

Can Less Informed Voters Control Public Policies?

Yusuke CHAMOTO, Etsuhiro NAKAMURA, Tadahiko MURATA



文部科学省私立大学社会連携研究推進拠点
関西大学政策グリッドコンピューティング実験センター

Policy Grid Computing Laboratory,
Kansai University
Suita, Osaka 564-8680 Japan
URL : <https://www.pglab.kansai-u.ac.jp/>
e-mail : pglab@jm.kansai-u.ac.jp
tel. 06-6368-1228
fax. 06-6330-3304

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Can Less Informed Voters Control Public Policies?

Yusuke CHAMOTO¹

Etsuhiro NAKAMURA^{2,4}

Tadahiko MURATA^{3,4}

Abstract

Can Voters Collectively Control Public Policies? According to the Downsian Median Voter Theorem, voters can control two competing parties in order to achieve an optimal policy outcome. However, compared with real party politics, there are several restrictive conditions in the classical Downsian spatial model and in this paper, we extend the model to incorporate the effects of party bargaining and investigate whether voters can control policy outcomes under these conditions.

The paper is organized as follows. In the first section, we review several models where voters select the party for whom to vote based on the distance between their ideal position and the final policy outcome. In the next section, we explain our model in terms of outcomes which are public policies, and voters' learning of how to control these outcomes by observing the behavior of the policies. In the third section, the results of simulation experiments are reported, and in the last section we summarize our findings and mention several extensions that we intend to perform in the future.

Keywords: Two-party competition, Multiagent-simulation, Policy-oriented voting, Voters' learning

¹ Graduate School of Informatics, Kansai University

² Faculty of Law and Letters, Ehime University

³ Faculty of Informatics, Kansai University

⁴ Policy Grid Computing Laboratory, Kansai University

1 Public Policy Control by Collective Voters

How do electorates collectively control public policies? In this section, we review two relatively distinct bodies of literature. First, theories of “institutional balancing” and “modifying institutions” according to which voters make their choices based on the final policy outcome are reviewed, followed by models of voting behavior when voters do not have perfect information. We then conclude this section by indicating the deficiencies in both bodies of literature and explain a possible extension which is developed in the next section.

1-a The Downsian Spatial Model and its Extension

The classical Downsian spatial model (Downs 1957) assumes that the policy of the winning party is implemented intact. Voters can therefore vote for the party closest to their ideals in policy space. As a result, two competing parties will converge to the ideal position of the median voter. The median can be considered optimal in the sense that no other possible position can beat it in terms of gaining the majoritarian vote.

If the mean and the median of the voters’ distribution are same (*i.e.*, the distribution is normal) then the policy position of the median voter is the point at which the sum of the squared loss of voters’ utility is minimal. This means that voters can be interpreted as collectively controlling competing parties in order to achieve a moderate and optimal policy outcome.

However, in the real policy selection process, the policy of the winning party is not always realized. Under the presidential system in the United States for example, neither the president nor congress can dominate the policy making process entirely. The effects of such “separation of power” structures on policy outcome must therefore be considered.

Besides presidential systems, in countries where coalition governments frequently appear, the policies of the largest party are not always realized. In addition, a parliament with strong viscosity does not always allow the governing party to implement its policies (Blondel

1973). For example, in the Japanese Diet, opposition parties have a strong influence on the law making process by making use of parliamentary institutions and customs (Mochizuki 1982). The dominant party, the Liberal Democratic Party (LDP), cannot change these institutions and customs while avoiding public criticism asserting that it repressively dominates all policy making processes using the unrestrained force of its majority status.

1-b Spatial Models and the Effects of Policy Making Structures on Voters' Decisions

Considering policy making structures such as those described above, the Downsian spatial model is much simpler than the real policy making process. Can voters simply achieve the optimal outcome just by voting for the spatially closest party?

Such outcome oriented voting has been investigated theoretically and empirically. It has been tackled theoretically by Alesina and Rosenthal (1995), Austin-Smith and Banks (1988), Quinn and Martin (2002), Baron and Diermeier (2001), and Merrill and Adams (2007).

Alesina and Rosenthal's analysis of midterm elections (Alesina and Rosenthal 1995) incorporates the structure of power separation. In their model, they assume voters know that policy outcomes would be pulled in a certain direction because of the effect of presidency. Thus, moderate voters might vote for relatively distant candidates in order to influence the final policy outcome.

