PG Lab ディスカッションペーパーシリーズ
第47号　2010年1月

ISSN 1880-9146

Discussion Paper Series
No.47 January, 2010

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Kenta KONISHI, Etsuhiro NAKAMURA and Tadahiko MURATA

文部科学大臣認定　共同利用・共同研究拠点
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Policy Grid Computing Laboratory,
The Research Institute for Socionetwork Strategies,
Joint Usage / Research Center, MEXT, Japan
Kansai University
Suita, Osaka 564-8680, Japan
URL: https://www.pglab.kansai-u.ac.jp/
http://www.kansai-u.ac.jp/riss/
e-mail: pglab.@ml.kandai.jp
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Agent Simulation

for Observing Cooperation Among Politicians

Kenta KONISHI[1], Etsuhiro NAKAMURA[2,3] and Tadahiko MURATA[3,4,5]

Abstract

When does cooperation among politicians occur? How do policy and non-policy consideration by voters and candidates affect the cooperation among politicians? In this paper, we try to answer these questions using a simple simulation approach. In our analysis, we consider the number of parties a proxy of cooperation among politicians and see how it is affected by voters' and candidates' preferences for the size of parties. In their seminal paper, Chhibber and Kollman (1998) show how nationalization of political parties, party aggregation in their word, occurs. We also extend the analytical framework to multimember districts and see how institutions work.

This paper proceeds as follows. In the first section, we briefly review literatures concerning the effects of electoral institutions, voters and candidates on the number of national level parties. Next, we show our simulation model and settings. Third, we present our simulation result and how party aggregation occurs. Finally, we summarize our findings and show the implications for the future research.

Keywords: Agent simulation, party formation, candidates, voters, multimember districts

1 Graduate School of Informatics, Kansai University

2 Faculty of Law and Letters, Ehime University

3 Policy Grid Computing Laboratory, Kansai University

4 The Institute for Socionetwork Strategies, Kansai University

5 Faculty of Informatics, Kansai University

1 The Number of National Level Parties

How does party aggregation occurs? In this section, we briefly review previous researches and identify factors that affect the number of national level parties. The first one is the electoral institutions. By the mechanical and psychological effects, electoral institutions shape the party system at the district level. The second literature focuses on representatives themselves. Parliamentary politics and electoral cooperation encourage politicians to work together and the resulting number of parties becomes small. The third literature insists that it is because of the voters' preference for the size of parties. Because voters prefer larger parties which have access to the national level policies and are able to provide pork, parties become larger reflecting such voters' side demand. We review the literatures above in order in this section.

1.1 Electoral Institutions

The most traditional and important theory concerning the number of parties is Duverger's Law. Duverger (1954) argued that the two-party system is likely to emerge under the simple plurality rule because of the mechanical and psychological effects of the electoral institution. He also argued that the proportional representation promotes the multi party system.

Cox (1994, 1997) further extend Duverger's Law by applying the strategic voting explanation. According to his argument, the number of parties converges to the district magnitude (M) + 1 when there are vote-seeking parties and strategic voters. It is called the "M + 1 rule".

Empirically, it is well investigated and confirmed that the electoral institution is the most important factor determining the number of parties. The comprehensive empirical work by Taagepera and Shugart (1989) revealed that the number of parties is a function of the magnitude of districts. Benoit (2000, 2001) shows that the district magnitude affects the number of parties under the multi member district system and the proportional electoral system, using Hungarian local assembly election data.

Although some studies suggest that the party system including the number of parties is not solely determined by the electoral system, they also admit that the district magnitude is very important determinant of party systems. For example, Amorim-Neto and Cox (1997) explore the number of parties in terms of the district magnitude and the number of social cleavages. Their result shows that the electoral institutions and the social cleavages affect the party system interdependently.

However, it should be noted that Duverers' law predicts the number of district level candidates. On the other hand, political parties are coalitions of representatives or candidates across different districts. We next review the literature why politicians from different districts need to cooperate.

1.2 Parties as Coalitions of Legislators

In their seminal paper, Chhibber and Kollman (1998) present two reasons that encourage the cooperation among candidates across different districts. The first one concerns electoral politics and the second one is the parliamentary politics. Here, we briefly review them.

