showing that the switching strategy is the only equilibrium strategy that survives iterative elimination of strictly dominated strategies. Let us consider a small creditor's best response conditional on x when all other small creditors follow the switching strategy around \tilde{x} and the large creditor plays its best response against this switching strategy, which we know is the switching strategy around $x_L(\tilde{x})$, by using (17). The net expected payoff for a small creditor from continued lending as opposed to stopped lending is given by

$$\begin{aligned} \pi(x, \tilde{x}) &= \int_{\underline{\theta}(\tilde{x})}^{\bar{\theta}(\tilde{x})} \phi\left(\sqrt{\alpha + \beta}(\theta - \xi(x))\right) \Phi(\sqrt{\gamma}(\theta - x_L(\tilde{x}))) d\theta \\ &\quad + \int_{\bar{\theta}(\tilde{x})}^{\infty} \phi\left(\sqrt{\alpha + \beta}(\theta - \xi(x))\right) d\theta - \kappa, \end{aligned}$$

where $\bar{\theta}(\tilde{x})$ and $\underline{\theta}(\tilde{x})$ denote the values of $\bar{\theta}$ and $\underline{\theta}$ when small creditors follow the switching strategy around \tilde{x} . Note that $\pi(\cdot, \cdot)$ is strictly increasing in its first argument and strictly decreasing in its second.

For sufficiently good signals, the net expected payoff for a small creditor from continued lending is positive, and continued lending is a dominant strategy for a small creditor, regardless of what the other creditors do. Let \bar{x}_1 be the threshold value of x , above which continued lending is a dominant strategy for a small creditor.

Since every creditor is rational, it does not use a dominated strategy. Furthermore, since each creditor knows that the others are rational, it infers that the others will not use a dominated strategy either. Thus, for a small creditor to stop lending above \bar{x}_1 cannot be a dominant strategy. As a consequence, to stop lending cannot be a dominant strategy for a small creditor whenever its signal is above \bar{x}_2 such that it solves $\pi(\bar{x}_2, \bar{x}_1) = 0$. Since the switching strategy around \bar{x}_2 is the best response to the switching strategy around \bar{x}_1 , even the small creditor who assumes the lowest possibility of the project's success believes that the incidence of continued lending is higher than that implied by the switching strategy around \bar{x}_1 and the large creditor's best response $x_L(\bar{x}_1)$. Since $\pi(\cdot, \cdot)$ is strictly increasing in its first argument, any strategy to stop lending for signals higher than \bar{x}_2 is strictly dominated. Proceeding in this way, we have the following decreasing sequence:

$$\bar{x}_1 > \bar{x}_2 > \bar{x}_3 > \dots > \bar{x}_k > \dots,$$

where any strategy to stop lending for signal $x > \bar{x}_k$ does not survive k steps of elimination of strictly dominated strategies. Common knowledge of rationality takes this procedure to the limit. Sequence $\{\bar{x}_k\}_{k=0}^{\infty}$ is monotone and bounded, so that the limiting point $\bar{x}_{\infty} = \lim_{k \rightarrow \infty} \bar{x}_k$ exists and is given by $\bar{x}_{\infty} = \sup\{x | \pi(x, x) = 0\}$. \bar{x}_{∞} is the largest solution to $\pi(x, x) = 0$. Any strategy to stop lending above \bar{x}_{∞} does not survive the iterative elimination of strictly dominated strategies.

A similar argument applies to the smallest solution to $\pi(x, x) = 0$, and thus there exists \underline{x}_{∞} such that $\underline{x}_{\infty} = \inf\{x | \pi(x, x) = 0\}$. Any strategy that continues lending below \underline{x}_{∞} does not survive iterative elimination. However, if $\pi(x, x) = 0$ has a unique solution, then the largest solution \bar{x}_{∞} must coincide with the smallest solution \underline{x}_{∞} . Hence, there is only one strategy that still remains after eliminating all strictly dominated strategies, implying that this switching strategy is the unique equilibrium strategy. ■

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