Other scholars have worked on models of coalition formation and parliamentary elections. In their models, voters first expect a post-electoral coalition and final policy outcome. Then parties expect such voters' strategic behavior and select their electoral platform in order to maximize their expected number of votes and the possibility of joining the coalition. The structure and the strategic calculation become so complex that intensive computer simulation are sometimes used to find the solution (Quinn and Martin 2002).

In addition to theoretical work, empirical studies have reported voters' behaviors regarding policy outcomes. Originally, some scholars reported that voters do not simply vote for the closest party. In other words, voters do not simply use proximity but rather use

a directional criterion (Rabinowitz and Macdonald 1989) or a combination of proximity and direction (Iversen 1994). However, these models assume that the source of directionality is psychological, *i.e.*, from a sense of friend or foe or from assertive leadership. In that sense, voters do not care about the final policy outcome.

Recently, several scholars have reinterpreted the directional model (Kedar 2005a 2005b). They interpret voters' use of directional criteria as an effort to control policy outcomes. Kedar (2005a 2005b), using multi-country opinion poll data, demonstrated that if the institutions in the policy making process are "moderating", then it encourages voters to use the directional criterion.

Similar phenomena have been observed in Japan. It is widely known that there were many "buffer players" in Japan under the one-party dominated era. "Buffer players" are defined as "voters who want the LDP to be the governing party, but also want the margin in the share of seats to be small" (Kabashima 2004. p75). In terms of their ideal policy positions, "buffer players" are closer to the LDP and should vote for this party if they use proximity. However, they also consider the fact that if the LDP wins by a large margin, then its more right-wing policies would be enacted without modification. Thus, "buffer players" try to control policy outcomes by voting for the opposition but expect that they will moderate the policy outcomes by making use of the viscosity in the Japanese Diet.

In summary, voters do not select parties solely on the basis of the proximity of their policies, but by using different criteria related to the moderating political institutions.

1-c Models of Voter Behavior in an Environment with Limited Information

As reviewed above, recent theoretical and empirical studies have shown that voters try to control policy outcomes using their votes with respect to the institutional environment. However, these studies also require strict assumptions according to which voters know the policy positions of each party and also know how policy outcomes emerge. Spatial models have been therefore received the criticism that the conditions on the amount of information possessed by voters are severe; voters usually do not have perfect information on the

locations of parties in policy space.

In this section, we review two models in which voters' informational requirements are relaxed. The most famous model of this type is Fiorina's retrospective voting model (Fiorina 1981).

Fiorina assumes that voters do not directly observe parties' policy positions. As a consequence they cannot use the policy proximity criterion. Instead, he insists that voters can observe policy outcomes and judge whether the incumbent party is sufficiently able to manage the economy, and claims that although voters are not perfectly rational, they can make reasonable decisions.

Fiorina's model became popular because of the plausibility of its assumptions and good empirical fit. However, it is not strictly based on the spatial framework. It therefore cannot be understood what kind of public policies appear in terms of a spatial context.

The online voting model is another in which informational requirements are relaxed. This model assumes that voters do not remember all information about political parties. Rather, they only retain *evaluations* of the parties and attempt to update these evaluations using the political information they receive, quickly discarding the information after updating. Voters do not use political information itself in elections, instead using only the evaluations as cumulative approximations of political information.

This online voting model also relaxes the assumption on the possession of information. In a strict sense however, this online voting model is not based on the spatial context either. Again, it cannot be understood where policy outcomes appear in policy space.

Both of these models, although having very plausible assumptions, have two insufficiencies in common. Firstly, they cannot predict where parties and policy outcomes are located in policy space because they do not embody the spatial framework. Secondly, they are also incapable of predicting how voters vote or update their evaluations if policy outcomes are formed as a compromise among parties.

2 Computational Extension of Policy-Voting Models in a Limited-Information Environment

As discussed above, spatial models assume strongly rational voters. However, if the assumptions on information are relaxed, the spatial framework collapses and the collective ability of voters to control public policies cannot be understood.

Our aim in this paper is to investigate whether retrospective-type voters can collectively control public policies in an environment with limited information. A simulation model is constructed for this problem and investigated using computational experiments.

2-1 Flow of Simulation

The basic setting of our computational experiment is a two party competition where two parties try to maximize their own support, and the support provided by public opinion reflects the bargaining power of the two parties in policy marking.

In the first stage, the parties and voters are generated according to a bivariate normal distribution.

In the second stage, voters stochastically determine their first party to support. The probability that the i -th voter v_i supports party j p_j at time t is represented by $P_i^t(p_j)$.