Electoral Incentives

Chhibber and Kollman pointed out that candidate need to associate themselves with national level parties in order to make clear their positions of national level issues. If voters mainly concern the local level issues, it is not useful for a candidate to join a national level party. However, if voters in a district also concern on the national level issues because of the growth of the central government activities for example, candidates will also try to affect national level policies by joining the national level parties.

In addition, there are several reasons for candidates to join a national level party, which we do not necessarily incorporate into our simulation. For example, if a candidate does not have any collaborators in congress, his policies will never be implemented. This also means that voters cannot believe the candidate's promise because it does not have any credibility (Osborne and Slivinski 1996, Levy 2004). In addition, candidates are able to share the cost of campaign by joining a larger party and use the party label (Aldrich 1995).

Parliamentary Politics

In addition to the direct electoral incentives, politicians need to cooperate in parliament in order to implement their policy. Typically the literature on logrolling suggests that politicians need to exchange votes, which means logrolling fosters coalitions of politicians (Carrubba and Volden 2000, Jackson and Moselle 2002). In order to have the majority of the parliament, politicians need to form a coalition with other politicians. Although logrolling might be an extreme example, it is apparent that the legislators want to join a larger party to have an access to policy making process if other conditions are equal (Desposato and Scheiner 2008). In other words, by trading their vote, they can achieve their policies in the expense of their ideological purity.

In sum, basically candidates have incentives to join larger parties and have the preference on the size of a party. However, there is a tradeoff between the ideological purity and power.

1.3 Voters' Preference for the Size of a Party

In this section, we consider another factor of nationalization of parties, voters. There are at least three explanations why voters prefer for a larger party in election.

Two electoral advantages of larger party candidates, national level policies and credibility which we have explained above, are effective when voters actually favor larger party candidates. As Chhibber and

Kollman emphasize, if national level policy has very important meaning and there are enough resources in central government, then voters favor larger parties that will have stronger impact on national level policies.

The third explanation is about clientalism. Clientalism is an extreme example on the preference of the size of parties. A definition of clientalism is “the proffering of material goods in return for electoral support, where the criterion of distribution that the patron uses is simply: did you support me?” (Stokes 2007). According to this idea, voters prefer a large party which has strong influences on public policies. Note that this is not the psychological effect of the Duverger’s law at the district level. Rather, the literature mentions the merit of a powerful party which is a coalition of politicians from different districts.

In sum, we can assume voters basically prefer larger parties, especially if the central government has enough ability to distribute material interests. Of course, programmatic appeals of candidates and parties are important. However, if their policy will not be implemented, the attractiveness is limited. Kitschelt (2000) provides a useful picture on the problem. Kitschelt insists that voter-party relationship is not based only on the programmatic linkage but also material clientalistic linkage and charismatic linkage. Both theoretically and empirically, it is impossible to invest several linkages simultaneously. Especially it is difficult for a party to have both clientalistic linkage and programmatic linkage.

Thus, from previous research, we assume that voters prefer a larger party but they also consider the programmatic position of the party.

1.4 Summary

We summarize our review here. The number of parties at the district level is basically restricted by electoral institutions. On the other hand, political party is a coalition of candidates across districts. We reviewed literature on the formation of the coalitions among politicians and present several conditions which foster the cooperation of politicians across different districts. We have also explained how voters evaluate the size of parties and when they prefer larger parties.

While there are several reasons that voters and candidates prefer larger parties, we still don’t know how these factors affect the party system. How are voters side demand and candidate side preferences interconnected? How do they encourage the cooperation among candidates? How do electoral institutions work on the relationships among them? When does candidates try to join a larger party? In order to investigate the relationship, we make use of the simulation approach.

2. A Simple Simulation Model of Party Formation

In this section, we show our simulation model and explain the procedure. We first introduce the basic

framework of our simulation. Then we explain the coalition formation algorithm used in our paper. Finally we show more specific details and parameter settings.