In the third stage, the parties change their policy platforms according to an adaptive rule. In this study, it was assumed that parties try to maximize their vote and follow the Hunter algorithm proposed by Laver (2005). This algorithm is described below.

HUNTER: if the last policy move increased support, make the same move; else, reverse heading and make a unit move on a heading chosen randomly from the arc ± 90 from the direction now being faced (Fowler and Laver, forthcoming) .

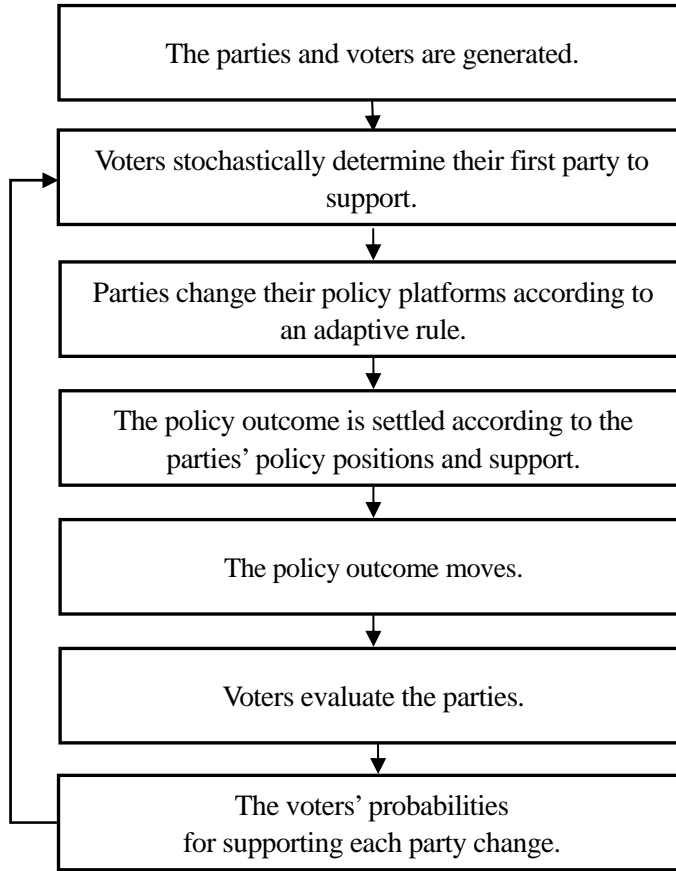


Figure 1 The flow of simulation

In the fourth stage, the policy outcome is settled according to the parties' policy positions and support. We define the party with the largest support as p_{1st} , its support rate as $r(p_{1st})$, and its policy position as $p_{1st}(x_1, y_1)$. For the party with the second largest support, these are likewise represented by p_{2nd} , $r(p_{2nd})$, and $p_{2nd}(x_2, y_2)$. The final policy outcome is determined as the weighted sum of the two party positions. That is, the final outcome o' is determined as follows:

$$\text{In case 1: } x_1 \leq x_2, y_1 \leq y_2 \quad o'(x_1 + |x_1 - x_2| \cdot r(p_{2nd}), y_1 + |y_1 - y_2| \cdot r(p_{2nd}))$$

$$\text{In case 2: } x_1 \leq x_2, y_1 \geq y_2 \quad o'(x_1 + |x_1 - x_2| \cdot r(p_{2nd}), y_2 + |y_1 - y_2| \cdot r(p_{1st}))$$

$$\text{In case 3: } x_1 \geq x_2, y_1 \leq y_2 \quad o'(x_2 + |x_1 - x_2| \cdot r(p_{1st}), y_1 + |y_1 - y_2| \cdot r(p_{2nd}))$$

In case 4: $x_1 \geq x_2, y_1 \geq y_2 \quad o' (x_2 + |x_1 - x_2| \cdot r(p_{1st}), y_2 + |y_1 - y_2| \cdot r(p_{1st}))$

In the fifth stage, policy outcome o_t moves to o'_t .

In the sixth process, voters evaluate the parties. The evaluation of parties is performed according to the Score Based on the Distance (hereafter SBD) and the Score Based on Learning (hereafter SBL). Each voter has SBD and SBL values for each party. The sum of SBD and SBL for each voter is set to one.