2.1 Basic Framework

In this subsection, we show the basic framework of our simulation. There are two approaches on the coalition formation models. We first explain the backward induction approach, which is mainly used for the coalition government formation. Then we explain the natural selection approach we use in this paper.

Backward Induction Approach

Backward induction approach, based on the game theory, is used for models of coalition government formation literature (Austin-Smith and Banks 1988). It divides government formation process into several stages and parties and voters try to find their optimal strategy considering the final outcomes. This is a very consistent approach assuming the rationality of agents. However, it is difficult to implement if the number of agents becomes larger. In simulation approach, Quinn and Martin (2002) used the simulation model of this approach for analyzing a coalition government formation. Laver and Benoit (2003) use a similar approach for the analysis of party switching behavior of legislators.

Natural Selection Approach

In this paper, we use natural selection approach. In this approach, we don't assume rationality of each agent but assume agents who fail to adapt to an environment will disappear as a result of competition. Although this approach is not theoretically well grounded, it is useful to analyze the evolution of agents and behavior of the system when number of agents is large. Schreiber (2001) use a similar approach. The summary of two approaches is compared in Fig 1.

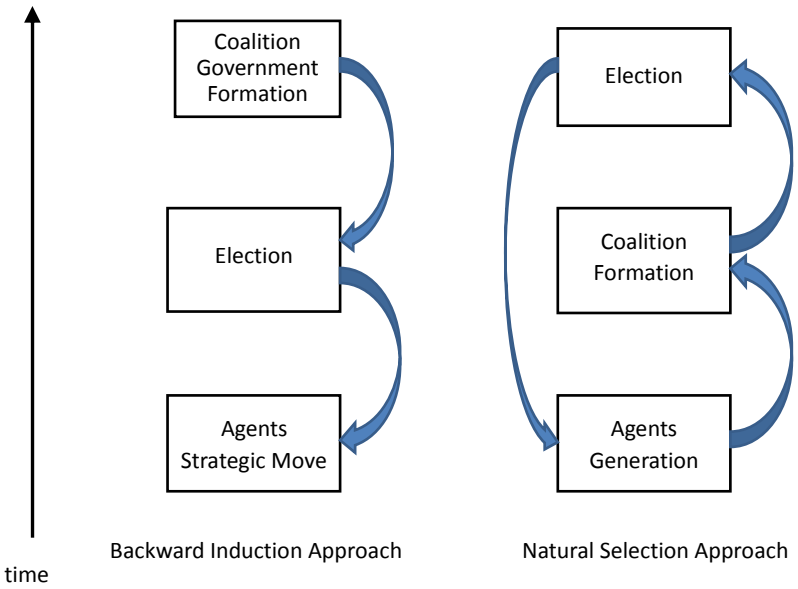

Figure 1. Comparison of Two Approaches

In the backward induction approach of the coalitional government formation, agents first calculate the coalition outcome given the expected electoral result and then maximize their votes given their strategic positioning and maximize the utility. However, in the natural selection approach, agents are first generated and make coalitions. Then they experience the election and some survive and the others die as the result of the electoral competition. Then new candidates are generated and repeat the process many times. Through these competitions, the electoral environment selects successful candidates.

2.2 Coalition Formation

As explained, we use the natural selection approach. However, still the procedure is vague. Especially, the coalition or party formation is the important element in our approach. Here, we review the literature and explain the specific procedure that we use in this paper.

Party formation is a special case of the coalition formation problem. Recently, variety of coalition formation models has been developed in game theory (Ray 2007, Humphreys 2008). Mainly these models are based on non-cooperative game theory.

Non cooperative game theory is so flexible as to include institutional structures and other factors during the negotiation. Therefore, some scholars have already applied the model specifically to political science context (Morelli 2004, Osborne and Tourky 2008.).

In this paper, we instead use cooperative game theory to analyze the party formation. Coalition made by cooperative game theory is stable in some senses. Therefore, it is straightforward to justify the final results. However, the number of possible coalitions is so numerous that we need to rely on the simulation approach. .