SBD is calculated from the distance between the voters and the parties. At time t , we denote the Euclidian distance between v_i and p_1 by $D_t(v_i, p_1)$ and that between v_i and p_2 by $D_t(v_i, p_2)$. Then $SBD_i^t(p_1)$ and $SBD_i^t(p_2)$ are calculated using the following two equations.

$$SBD_i^t(p_1) = \frac{D_t(v_i, p_2)}{D_t(v_i, p_1) + D_t(v_i, p_2)} \quad (1)$$

$$SBD_i^t(p_2) = \frac{D_t(v_i, p_1)}{D_t(v_i, p_1) + D_t(v_i, p_2)} \quad (2)$$

SBL is defined as follows. The distance between v_i and policy outcome o_t is denoted by $D_t(v_i, o_t)$. The difference between $D_t(v_i, o_t)$ and $D_{t-1}(v_i, o_{t-1})$ is

$$\Delta D_t = |D_t(v_i, o_t) - D_{t-1}(v_i, o_{t-1})|. \quad (3)$$

At time t , if v_i supports p_j and $D_t(v_i, o_t) < D_{t-1}(v_i, o_{t-1})$, then $SBL_i^t(P_j)$ increases according to equation (4), while the score of the other parties $SBL_i^t(P_{others})$ decreases according to equation (5).

$$SBL_i^t(P_j) = SBL_i^{t-1}(P_j) \cdot \left(1 + \frac{\Delta D}{D_{t-1}(V_i, O_{t-1})} \cdot rwd\right) \quad (4)$$

$$SBL_i^t(P_{others}) = SBL_i^{t-1}(P_{others}) \cdot \left(1 - \frac{\Delta D}{D_{t-1}(V_i, O_{t-1})} \cdot \frac{rwd}{n-1}\right) \quad (5)$$

n is the number of parties. In both equations, rwd is a nonnegative parameter tuning the speed of learning.

On the other hand, if at time t , v_i supports p_j and $D_t(v_i, o_t) > D_{t-1}(v_i, o_t)$, then $SBL_i^t(P_j)$ decreases according to equation (6) and the score for the other parties, $SBL_i^t(P_{others})$, increases according to equation (7).

$$SBL_i^t(P_j) = SBL_i^{t-1}(P_j) \cdot \left(1 + \frac{\Delta D}{D_{t-1}(V_i, O_{t-1})} \cdot rwd\right) \quad (6)$$

$$SBL_i^t(P_{others}) = SBL_i^{t-1}(P_{others}) \cdot \left(1 - \frac{\Delta D}{D_{t-1}(V_i, O_{t-1})} \cdot \frac{rwd}{n-1}\right) \quad (7)$$

In the seventh stage, the voters' probabilities for supporting each party change. The probability for supporting each party is determined by the evaluation of each party. The following three rules were used in the experiment.

Rule 1. Learning: $P_i^t(p_j) = SBL_i^t(p_j)$

Rule 2. Learning & Distance: $P_i^t(p_j) = \frac{SBL_i^t(p_j) + SBD_i^t(p_j)}{2}$

Rule 3. Distance: $P_i^t(p_j) = SBD_i^t(p_j)$

Stage two to stage seven are iterated a predetermined number of times. It is then examined whether the voters are able to control the policy outcomes, and where the parties' policies are located in policy space.

2-2 Political Interpretation of the Experimental Setting

In our experiment, three different rules describing voters' assessments of parties were employed. The "Distance" rule can be considered as a stochastic version of the proximity criterion in the spatial models. The "Learning" rule is similar to the retrospective voting algorithm and the online voting algorithm. The "Distance and Learning" rule is a combination of the first two rules.

The "Learning" rule is actually based on the notion of reinforcement learning. There are only a few applications of reinforcement learning in political science. Fowler (2006) uses the algorithm to explain electoral participation. As an algorithm, it is natural and flexible in describing the human leaning process.

In terms of political science, the algorithm we employ is similar to the retrospective voting and online voting models. The objective of our work is similar to that of Merrill and Adams (2007). In their work, political competition is simulated with directional voting and proximity voting, and with policy modification. They investigate whether voters' modes of party evaluation affect the final policy outcome.

However, there are two differences between Merrill and Adams' work and our own. The first difference, as described above, is that we use the notion of retrospective voting to represent voters' efforts to control the outcome while Merrill and Adams use directional voting. The second difference is that in Merrill and Adams' work, all voters know how party bargaining will be settled as a function of the seat share while in our work, voters do not have any knowledge regarding the bargaining process and merely observe the results and learn from them. Our model therefore assumes much less regarding the voters' rationality.