2.2.1 Definitions

Consider $N = \{1,...,n\}$ agents form coalitions. Each coalition is denoted by S_k and the set of coalitions is also denoted by $\Pi = \{S_1,...,S_K\}$. Here S_k satisfies $S_k \cap S_j = \phi$ and $\cup_{k=1}^{K} S_k = N$.
Utility of voter v_i for a candidate *i*

$$u_{v_i}(S) = |S| - a_{v_i}^2 \sum_{j \in S} (P_{v_i} - P_j)^2 = \sum_{j \in S} \{1 - a_{v_i}^2 (P_{v_i} - P_j)^2, \quad a \geq 0.$$

Here P_{v_i} represents a bliss point of voter v_i, P_j represents bliss point of candidate *i*, |*S*| is a size of coalition which candidate *i* belongs to, a_{v_i} is a parameter which determines the weight between the policy elements and the size element. There, voters evaluate candidates in terms of their policy position and the size of the party she belongs to.

We assume candidate utility function as follows.

$$u_i(S) = B_i(S) - C_i(S)$$

where $B_i(S)$ represents a utility for *S*, $C_i(S)$ is a cost of joining *S*.

Now $B_i(S)$ is just an increasing function of the size of a party. In order to simplify the model, we assume $B_i(S) = |S| - 1$. On the other hand, we assume there is a psychological cost if they cooperate with ideologically distant candidates. Therefore, $C_i(S) = \sum_{j \in S} (P_i - P_j)^2$, $j \neq i$.
Therefore,

$$u_i(S) = |S| - 1 - a_i^2 \sum_{j \in S, j \neq i} (P_i - P_j)^2 = \sum_{j \in S, j \neq i} \{1 - a_i^2 (P_i - P_j)^2\}.$$

Note that a smallest coalition, singleton, is $u_i(i) = 0$.

2.2.2 Coalition Formation Models

The setting above is called NTU game. If it is the game of the coalitional government formation, it is possible to justify the assumption of transferable utility. However, because this is a coalition of candidates, it might be difficult to justify the transferability of utilities across agents.

2.2.3 D-hp Stable set

In this paper, we use the concept of D-hp stable set proposed by Apt and Witzel (2009). D-hp stable set can be obtained through the finite iteration of the following operation called Merge and Split.

Assume for a subset of agent $A \subset N$. There exists coalitions $S = \{S_1, ..., S_l\}$ and $R = \{R_1, ..., R_n\}$. Relational operator $>$ is defined for the S and R as $S > R \Leftrightarrow u_i(S) \geq u_i(R)$, $\forall i \in A$ and at least one strict inequality hold.

If Π^* satisfies the following two conditions, we call it D-hp Stable set.
For one or several coalition $S^* = \{S_1^*, ..., S_l^*\} \subseteq \Pi^*$.
(1) For any partitions of S_i^*, $\{R_1, ..., R_m\}$ and $\cup_{k=1}^m R_k = S_k^*$, $\{R_1, ..., R_m\} \rhd_{not} S_i^*$
(2) For any $L \subseteq 1, ...l$, $S^* = \{S_1^*, ..., S_l^*\}$, $\cup_{j \in L} S_j^* \rhd_{not} \{S_j^* \mid j \in L\}$.

That is, D-hp Stable set is stable against the merge and division of a set. In our context, this means that it is stable against the merge and dissolution of parties and factions within a party.

Apt and Witzel（2009）shows D-hp Stable set is obtained by the finite iteration of the following merge and split.

Merge and Split Algorithm (Apt and Witzel 2009)
(1) For any $L \subseteq 1, ...l$ and $S^* = \{S_1^*, ..., S_l^*\}$, if there exists $j \in L$ such that $\cup_{j \in L} S_j^* \rhd_{not} \{S_j^* \mid j \in L\}$, Then $\{S_j^* \mid j \in L\} \rightarrow \cup_{j \in L} S_j^*$.
(2) For any partitions of S_i^*, $\{R_1, ..., R_m\}$ and $\cup_{k=1}^m R_k = S_k^*$, if there exists $\{R_1, ..., R_m\}$ such that $\{R_1, ..., R_m\} > S_i^*$, $S_i^* \rightarrow \{R_1, ..., R_m\}$.