3 Experiments and Results

Experimental results are reported in this section. The basic settings of the experiments are as follows. The number of parties is two, and the number of voters is 1000. The initial

values of $SBL_i^0(p_j)$ and $SBD_i^0(p_j)$ are both 0.5, and the initial value of d is 0.01. There are 100 trials, and each trial has 100,000 steps. The value of rwd is 10. Voters are randomly generated in accordance with a bi-variate normal distribution with mean (0, 0) and standard deviation 1.0. Voters' policy positions do not change during the experiment.

The experiments are based on the behavioral rules of (a) Learning, (b) Learning & Distance, and (c) Distance. In addition, we assumed that (1) all voters always support one of the two parties, or (2) all voters have some probability of being independent, which is proportional to the distance between their ideal positions and the policy outcome. The latter case may be considered as representing the case that voters can abstain. The former case is denoted “no abstentions” and the latter case “abstentions possible”. Note that even in the latter case, voters only observe the policy outcome and do not observe the policy positions. Results are reported for each of the six combinations of these cases.

3-1 Movement of Policy Outcomes

First, the distance between the policy outcome and the mean of the ideal positions of all voters is reported. Figure 2 represents the “no abstentions” cases and Figure 3 represents the “abstentions possible” cases. These figures are based on the means taken over a hundred trials. In Figure 2, voters have little influence on the policy outcome towards the mean using the Learning rule. The convergence is the fastest under the Distance rule. The Learning & Distance rule is in between. The speed of convergence under the Learning & Distance rule is not as fast as under the Distance rule. However, in the final state, the distance is almost same as under the Distance rule.

Figure 3 shows the results for the “abstentions possible” cases. All three rules enable voters to influence the policy outcome to be close to the mean of the voters' ideals. The result of the Learning rule is particularly striking. While the policy outcome is far from the center in the case of “no abstentions”, if voters withhold their support according to the distance between their ideals and the policy outcome, they can allow parties to make moderate policies. This is because parties try to move to gain votes from voters who

abstain.

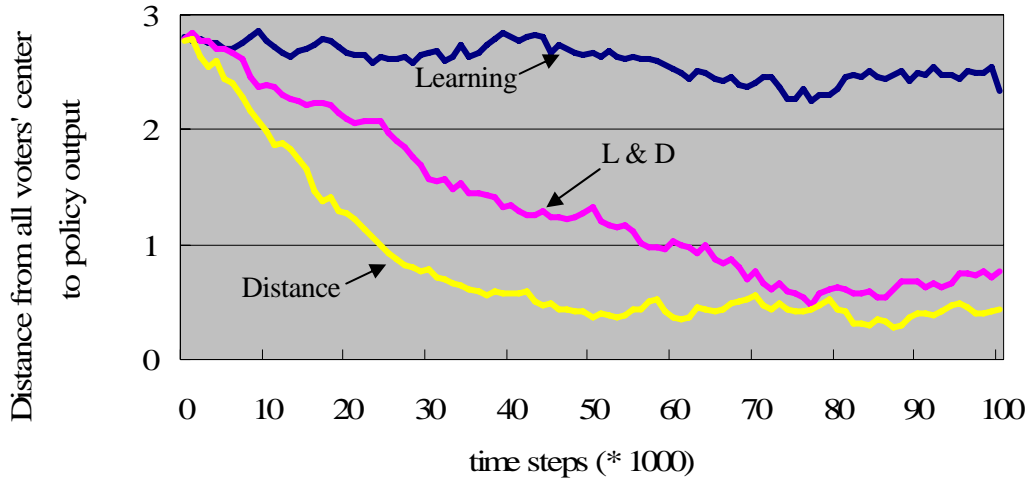


Figure 2 Transition of the distance from the all voters' center to the policy output (Each voter supports either party inevitably)

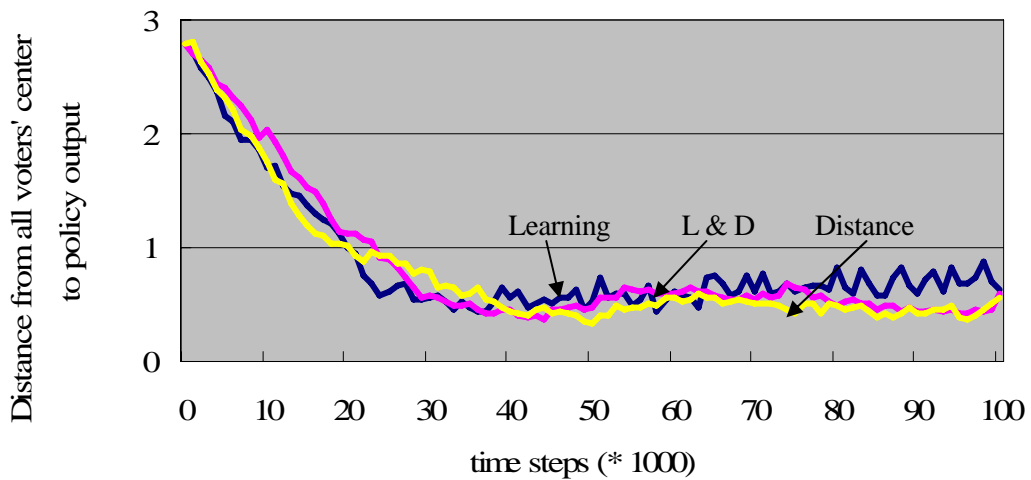


Figure 3 Transition of the distance from the all voters' center to the policy output (Each voter may abstain according to distance from her to the policy output.)

In this specific setting, it is expected, and can be theoretically calculated in advance that the Distance rule is the best algorithm, and we therefore use this algorithm as a benchmark. In fact, the results indicate that the performance of the Distance rule is the best – policy outcomes are closest under this rule. On the other hand, if abstention is introduced then the

Learning rule also shows excellent performance. This result has an interesting implication on the democratic theory of political competition and electoral participation. That is, if policy outcomes are far from the center, the implication of the models is that voters should abstain. In general there are two methods of expressing frustration with a democratic organization, denoted “voice” and “exit”. In electoral politics, “voice” has been treated as the only way to move the policy. Increasing participation has always been considered desirable.

However, in our experiment “exit” may be a desirable option when the focus is placed on the system level output. If voters do not have enough information, “voice” is not a good strategy. Because voters always support one of the two parties, the party receiving support can misinterpret this as indicating that its policy position is truly supported in the case that voters support it merely as a criticism of the alternative. On the other hand, if voters can abstain, both parties’ support may be simultaneously reduced and it is possible for the parties to understand that their policy positions were not supported. The parties may then try to maximize their share of the vote by moving their policy positions, thus enabling voters to control the public policies.

3-2 Party Policy Positions

Figure 4 shows the mean distance between the mean of the voters’ ideal positions and party positions in the case of no abstentions and Figure 5 shows the distance in the case that abstentions are possible. In both cases, the Learning rule takes both parties far from the center. The Distance rule takes both parties close to the center, and again, the Learning & Distance rule is in between the two extremes. It is not surprising that the two parties obeying the Distance rule and the Distance & Learning rule move to the center because voters are dense there

When voters use rules other than the Distance rule, the parties move off from the center. In our model, voters do not need to cement parties in the center because the policy outcome can be a moderate one even when the parties’ policy positions themselves are extreme.

Here we explain why this occurs. When one party moves away from the center, the policy outcome is pulled in its direction. Thus, the other party must also move off center because it is likely to increase its support among voters by moderating the outcome. The adaptive “Hunter” rule in this case makes both parties move off from the center. Thus, if the parties’ adaptive algorithm is slightly modified, this “moving off the center” might be prevented, even under the Learning rule.

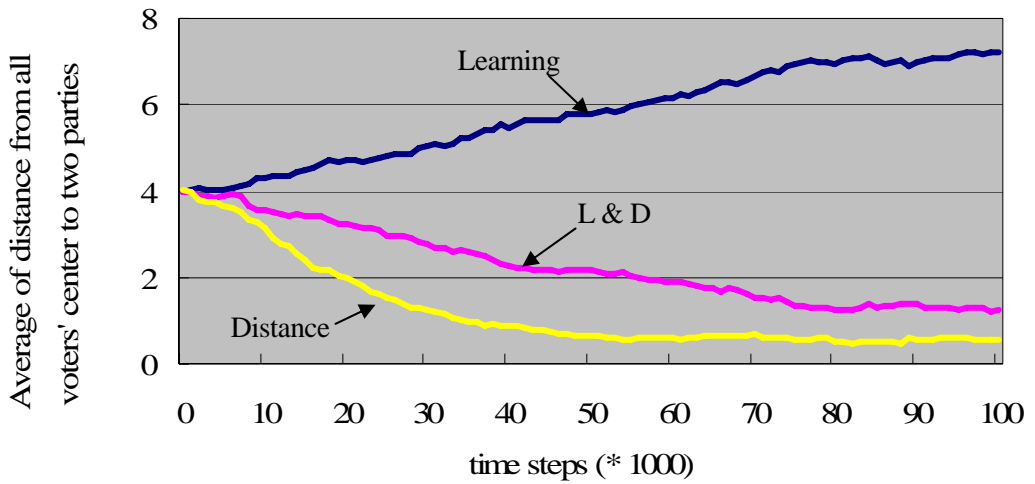


Figure 4 Transition of the average of the distance from the all voters’ center to two parties (Each voter supports either party inevitably)