The final result is stable against any merge and split operation. In addition, D-hp stable set always exists and can be found through the iteration. However, one problem is that the resulting D-hp stable

coalition is not unique. Rather, it is common that there are several D-hp Stable coalition profiles. In order to mitigate the problem associated with this multiple solutions, we run several trials and average the results.

2.3 Parameter Settings and Simulation Procedure

The detailed procedure of our simulation is as follows. Our policy dimension is two. The number of seats is 48. The number of voters is 4800. The number of seats per district is from 1 to 4 depending on the electoral rule. We ran the simulations separately in order to see the effect of institutions. The number of candidates at each district is the number of seats + 1, assuming district level Duverger's law perfectly works.

Step 1. Generate Districts, Voters and Candidates

We first generate districts' mean from standard bivariate normal distribution. Given the mean, voters are generated from the normal distribution whose variance is one. A thousand voters are assigned per a seat. That is, in multimember district setting, 2 to 4 thousand voters are in one district. Each candidate has her ideal policy position, alpha and a tag of the district number.

Step2. Make Coalitions

Candidates form coalitions according to the Merge and Split algorithm. Candidates from the same district will not join the same coalition making use of the tags they have.

Step3 Election

Voters vote for a candidate who gives highest utility.

Step4. Generate New Candidates

Unseated candidates disappear. New candidates are generated.

Step 5. Repeat step 2 to step 4

We carry out two simulations. In the first simulation, all parameters are fixed through the iteration. In the second simulation, candidates' alphas are randomly generated in order to see the result of electoral competition.

Parameter Setting in Simulation 1

We use candidates' alpha = 0.7 and 2, voters' alpha = 0.7 and 2 and district magnitude 1 to 4. Therefore, total 16 patterns of simulations are performed.

Parameter Setting in Simulation 2

We randomly generate candidates' alphas from Uniform distributions. We first generate alphas from the distribution U[0,0.7] , then we try U[0,2]. Therefore, total 16 patterns of simulations are performed.

2.4 Summary

Voters prefer larger parties but give them penalties if ideological positions of some members are far from them. In addition, each party has its valence, which is obtained as a result of candidates' negotiation. Therefore, this simulation model can be considered as an endogenous valence competition model with the penalty of ideological diversity.

3. Result of the Simulations

In this section, we show the result of our simulations. We first show the result when candidates' alpha and voters' alpha are fixed. Second, we randomly generate candidates' alphas to see what type of candidates survive through elections.

We ran 20 trials changing initial conditions. We repeat 5 elections in every trial and report the average number of parties of the final electoral results of the 20 trials. The number of elections is not large enough due to the short of time and computational power. Therefore, this result is not perfect at all. However, still we can see how our simulation works.

3.1 Result when candidates' alpha is fixed

Table 1 shows the result when candidates' alpha is fixed. As seen there, the number of parties is so large that we cannot consider it reflects real political situations. However, it can be seen as a proxy of the degree of the collaboration among candidates.

Table 1 Average number of parties

# of seat 1		candidates alpha	
voters alpha		0.7	2.0
	0.7	23.7	44.5
	2.0	26.1	50.9
# of seats 2		candidates alpha	
voters alpha		0.7	2.0
	0.7	15.3	28.2
	2.0	18.5	33.6
# of seats 3		candidates alpha	
voters alpha		0.7	2.0
	0.7	13.6	24.3
	2.0	16.7	27.1
# of seats 4		candidates alpha	
voters alpha		0.7	2.0
	0.7	14.6	23.4
	2.0	16.5	24.1

Table 2 Average number of candidates

# of seat 1		candidates alpha	
voters alpha		0.7	2.0
	0.7	4.1	2.2
	2.0	3.7	1.9
# of seats 2		candidates alpha	
voters alpha		0.7	2.0
	0.7	4.7	2.5
	2.0	3.9	2.1
# of seats 3		candidates alpha	
voters alpha		0.7	2.0
	0.7	5.3	3.0
	2.0	4.3	2.7
# of seats 4		candidates alpha	
voters alpha		0.7	2.0
	0.7	4.1	2.6
	2.0	3.6	2.5

As expected, candidates' alpha has stronger effects on the number of parties. It is natural because candidates themselves form parties. Even though voters have preference on the size of parties, voters cannot control it because all candidates share the same alpha. Nonetheless, voters' alpha also affects the number of parties through elections although the effects are very limited.