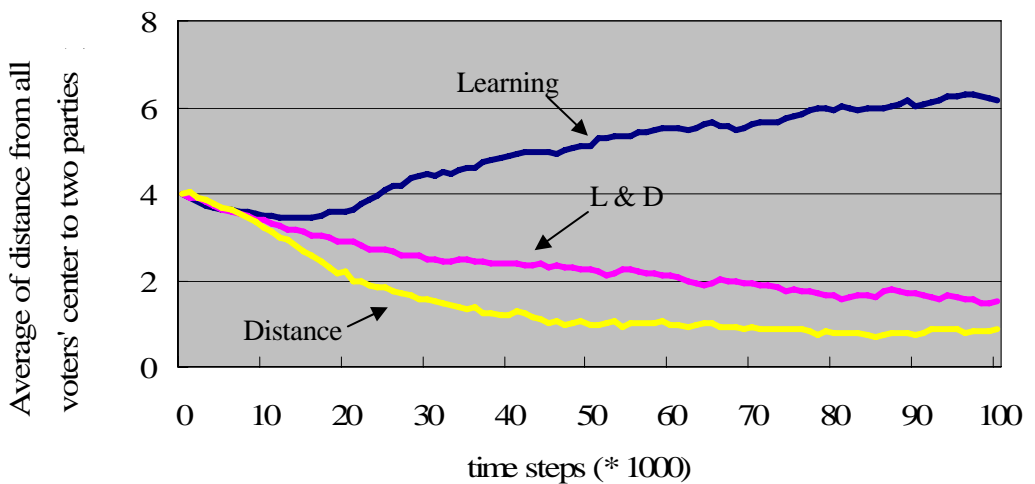


Figure 5 Transition of the average of the distance from the all voters’ center to two parties (Each voter may abstain according to distance from her to the policy output.)

3-3 Party Support

Lastly, Figure 6 shows who supports each party's policy under the Learning rule and the "abstentions possible" case. Snapshots were taken every 20,000 iterations. As may be seen, voters vote for the closer party. This feature appears at a relatively early stage of iteration.

This phenomenon prompts two comments. Firstly, voters may make reasonable choices if they learn the outcome based on their experiences. Even when they lack substantive knowledge regarding party policy positions and the policy outcome is a mixture of both parties' positions, the resulting choice made by voters is similar to that under the Distance rule. In the coalition making models reviewed above, highly rational voters are assumed. However, if the voters learn the relationship between their choices and policy outcomes, it is possible for them to act as if they knew the parties' policy positions.

Secondly, if the proximity model or directional model is fitted to this data, the estimation result will be satisfactory. Therefore, in the empirical studies of public opinion, if such data is available and the proximity model or directional model is applied to it, the estimation results will be good enough to demonstrate these models too. However, as shown in our experiments, voters may actually know nothing about the policy positions, and only have information regarding the policy outcome. That is, our learning model, the spatial model, and the proximity model are indistinguishable empirically. Voters with perfect information may vote for the closest party. However, even when voters vote for their closest party, they may not be rich in political information.

4 Conclusion

We now return to the original question: can less informed voters collectively control public policies? The answer to this question is mixed. However, it is more affirmative. Even when voters can observe a policy outcome which is formed as a combination of the policy positions of multiple parties, the voters can control the outcome if they are able to abstain. In other words, if voters have a method of expressing their frustration with the entire party

system, they can control the public policy.

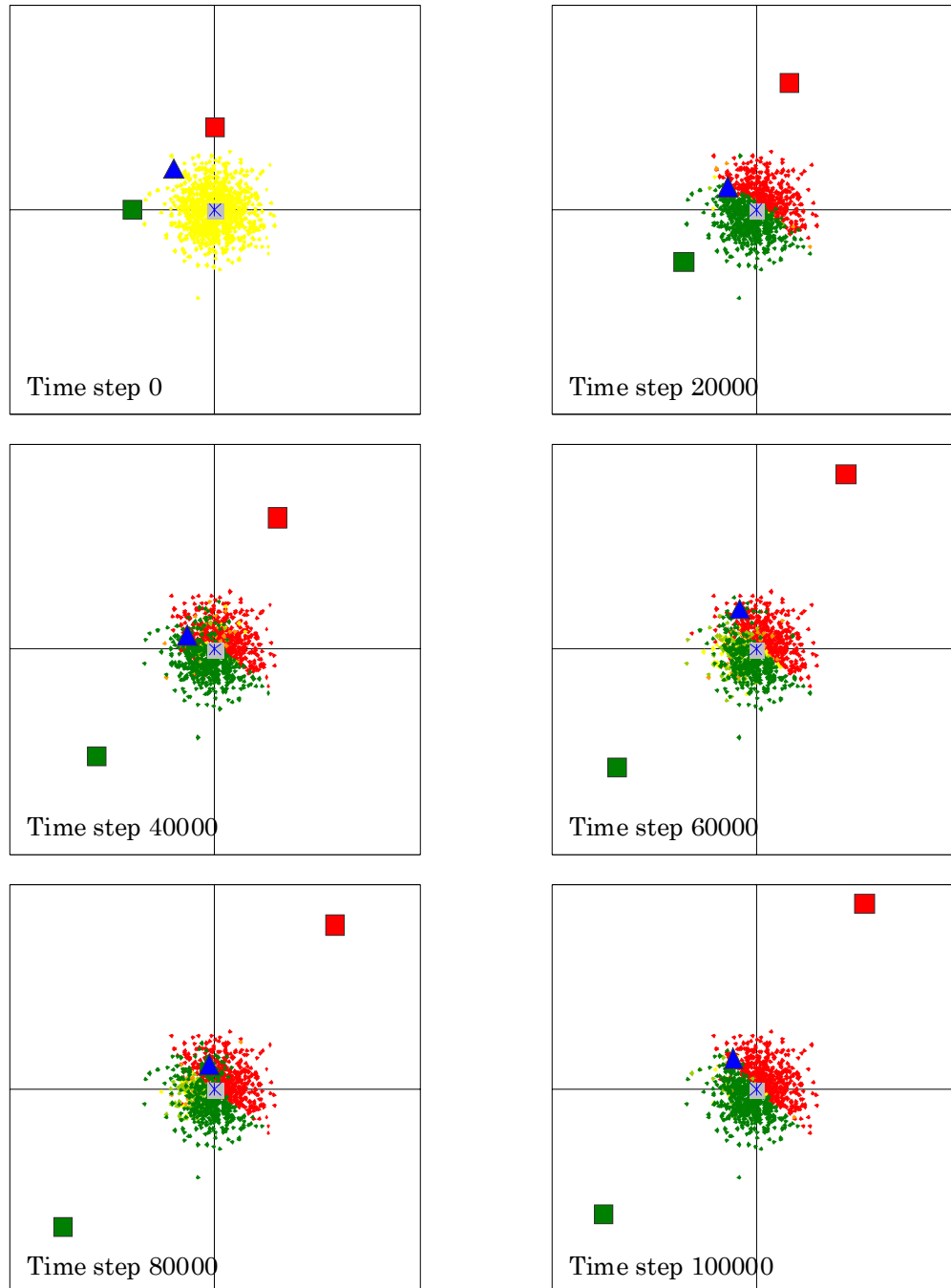
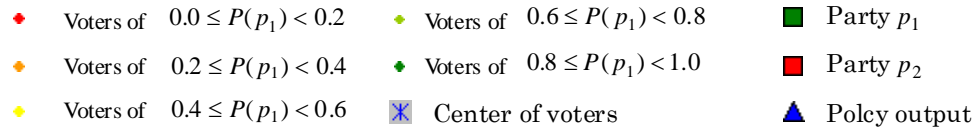


Figure 6 Transition of voters' support and policy output

Our experiments have demonstrated the powerful capabilities of adaptive voters. However, the model must be extended in order to assert the generality of the findings. In the experiments presented, the simplest situation in which only two parties are in competition was handled. The case of competition among three or more parties will be the subject of our next investigation.

Although the experimental setting in this paper was deliberately chosen to ensure that the number of parties for example, there is a possibility that the performance of the learning algorithm may exceed that of the others. In future work, we will investigate various settings in order to demonstrate the generality of reinforcement learning.

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