In this simulation, we cannot distinguish the effects of the diversity of voters' preferences across districts and electoral institutions. In addition, the total number of candidates is $(48/M)*(M+1)$ where M is the number of seats in a district. Therefore, depending on the seats per a district, the number of candidates varies. It is another deficit in our simulation.

Because of these problems, it is difficult to mention the effects of electoral institutions precisely. However, as shown in Table 2, the average number of candidates in a party is slightly larger in multimember district settings. A hypothetical explanation of the phenomena is that, because of the higher electoral threshold in SMD, candidates need to precisely reflect the district mean and have larger penalties if they collaborate with candidates from different districts in SMD case. On the other hand, such strong electoral pressure does not exist in multimember district. Therefore, under the multimember district environment, it is easier for candidates to cooperate with other candidates from different districts.

3.2 Result when candidates' alpha is random

Table 3 shows the result when candidates' alphas are randomly generated from uniform distribution. Table 4 also shows the average of alphas of the elected candidates.

Table 3　Average number of parties

# of seat 1		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	15.8	34.4
	2.0	17.2	38.2
# of seats 2		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	13.0	20.3
	2.0	14.5	25.1
# of seats 3		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	13.1	18.0
	2.0	13.5	21.6
# of seats 4		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	13.2	18.1
	2.0	13.8	20.7

Table 4　Average candidate alphas

# of seat 1		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	0.37	1.10
	2.0	0.38	1.14
# of seats 2		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	0.39	1.12
	2.0	0.39	1.16
# of seats 3		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	0.39	1.07
	2.0	0.39	1.14
# of seats 4		candidates alpha	
voters alpha		U[0,0.7]	U[0,2]
	0.7	0.38	1.01
	2.0	0.40	1.11

As Table 3 suggests, basic feature of the model are not so different from Table 1. It is partly because the number of elections is small. However, still we can admit that the effect of the candidate alpha is stronger. At least, we can insist that voters cannot easily control the candidate alphas through election.

As shown in Table 4, the average values of candidate alphas are close to their means of the probability distributions. All of them are slightly larger than their theoretical means, even when voters alpha is 0.7 and candidate alphas are generated from U[0,2]. This means that generally speaking idealistic candidates are likely to survive slightly more. However, still we need to investigate the movements of alpha more in order give a clear conclusion.

3.3 Summary

Here, we summarize our results. Fist, both voters' and candidates' preferences for the size of parties have effects on the number of parties. Especially, candidates' preference has stronger influence on the number of parties. When candidate alpha is fixed, voters do not have a method to prevent candidates' collaboration because all candidates share the same alpha.

This holds true when candidates' alphas are random. Basically the same feature remains. Concerning candidates' alpha, the average alpha of the elected candidates is close to the mean of the probability distribution although elected candidates are a bit more policy-oriented.

4. Conclusion

When does cooperation among politicians occur? Although our simulation is very limited, we first confirmed that, when candidates and voters prefer the larger party, candidates cooperate to form a party. In addition, we showed that it is easier for candidates to cooperate with others in multimember district. Then when do candidates try to cooperate? We tried to find the connection between voters' and candidates' preference for large parties through electoral competition. However, we could not find any connection between them still.

The voters' and candidates' preferences for the size of parties and their effects on party formation have very important implications in politics. Reform on electoral institutions has been attempted in many countries in order to prevent clientalism and encourage policy-based electoral competition. In addition, ordinary citizens naively believe that candidates will be more policy oriented if citizens themselves get more interested in policy.

However, these may be just illusions. So far, our simulation suggests that candidates' alphas largely depend on the probability distribution that they generate. In political science context, the pool of candidates is more important determinants of the cooperation among politicians. Yet our simulation is very limited and more comprehensive examination is needed in order to give a concrete conclusion.

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